FORMULA SHEET

+1 Physics

Akshansh Chaudhary

PHYSICS FORMULAE

Chapter ELECTROSTATICS

(i) Coulomb's law : $F = \dfrac{q_1 q_2}{4\pi\epsilon_0 r^2}$

✓ $\epsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

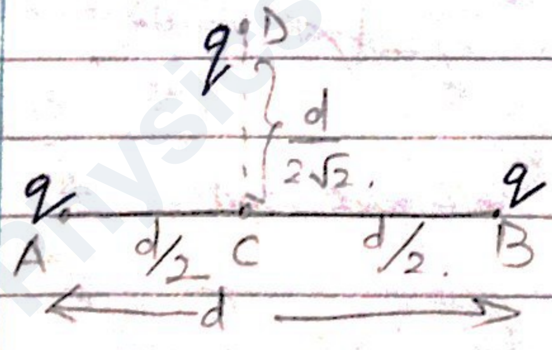
✓ $1C = 3 \times 10^9$ esu.

✓ Valid only for point charges

✓ Use POS to find net force in vector form

✓ $\dfrac{\epsilon_{medium}}{\epsilon_{0\,vacuum}} = K = \epsilon_r$ → relative permittivity → dielectric constt

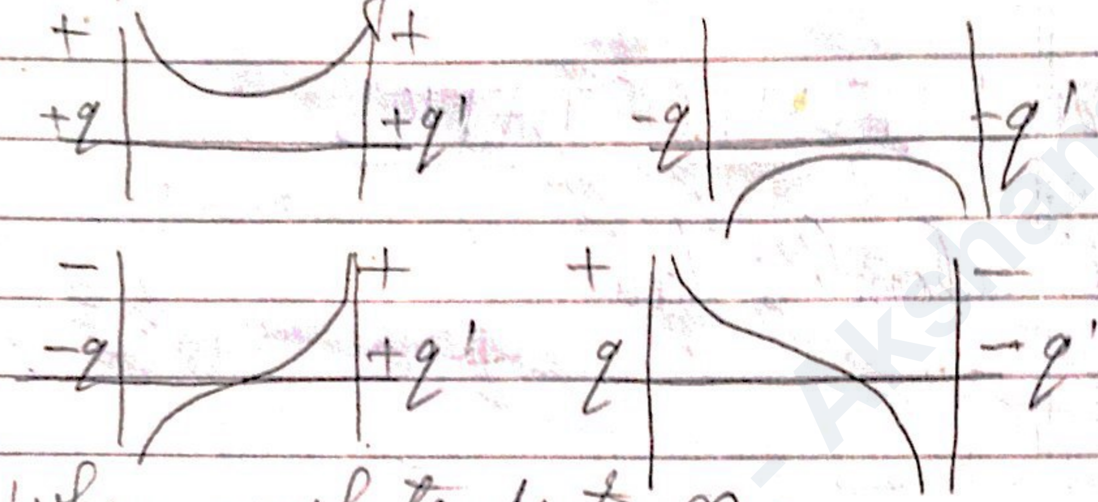
✓



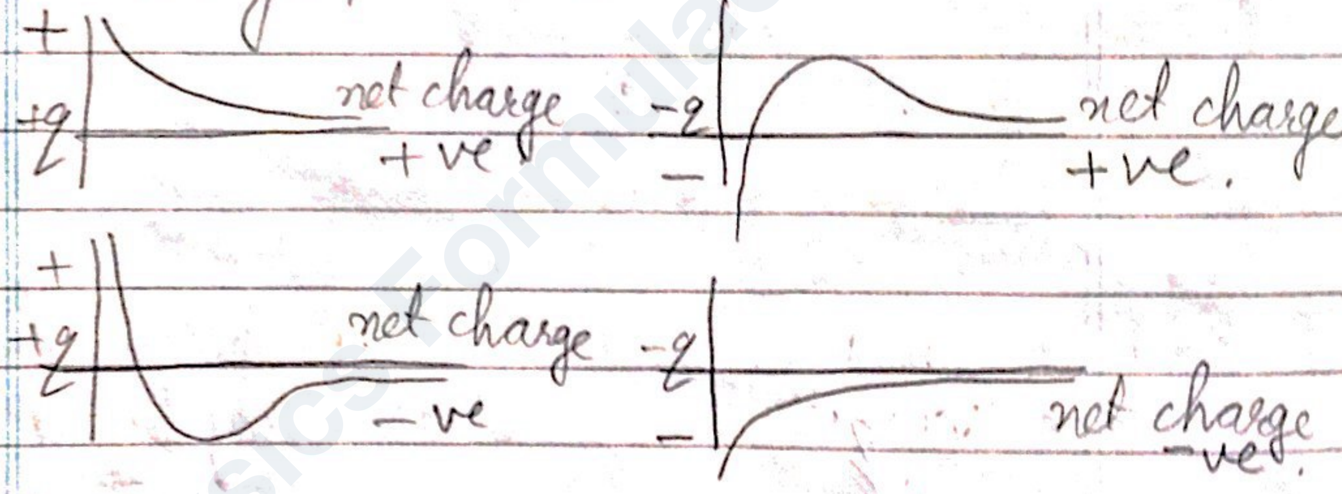
: If a particle of charge 'q' is placed on the ⊥ bisector of AB at a distance of $d/2\sqrt{2}$, it experiences max force from A & B. ($|\vec{F}|$ is independent of nature of charges at A & B)

