FORMULA SHEET

+2 Physics

Akshansh Chaudhary

My Formula Page

Chapter – Capacitors

* Capacitance: Capacity to store charge or Electric lines of force or EPE.
* Capacitance depends on: Geometry, medium.
* $WD = q(\Delta V)$
  ↳ +ve if +ve charge leaves +ve plate of capacitor.
  −ve if +ve charge enters +ve plate of capacitor.
* Effective field inside a ∥ plate capacitor →
  $$\vec{E} = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0} \quad (+ve \text{ to } -ve)$$
* $q = CV$: $C \rightarrow$ capacitance ; $q \propto V$, $\cancel{q \propto C}$, $\cancel{C \propto \frac{1}{V}}$
* $C = \frac{A\epsilon_0}{d}$ → area, → distance b/w plates

SI – Farad
* EPE stored inside capacitor :–
  $$EPE = E_{stored} = \frac{q_{(t)}^2}{2C} = \frac{1}{2}CV_{(t)}^2 = \frac{1}{2}q_{(t)}V_{(t)}$$
  apply → (t) or (equivalent)
* Force b/w plates of a charged capacitor having charges $q, -q$ at an instant : $F = \frac{q^2}{2A\epsilon_0}$
* $WD_{battery} = \Delta EPE + \text{Spark/Heat}$.
  $\Rightarrow q(\Delta V) = EPE_f - EPE_i + \text{Spark/Heat}$.
* If capacitor(s) is initially uncharged, then,
  $q(\Delta V) = EPE_f - 0 + \text{Spark/Heat}$
  $\Rightarrow q(\Delta V) = \frac{1}{2}q(\Delta V) + \frac{1}{2}q(\Delta V)$
  i.e. Half of the WD is stored as EPE inside capacitors & 1/2 is liberated as heat/spark.

★ Series connection of capacitors :-

• $C_{eq} = \sum_{i=1}^{n} \frac{1}{C_i}$

• $V_{eq} = V_{battery}$ = Sum of PD b/w all capacitors (valid everywhere)

• In series connection, current/charge is same, but voltage changes. So, distrib^n of voltage across the circuit is done as inverse of ratio of capacitance :-

$PD_{C_1} : PD_{C_2} : \ldots :: \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3} \ldots$

$\Rightarrow \frac{PD_{C_1}}{PD_{C_2}} = \frac{C_2}{C_1} \Rightarrow PD_{C_1} = PD_{C_2} \times \frac{C_2}{C_1}$

★ Parallel combin^n of capacitors :-

• $C_{eq} = \sum_{i=1}^{n} C_i$

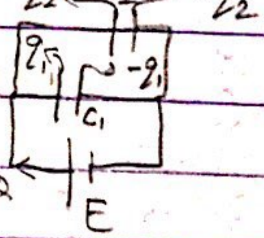
• In ∥ connection, PD across capacitors in ∥ remains same, but current varies. So, distrib^n of current across the circuit is done just as the ratio of capacitance i.e.

final charges $Q_{C_1} : Q_{C_2} : \ldots :: C_1 : C_2 : \ldots$ } Holds everywhere : capacitor charged/uncharged.

★ Understanding example :-

$q_1 = Q\left(\frac{C_1}{C_2+C_1}\right)$

$q_2 = Q\left(C_2/_{C_1+C_2}\right)$



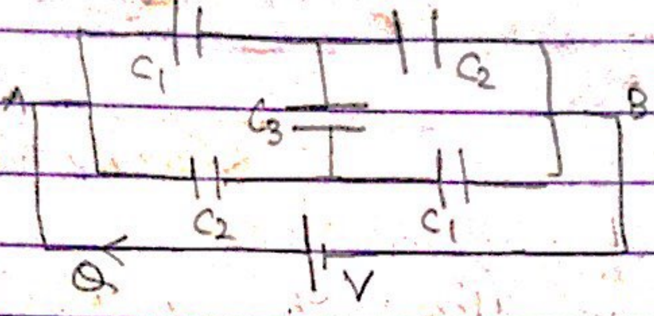
★ Method to find charges on capacitors or solve any term of a loop : KVL, POS, symmetry or accumul^n/distrib^n pattern.

POS → only when battery has resistor connected to it

★ If capacitors are in ∥, the governing factor will be the capacitor having lower breakdown voltage.