✓ U vs x graph :–

- In b/w charges



- When graph tends to ∞.



✓ $F = -\dfrac{dU}{dx}$, for +ve charge

$= \dfrac{dU}{dx}$, for -ve charge (make F +ve)

- F = 0 when slope of tangent in U vs x graph = 0.

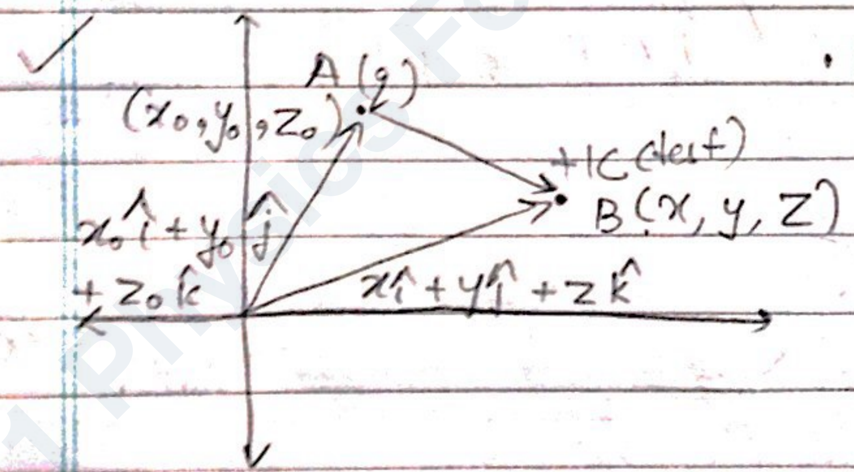
- Previous approach of U vs x graph is applicable iff net charge ≠ 0.
  If net charge = 0, asymptotes can't be defined.

(ii) Electric field :- $(\vec{E})$

✓ $\vec{E} = \lim_{q_0 \to 0} \left(\frac{\vec{F}_{on\, q_0}}{q_0} \right) \frac{N}{C}$.

✓ $\vec{E} = \frac{|q| \, \vec{r}}{4\pi\epsilon_0 |\vec{r}|^3}$, $|\vec{E}| = \frac{q}{4\pi\epsilon_0 r^2}$

✓



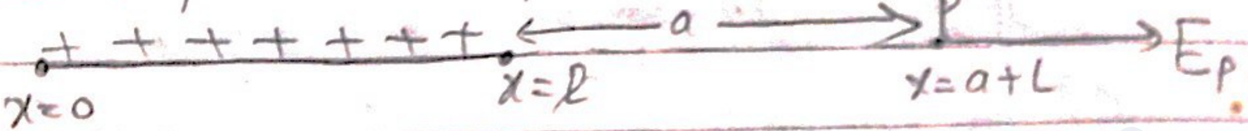
- Electric field vector at pt. B due to a charge q kept at A(x_0, y_0, z_0)

$$\vec{E}_B = \left(\frac{\pm q}{4\pi\epsilon_0} \right) \frac{(x - x_0)\hat{i} + (y - y_0)\hat{j} + (z - z_0)\hat{k}}{\left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \right]^{3/2}}$$

x posn of +1C test charge

x posn of charge q

Electric field at a point on a LCD.