• If connected in series, governing factor is lower breakdown charge

* Cross Symmetry in wheatstone bridge :-

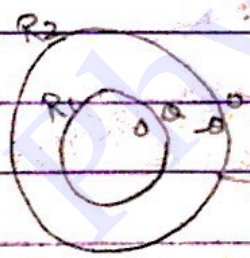


$$\frac{Q}{V} = \frac{2C_1C_2 + C_2C_3 + C_3C_1}{C_1 + C_2 + C_3}$$: direct formula

* Folding of a circuit can be done only if
(a) line of mirror symmetry is present.
(b) Battery is connected b/w any 2 pts lying on the mirror symmetry line

** SPHERICAL CAPACITORS

* $$C = \frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1}$$



→ 2 spherical concentric shell conductors

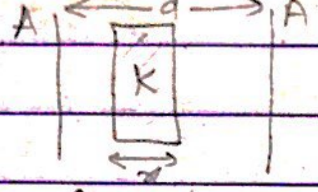
* $C = 4\pi\epsilon_0 (r)$ → radius : Capacitance of a spherical capacitor

* Magnitude of induced charges on dielectric :-

$$Q_i = Q\left[1 - \frac{1}{K}\right]$$

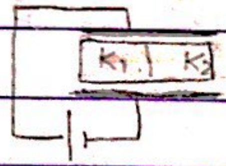
- For perfect conductor, $K \to \infty \Rightarrow Q_i = Q$
- For perfect insulator, $K = 1 \Rightarrow Q_i = 0$.
- All parameters with ϵ_0 are changed to $K\epsilon_0$ in problems of dielectric eg : $C = \frac{A\epsilon_0}{d} \rightarrow C = \frac{KA\epsilon_0}{d}$.

* Capacitance of a ∥ plate capacitor with dielectric (neutral) put inside it, whose length is x



$$C = \frac{KA\epsilon_0}{K(d-x)+x}$$

* Fringe effect or end effect ≡ tilting of field lines at the edges of ∥ plate capacitor.
* If a charge Q is made to flow through n plates connected in series with dielectric in b/w them (in series) then charge on the interface of n^{th} & $(n+1)^{th}$ dielectric is given as, $Q_{flow}\left(\frac{1}{K_{n+1}} - \frac{1}{K_n}\right)$

* 

$$C = \left(\frac{A}{2}\right)\frac{\epsilon_0}{d}(K_1 + K_2)$$

* In case of many dielectrics,

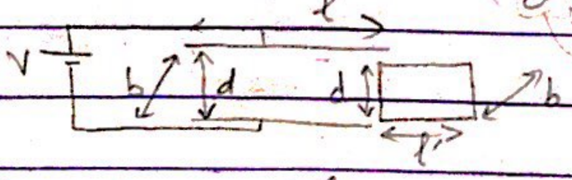
In series, $\frac{1}{C_{eff}} = \frac{1}{dC_1} + \frac{1}{dC_2} + \text{-----} = \int \frac{1}{dC}$

In ∥, $C_{eff} = dC_1 + dC_2 + \text{---} = \int dC$

* Force experienced by a dielectric slab when it is just outside or at some distance inside the ∥ plate capacitor, $|\vec{F}| = \frac{1}{2}\frac{b\epsilon_0}{d}V^2(K-1)$

b → breadth

d → height

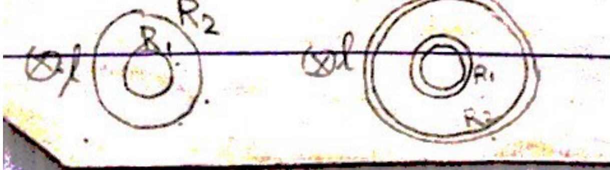


* Field inside a capacitor is independent of dielectric connected if battery is connected

* Cylindrical capacitance b/w 2 cylindrical shells (thick/thin) having a common axis → $C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{R_2}{R_1}\right)}$

l → length of capacitor

→ see how to take them in case of thick capacitors ←



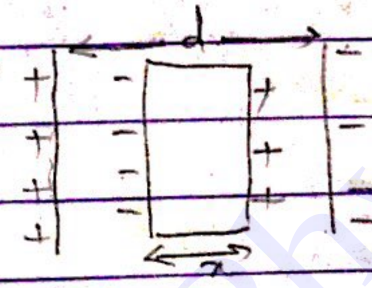
• If $l = 1$ & $R_2 >>> R_1$, $C_l = \dfrac{\pi\epsilon_0}{\ln\frac{R_2}{R_1}}$

* Capacitance b/w 2 solid spheres of same radius R.
$C = 2\pi\epsilon_0 R$, independent of distance b/w spheres.