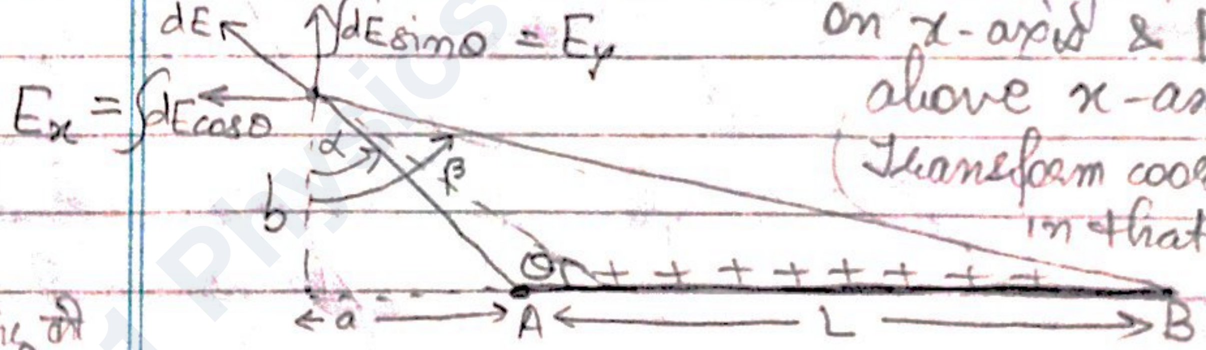


$$\vec{E}_P = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{a+l}\right]$$

shortest distance, largest distance

- Its a finite LCD.
- $\vec{E}_P$ is along the length of LCD
- In case of infinite LCD or SCD, $\vec{E}$ is not defined on the line/surface $\because$ $(\text{distance})^2$ b/w charges $\to 0$

E at any point in space due to a finite LCD (Valid only if LCD is on x-axis & pt. is above x-axis (Transform coordinates in that case))



line की वजह से x-dir^n में field

$$E_x = \frac{\lambda}{4\pi\epsilon_0 b}(\cos\alpha - \cos\beta)$$

$$E_y = \frac{\lambda}{4\pi\epsilon_0 b}(\sin\beta - \sin\alpha)$$

Put proper sign convention on α & β.

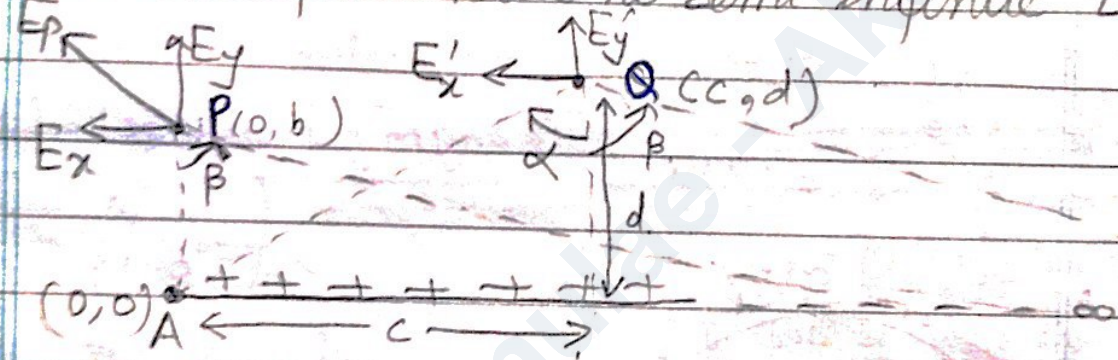
line की वजह से y-dir^n में field.

✓ Electric field due to an infinite LCD.

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$; r : $\perp$ distance of pt. from the line.

- It has cylindrical symmetry

✓ Electric field due to semi-infinite LCD.



- For P $\quad E_x = \frac{\lambda(1-0)}{4\pi\epsilon_0 . b}$, $E_y = \frac{\lambda(1-0)}{4\pi\epsilon_0 b}$

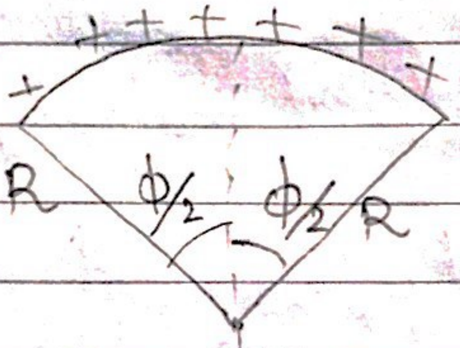
$$E_P = \frac{\lambda\sqrt{2}}{4\pi\epsilon_0 b}$$ 45°

- For Q $\quad E_x' = \frac{\lambda}{4\pi\epsilon_0 d}\left(\frac{d}{\sqrt{c^2+d^2}} - 0\right) = \frac{\lambda}{4\pi\epsilon_0\sqrt{c^2+d^2}}$,

$$E_y' = \frac{\lambda}{4\pi\epsilon_0 d}\left[1 + \frac{c}{\sqrt{c^2+d^2}}\right].$$

$$E_Q = \sqrt{(E_x')^2 + (E_y')^2}$$

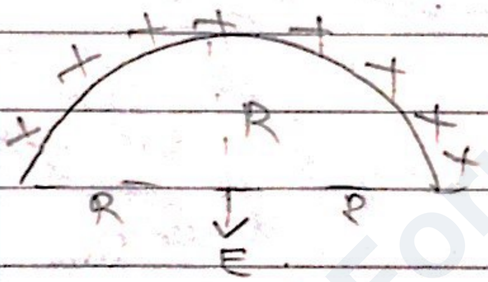
Electric field because of a charged arc (U I C D) at centre of the arc :



$$E_{net} = \frac{\lambda}{2\pi\epsilon_0 R} \sin\frac{\phi}{2}.$$

(along angle bisector of arc)

•

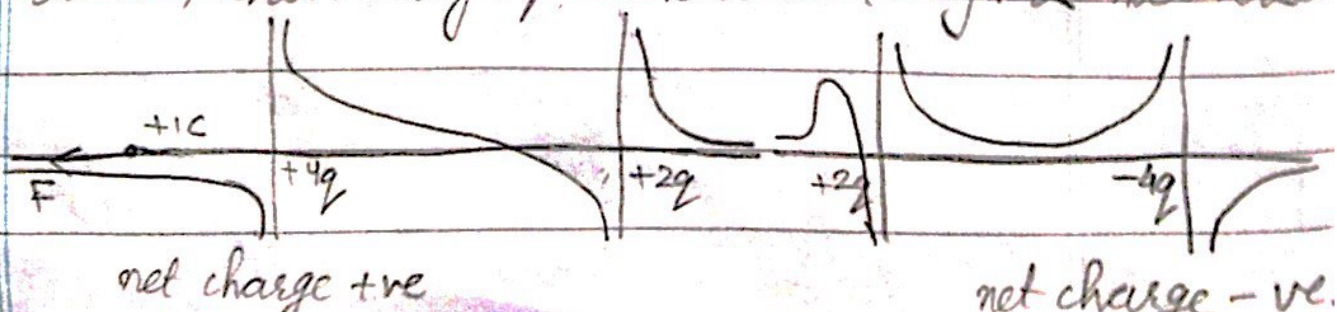


$$E = \frac{\lambda}{2\pi\epsilon_0 R} \sin\left(\frac{180}{2}\right)$$

$$= \lambda/2\pi\epsilon_0 R$$

E vs x graph,

- For +ve charge $+\infty$ in front & $-\infty$ at back
- For -ve charge, $-\infty$ in front & $+\infty$ at back
- For asymptotes, see net charge. Now, take +1C charge at ends. If force acts in +X dirn, then asymptote ends +vely. & vice versa.



net charge +ve

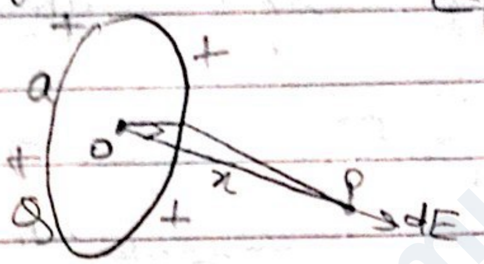
net charge -ve.

- Findings from E vs x graph:
- ✓ Pt. where it cuts the axis, $E = 0$.
- ✓ If at a pt in V vs x graph, slope $= 0$, then at that pt, value of E in E vs x graph $= 0$

✓ Electric field due to a uniformly charged ring, on its axis

$$\vec{E_p} = \frac{\underbrace{Q}_{\text{total charge}}\ \underbrace{x}_{\text{distance away from centre of ring along axis}}}{4\pi\epsilon_0 (\underbrace{a}_{\text{radius}}^2 + x^2)^{3/2}}$$

along OP



* Field will be max along axis at a distance of $a/\sqrt{2}$ from centre of ring.

- If $-Q'$ is placed at P,

$$F = \left[\frac{Qx}{4\pi\epsilon_0 (a^2 + x^2)^{3/2}}\right](Q') \rightarrow \text{oscillatory motion.}$$

If $x <<< a$, $F = \dfrac{QQ'x}{4\pi\epsilon_0 a^3}$, SHM ($F \propto x$)

$$\hookrightarrow T_P = 2\pi\sqrt{\frac{m}{QQ'}(4\pi\epsilon_0 a^3)}$$

(iii) Electric flux: (ϕ)

✓ $\phi = \displaystyle\int_{\text{open surface}} \vec{E}\cdot\vec{dA} = \oint_{\text{closed surface}} \vec{E}\cdot\vec{dA} = \underbrace{\vec{E}\cdot\vec{A}}_{\text{in uniform field } \vec{E} \text{ \& if } \vec{A} \text{ is defined}}$