* Hemispherical capacitor has capacitance

$$C_h = \frac{1}{2}\times 4\pi\epsilon_0\left(\frac{R_2 R_1}{R_2 - R_1}\right)$$

$$= \frac{1}{2}\times \text{Capacitance of full spheres.}$$

* 

When a conducting slab fills the space (partially) b/w ‖ plate capacitors,

$$C = \frac{A\epsilon_0}{d\left(1-\frac{x}{d}\right)} = \frac{C_0}{1-x/d}$$

CHAPTER : ELECTRIC CURRENT

* $R \times C$ i.e Resistance × Capacitance = Time constt for any R-C circuit. It's denoted by τ. SI → Sec.

* For std ↑ f^{ns} ; $X = Y(1 - e^{-t/\tau})$ } valid only when starting value = 0

  where, Y is max value attained by the f^n of X

  For RC circuit, $\tau = RC$: (for Power across Resistor, $\tau = \frac{RC}{2}$)

  Std ↑ f^{ns} attain their 63% of max. value at $t = \tau$

* For std ↓ f^{ns} ; $X = Y(e^{-t/\tau})$ } Valid only when value at end = 0.

  The same applies here also as above.

  Std ↓ f^{ns} attain 37% of their max. value at $t = \tau$ i.e after one time constant

* For LR circuit; $\tau = \frac{L_{eff}}{R_{eff}}$ eff : effective

* If charge enters the high potential surface, WD is -ve & power is absorbed.

* If charge leaves high potential end, WD is +ve & power is released.

  → Reverse would happen for -ve potential (low) end.

* $P(t) = i(t)^2 R = V(t)\, i(t) = \frac{V(t)^2}{R}$

* General equation w.r.t time of any f^n given by Norton - Thevenin Theorem:-

  $$f(t) = A + Be^{-t/\tau} \quad ; \quad \tau = R_{eff} \times C_{eff}$$

  A & B can be found by taking values at $t = 0$ (= A+B) & $t \to \infty$ (= A)

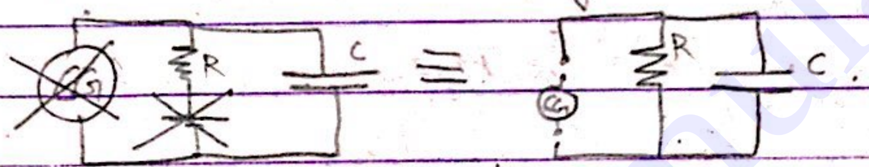
  or $\tau = \frac{L_{eff}}{R_{eff}}$ { depending upon the circuit }

★ Norton-Thevenin Theorem: It helps in finding τ in any numerical

Method 1) Short all the batteries present in circuit.

2) Open all the current generators (CG)

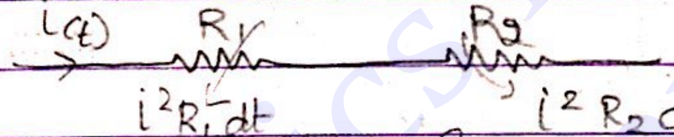
ie: eg



3) Then apply KVL to get net $R_{eff} \times C_{eff}$.

Note: We can never apply KVL in a loop containing current generators

★



Ratio of differential heat in dt time = $\frac{R_1}{R_2}$.
or total heat . or total time

ie In series $\frac{H_1}{H_2} = \frac{R_1}{R_2}$.

In parallel, $H_1 : H_2 : H_3 \ldots :: \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} \ldots$

★ Useful for numericals :- In RC circuits, charge or voltage of capacitors can't change suddenly. (Although current values can change by changing switch alignment).

Note :- Power (t) :-

Across R : $\tau = RC/2$

Across battery : $\tau = RC$

Across capacitor : τ = not defined

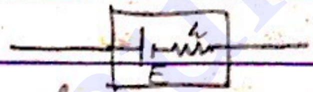
Across non ideal batt. : τ = not defined

* Equivalent capacitance b/w 2 non-parallel sheets, having an angle θ b/w them is given as.

l : length of sheets

b : breadth of sheets

$$C = \frac{b\epsilon_0}{\tan\theta} \ln\left|\frac{l + h/\tan\theta}{h/\tan\theta}\right|$$

* For a non ideal battery, 

net emf provided $= E - (ir) \rightarrow$ losses.

* Maximum power transfer theorem : If, in case of a non-ideal battery, $R = r \rightarrow$ internal resistance, then, Power (given by batt. or absorbed by R or heat given by R) will get maximised ; current gets maximised

* For POS to be applicable in current calculation, the segments for each of the battery should have a single resistance atleast in series with the battery.

* Cell combination:

In series, $E_{eq} = \varepsilon_1 + \varepsilon_2 + \ldots$ (with polarity)

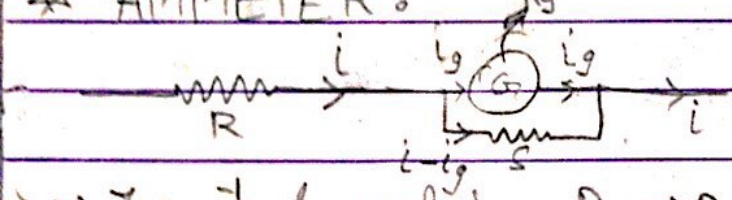
$r_{eq} = r_1 + r_2 + \ldots$

In $\parallel$, $E_{eq} = \dfrac{\dfrac{\varepsilon_1}{r_1} + \dfrac{\varepsilon_2}{r_2} + \ldots}{\dfrac{1}{r_1} + \dfrac{1}{r_2} + \ldots}$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots$$

★ Magnetic dipole moment $(\vec{\mu}) = I \cdot A$; Dir^n: by RHTR
(A = area)

★ AMMETER :



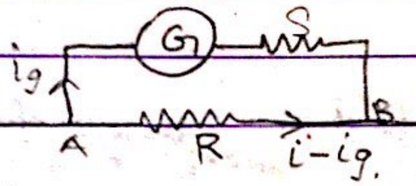
$\Rightarrow i = i_g\left(\frac{R_g+S}{S}\right)$ → calibr^n factor

★ $i_g R_g = (i - i_g) S$.

★ $R_A = \frac{R_g S}{R_g + S} \approx S$

→ For ideal working, $R_A \to 0$ or $S \to 0$ or $i_g \to 0$
or Ammeter is short circuited.

★ VOLTMETER



★ $(i - i_g)R = i_g(R_g + S)$

★ $V_{AB} = i_g(R_g + S)$ → calibr^n factor ($R_g + S$)

→ For ideal working, $R_V \to \infty$, $i_g \to 0$, $S \to \infty$ or
circuit is open circuited.

★ Potentiometer : Do from notes.

★ Drift velocity : $|\vec{V_d}| = \frac{e\tau}{2m_e} \times E$; $\vec{V_d} = \frac{e\tau}{2m_e}(-\vec{E})$

(E → field; τ → avg relax^n time (time gap b/w 2 successive col^{ns}))

★ $i = n e^{-1} A V_d$

(n → no. of e^-s p.u vol; A → area of crossection (uniform))

★ j = current density $= \frac{i}{A} = \frac{i}{A\cos\theta}$ → To make dir^n of j ⊥ to flow of i.

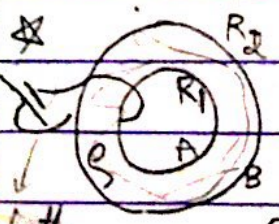
OHM'S LAW

$$j = \left(\frac{ne^2\tau}{2m_e}\right)E \Rightarrow \left(\vec{j} = \sigma \vec{E}\right)$$

(σ → electrical conductivity of material)

✓ Pt. property ✓ Vectorial rel^n ✓ $\hat{j} = \hat{E}$.