$= \dfrac{q_{enclosed}}{\epsilon_0}$

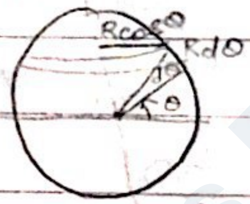
✓ Flux passing a hemispherical shell in uniform $\vec{E_0}$ $\quad \phi = E_0 \pi R^2 \ \frac{Nm^2}{C} = \frac{q}{2\epsilon_0}$

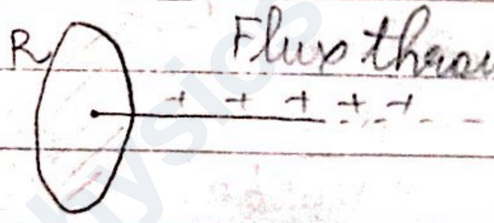
• If lid is closed, $\phi = 0$.

✓ For a spherical shell, $\phi = \frac{q}{\epsilon_0}$

✓ Gauss Law, $\phi_{\text{air or vacuum}} = \frac{q_{\text{enclosed}}^{\text{net}}}{\epsilon_0}$, $\phi_{\text{medium}} = \frac{q_{\text{enclosed}}^{\text{net}}}{\epsilon_0 K}$

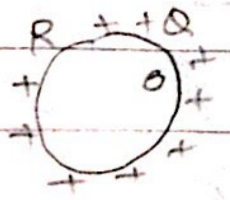
✓ Integral for finding SA of whole sphere, hemisphere or any bulge, $= \int_{0}^{\theta} 2\pi (R\cos\theta) R\,d\theta$ → put proper limits



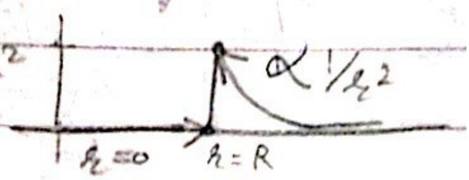
✓ 

Flux through a disc ($r = R$) due to a semi-infinite lined charge $= \frac{\lambda R}{2\epsilon_0} = \phi$

✓ Field due to a spherical conducting shell

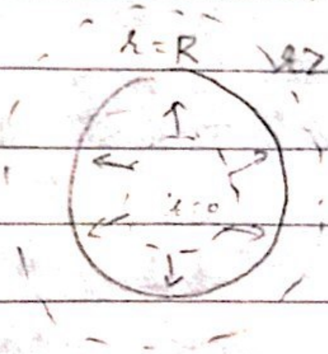


	$E(r)$
$0 \le r < R$	0
$r = R$	not defined
$r > R$	$\frac{Q}{4\pi\epsilon_0 r^2}$



✓ $E(r)$ for an insulating solid sphere

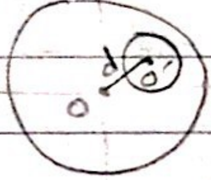
U VCD $= \rho\ C/m^3$



	$E(r)$
$0 \le r \le R$	$\left(\frac{\rho}{3\epsilon_0}\right) r$
$r > R$	$\left(\frac{\rho R^3}{3\epsilon_0}\right)\frac{1}{r^2}$

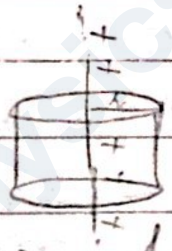
$\boxed{r = R,\ \frac{\rho R}{3\epsilon_0}}$

✓ U insulating solid sphere (R_1), $\rho \to$ U VCD, cavity of (R_2) cut.



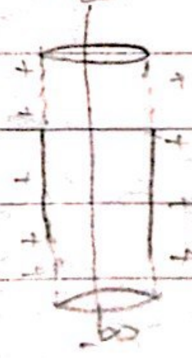
$\vec{E}_p = \frac{\rho d}{3\epsilon_0}$ along OO'

★ ✓ Electric field due to infinite LCD.
(cylindrical symm.)

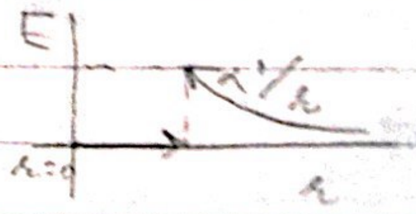


$E = \frac{\lambda}{2\pi\epsilon_0 r}$ — distance from line (radius of cylinder)

✓ $E(r)$ for an infinite conducting cylindrical shell, radius $= R$.