★



: Spherical concentric shells : $R_{AB} = \frac{\rho}{4\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

: Concentric cylinders : $R_{AB} = \frac{\rho}{2\pi l} \ln \frac{R_2}{R_1}$

batt. connected b/w inside & outside shells ⊗ l

★ If batt. is connected b/w base area (For cylinders) : $R = \frac{\rho l}{\pi(R_2^2 - R_1^2)}$

* $\rho = \dfrac{1}{\sigma} = \dfrac{2m_e}{ne^2\tau}$

$\rho_{T^\circ C} = \rho_{0^\circ C}(1+\alpha T)$: linear variation of ρ with Temp (T)

α : Temp. coeff of resistivity ($/^\circ C$)

$R_{T^\circ C} = R_{0^\circ C}(1+\alpha' T)$

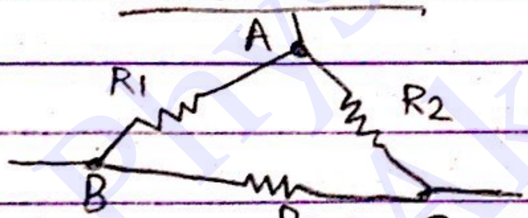
α' : Temp. coeff. of Resistance

* In general case,

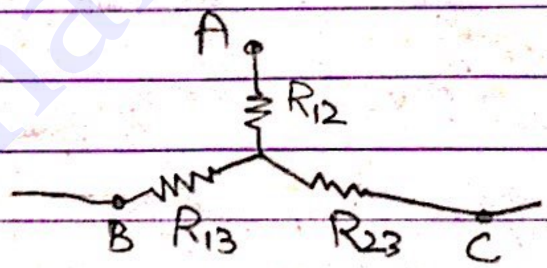
$$\alpha = \frac{\rho_{100^\circ C} - \rho_{0^\circ C}}{100\,\rho_{0^\circ C}} \quad ; \quad \alpha' = \frac{R_{100^\circ C} - R_{0^\circ C}}{100\,R_{0^\circ C}}$$

* Color coding : Do from notes.

* Star Delta Method



$\equiv$

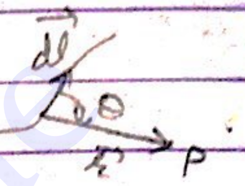


Delta to star :- $R_{12} = \dfrac{R_1 \cdot R_2}{R_1+R_2+R_3}$, Similarly others

Star to delta :- $R_{12} = \dfrac{R_{12}+R_{23}+R_{31}}{R_3}$

Ch : MAGNETIC EFFECTS

★ Biot-Savart law:-



$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{r^2}$ → from dl to $\vec{r}$

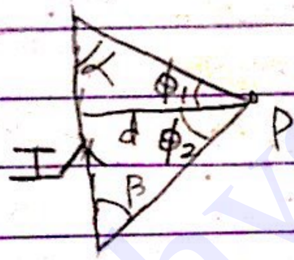
$= \frac{\mu_0}{4\pi}\left(\frac{q v \sin\theta}{r^2}\right)$, $d\hat{B} = (\hat{v} \times \hat{r})$

$d\hat{B} = \hat{dl} \times \hat{r}$.

★ Magnetic flux $\phi = \vec{B} \cdot \vec{A}$ (in Weber)

★ $\mu = \mu_0 \mu_r$

★ Magnetic field due to a thin, finite, linear, current carrying wire



$\vec{B_P} = \frac{\mu_0}{4\pi}\left(\frac{I}{d}\right)(\sin\phi_1 + \sin\phi_2)$

or $\vec{B_P} = \frac{\mu_0}{4\pi}\left(\frac{I}{d}\right)(\cos\alpha + \cos\beta)$

★ So, for infinite wire, $\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{2I}{r}\right)$

for semi-infinite wire,

$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{I}{r}\right)(1 + \sin\phi_1)$

★ Magnetic field due to an infinite sheet i.e at any point due to a surface current

$B = \frac{\mu_0 j}{2}$

★ Magnitude of magnetic field at centre of cube is given as

$|\vec{B}| = \frac{\mu_0}{4\pi}\left(\frac{I}{a\sqrt{3}/2\sqrt{2}}\right)$

* Magnetic field of complete ring at its centre point $|\vec{B}| = \frac{\mu_0}{4\pi}\left(\frac{2\pi I}{r}\right)$

* Magnetic field due to a current carrying arc at centre of arc $|\vec{B}| = \frac{\mu_0}{4\pi}\left(\frac{2\pi I}{r}\right) \times \frac{\theta}{360°}$

* If 2 semi-infinite wires, one entering and other leaving a loop (circular, star or any figure), directed towards the centre, then, magnetic field due to them $= 0$.

* Magnetic field on the axis of a current carrying circular loop.