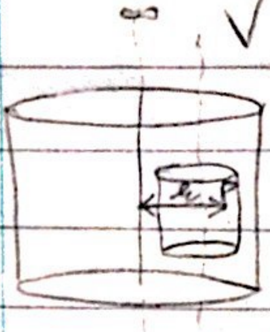


	$E(r)$
$0 \le r < R$	0
$r = R$	not defined
$r > R$	$\frac{\lambda}{2\pi\epsilon_0 R}$



Taking charge of cylinder on the infinite line (the line's λ)

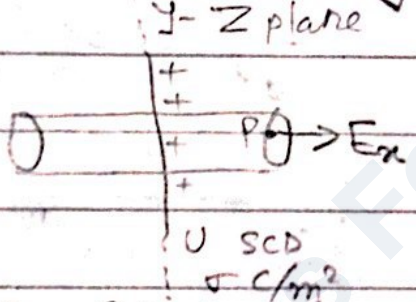
✓ Field inside cavity made in a uniformly charged insulating cylinder (U)



$E_P = \frac{\rho d}{2\epsilon_0}$ along line joining axis to axis

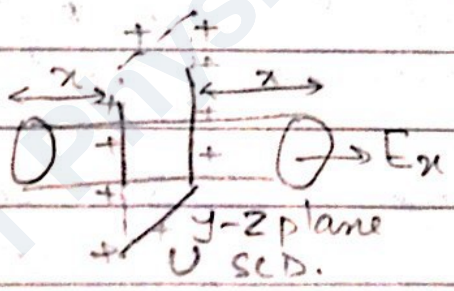
- If cylinder is not kept ∥ to axis of bigger cylinder, take resultant of E of both.

★ ✓ Field due to an infinite conducting or non conducting sheet



$E_x = \frac{\sigma}{2\epsilon_0}$; holds iff we take a pt. P closer to the sheet

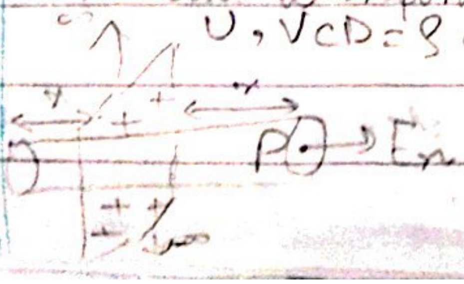
★✓ Field due to infinite conducting plate



$E_x = \frac{\sigma}{2\epsilon_0}$ → For ∥ plate, $E_x = \frac{\sigma}{\epsilon_0}$ → For field b/w 2 ∞ conducting plates.

- Inside conducting plate, at any pt., $E = 0$.

✓ Field due to infinite insulating plate of thickness t

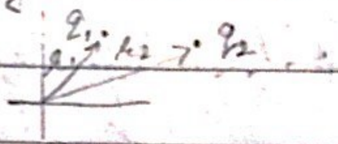


, $E_x = \frac{\rho t}{2\epsilon_0}$ → E at pt. P, outside the plate

- $E_i = \frac{\rho x}{\epsilon_0}$ } Field at a pt. P inside plate at dist = x

(iv) Dipole moment ($\vec{p}$) : gives total effect on a sys when acted by $\vec{E}$.

✓ $\vec{p} = \sum_{i=1}^{n} q_i \vec{r_i}$ (q_i → with sign)



✓ For 2 pt. charges of dipole,

$|p| = q\,d$ (q → magnitude; d → Distance b/w 2 charges.)

dir^n of $\vec{p}$ ← $\hat{p}$ = −ve to +ve.

* ✓ Field on the axial line of a dipole → not valid when pt is in b/w dipole

$$\vec{E_{a}} = \frac{2\vec{p}r}{4\pi\epsilon_0 (r^2-a^2)^2}, \quad \frac{2\vec{p}}{4\pi\epsilon_0 r^3} \text{ (For } a \lll r)$$

($\vec{E_a}$ → axial; first formula → valid + ots; 2a = length of dipole)

* ✓ Field on equitorial line of dipole (r → distance from centre of dipole)

$$\vec{E_{e}} = \frac{\vec{p}}{4\pi\epsilon_0 (r^2+a^2)^{3/2}}, \quad \frac{\vec{p}}{4\pi\epsilon_0 r^3} \text{ (For } a \lll r)$$

($\vec{E_e}$ → equitorial; a → length of dipole (2a))

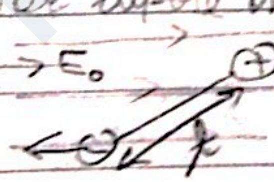
* ✓ Field at any pt. due to a dipole

$$\vec{E_{net}} = \vec{E_a} + \vec{E_e}, \quad |\vec{E}_{net}| = \sqrt{(E_a)^2 + (E_e)^2}$$

angle b/w E_a & E_{net} → $\tan\phi = \frac{E_e}{E_a} = \frac{\tan\theta}{2}$, $= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$

(θ → angle made with centre of dipole)

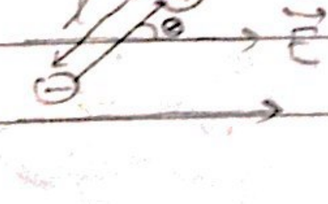
✓ For dipole in uniform electric field



For stable equi., TP of small oscill^{ns}

$$= TP = 2\pi\sqrt{\frac{ml}{2qE_0}}$$

(l → length of dipole; m → mass; E_0 → field)



✓ $\tau = \vec{p} \times \vec{E}$

Torque produced when a dipole is present inside an electric field.

(V) Electric Potential (V): $N\,m/C$

✓ $V = \frac{W}{q} = \frac{q}{4\pi\epsilon_0 r}$ → with sign $= E\cdot r$

charge carried

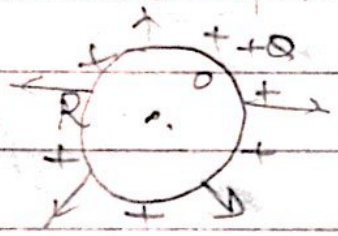
✓ $WD_{ag} = q\,(V_f - V_i) = \int_i^f (-\vec{E})\cdot(\vec{ds})$

with sign

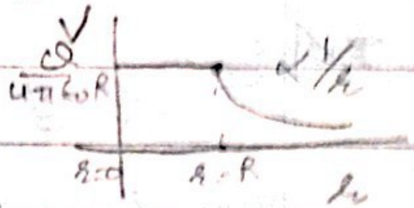
✓ $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$; $\boxed{E = -\frac{dV}{dr}}$

✓ $|\Delta V| = |\vec{E}|\,d$ → distance • Holds only when E is uniform & d is taken in dirⁿ of E

✓ Potential of thin conducting shell.

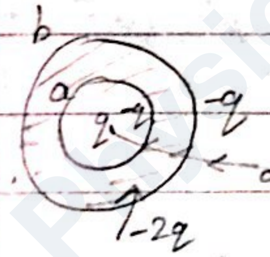


	$V(r)$
$0 \leq r \leq R$	$Q/4\pi\epsilon_0 R$
$r \geq R$	$Q/4\pi\epsilon_0 r$

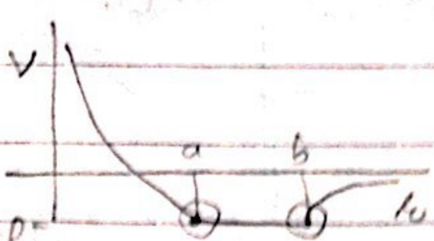


✓ PE is stored inside E(field). So, if, E = 0, PE there = 0.

✓ V(r) for diff cases:-



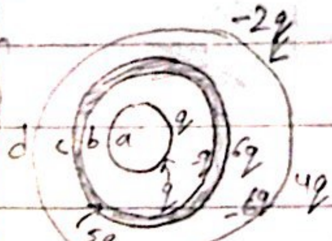
$0 < r \leq a$	$-q/4\pi\epsilon_0 b + \left(\frac{q}{4\pi\epsilon_0 r} - \frac{q}{4\pi\epsilon_0 a}\right)$
$0 \leq r \leq b$	$-q/4\pi\epsilon_0 b$
$r \geq b$	$\frac{-q}{4\pi\epsilon_0 r}$



* POS can also be used to evaluate this.

✓

For understanding



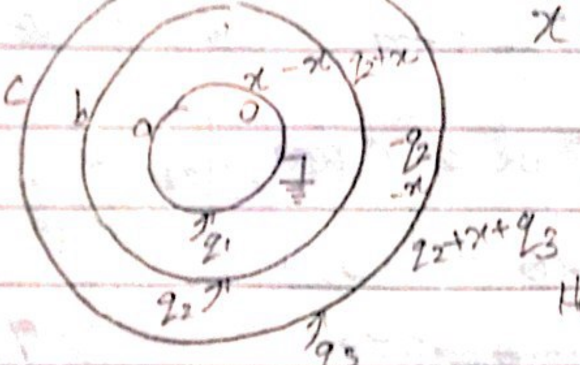
$0 \leq r \leq a$	$\frac{4q}{4\pi\epsilon_0 d} + \left(\frac{6q}{4\pi\epsilon_0 c} - \frac{6q}{4\pi\epsilon_0 d}\right) + \left(\frac{q}{4\pi\epsilon_0 a} - \frac{q}{4\pi\epsilon_0 b}\right)$
$a \leq r \leq b$	$\frac{4q}{4\pi\epsilon_0 d} + \left(\frac{6q}{4\pi\epsilon_0 c} - \frac{6q}{4\pi\epsilon_0 d}\right) + \left(\frac{q}{4\pi\epsilon_0 r} - \frac{q}{4\pi\epsilon_0 b}\right)$
$b \leq r \leq c$	$\frac{4q}{4\pi\epsilon_0 d} + \left(\frac{6q}{4\pi\epsilon_0 c} - \frac{6q}{4\pi\epsilon_0 d}\right)$
$c \leq r \leq d$	$\frac{4q}{4\pi\epsilon_0 d} + \left(\frac{6q}{4\pi\epsilon_0 c} - \frac{6q}{4\pi\epsilon_0 d}\right)$
$d \geq r$	$\frac{4q}{4\pi\epsilon_0 r}$

same

$V_f - V_i$

✓ $W.D._{\text{conservative force}} = -(\Delta U)$, change in PE or EPE

✓ The charge on the earthed spherical shell in a concentric shell network



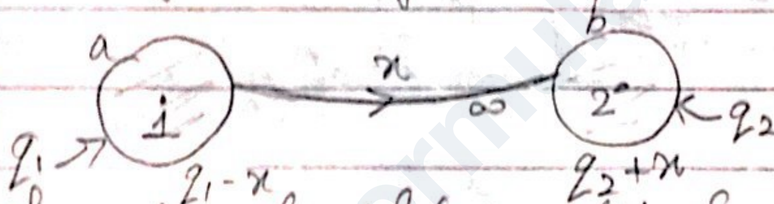
$$x = \frac{\sum (-1)\left(\frac{\text{Total charge}}{\text{radius}}\right)}{\frac{1}{\text{radius of earthed surface}}}$$

(Total charge: of outside surface)

Here, $x = \dfrac{-\frac{q_2}{b} - \left(-\frac{q_2}{c}\right) - \frac{q_2}{c} - \frac{q_3}{c}}{1/a}$

* Above trick holds only when all conductors are thin & earthing of a single conductor is done.

✓



$$x = \frac{\frac{q_1}{a} - \frac{q_2}{b}}{\frac{1}{a} + \frac{1}{b}}$$

Charge transferred b/w 2 solid spheres of radius a & b & charges q_1, q_2, infinitely separated.

✓ Mutual Energy $(E_M) = \dfrac{q_1 q_2}{4\pi\epsilon_0 r}$ → distance b/w charges

* V should be same of both bodies

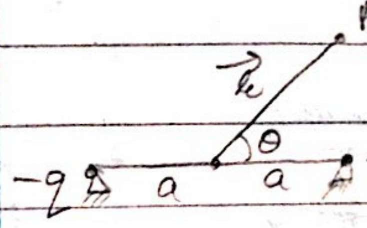
✓ Self Energy $(E_s) = \dfrac{q^2}{8\pi\epsilon_0 r}$ = E stored inside field lines of charge distribⁿ

✓ Energy stored (General formula) $= \int_{\text{complete volume}} \left(\frac{1}{2}\epsilon_0 E^2\right) dV$ → volume

(field; $\frac{1}{2}\epsilon_0 E^2$ → Energy per unit vol.)

✓ $\Delta EPE = \Delta E_M + \Delta E_s$.

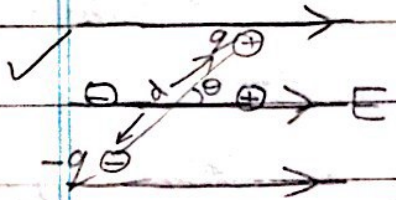
✓ Potential due to an electric dipole



$$V_P = \frac{2aq\cos\theta}{4\pi\epsilon_0 r^2} = \frac{|\vec{p}|\cos\theta}{4\pi\epsilon_0 |\vec{r}|^2} \rightarrow \frac{\vec{p}\cdot\vec{r}}{|\vec{p}||\vec{r}|}$$

• angle θ is from $\vec{p}$ to $\vec{r}$ (not necessary acute)

✓ PE of a dipole kept in a U field $\vec{E}$



$$U_\theta = U_0 + pE(1-\cos\theta)$$

U_θ → PE of a dipole kept at ∠ = θ

$$U_0 = -pE - \frac{q^2}{4\pi\epsilon_0 d}$$

U_0 → taking zero reference at ∞.

$$\Rightarrow U_\theta = -\frac{q^2}{4\pi\epsilon_0 d} - pE\cos\theta$$ → zero reference at ∞ again

d → length of dipole