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi I a^2}{(a^2+x^2)^{3/2}}$$

x: distance from centre of loop
a: radius of loop.

when $x >>> a$,

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2\vec{\mu}}{x^3}\right)$$

(Identical result to $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}$)

$\vec{\mu}$ → magnetic dipole moment $= I \cdot A$

* $\vec{\mu} = \frac{q}{2m}(\vec{L})$

$\vec{L}$ → angular momentum $= I\alpha$.

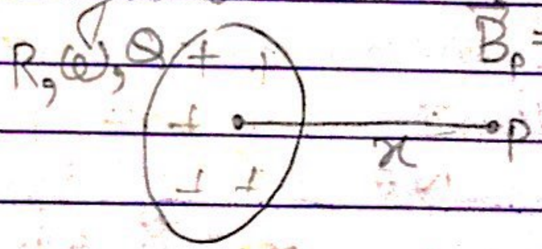
✓ above eq^n holds iff body is purely insulating
✓ distrib^n of charge is symmetric
✓ distrib^n of mass has got same nature as distrib^n of charge
✓ Body is in a state of pure rot^n.

✱ Magnetic field at the centre of a spiral

$$\vec{B} = \frac{\mu_0}{4\pi} \times \frac{2\pi I}{(b-a)} \ln\frac{b}{a} \; ; \; \vec{\mu} = I \cdot N\left(\frac{\pi}{(b-a)}\frac{(b^3-a^3)}{3}\right)$$

where b: larger radius, a: smaller radius

✱ Magnetic field on the axis of a uniformly charged disc :-



$$\vec{B}_P = \frac{\mu_0}{4\pi} Q(\omega)\left[\ln\left|\frac{\sqrt{R^2+x^2}+R-x}{x}\right| - \frac{R}{\sqrt{R^2+x^2}}\right]$$

✱ $\vec{\tau} = \vec{\mu} \times \vec{B}$: By RHTR (fingers in dir^n of movt)

✱ Magnetic PE $= -\vec{\mu}\cdot\vec{B}$ + const^{t}

✱ $W_{ag} = MPE_f - MPE_i$

✱ Ampere's Circuital law :- $\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enclosed}$.

✱ Magnetic field due to a thin, infinite current carrying cylindrical shell :-

$r > R$: $B = \left(\frac{\mu_0}{4\pi}\left(\frac{2\pi I}{r}\right)\right)$; $0 \le r \le R$: $B = 0$; $R = r$: $B = \infty$

✱ Magnetic field because of solid cylinder

$0 \le r \le R$; $B(r) = \frac{\mu_0}{4\pi}\left(\frac{2\pi I r}{R^2}\right)$ or $\left(\frac{\mu_0 j}{2}\right) r$; $j = \frac{I}{\pi R^2}$

$r > R$; $B(r) = \frac{\mu_0}{4\pi}\left(\frac{2\pi I}{r}\right)$

$r = R$; $B(r) = \frac{\mu_0}{4\pi}\left(\frac{2\pi I}{R}\right)$

✱ Magnetic field inside cavity

$\vec{B} = \frac{\mu_0 j d}{2}$; valid only when their axis are parallel.

✲ Magnetic field due to a long ideal solenoid

- At a point deep inside the solenoid: $B = \mu_0 n I$
- At a point near its ends: $B = \frac{1}{2}\mu_0 n I$, n → no. of turns per length

✲ Magnetic field due to a non-ideal solenoid

on its axis; $\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{2\pi I}{l}\right) N (\sin\alpha + \sin\beta)$

✲ Magnetic field inside an ideal toroid

$$B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2I}{R}\right) N = \mu_0 n I$$

$N = n(2\pi R)$

✲ Magnetic force on a moving charge particle:

LORENTZ FORCE :- $\vec{F}_m = q(\vec{v} \times \vec{B})$

$\vec{F}_m$ → magnetic; q → with sign; $\vec{v}$ → vel.

✓ $\vec{F}_m \perp \vec{v}$ & $\vec{F}_m \perp \vec{B}$: Conclusions:

∵ ✓ $\vec{F}_m \cdot \vec{vel} = 0$

✓ Instt. Power = 0.

✓ $WD_{\substack{\text{on } \vec{F}_m \\ i \to f}} = 0$ (i.e $\vec{F}_m$ can't do work on moving particle)

✓ $a_T = 0$ (tangential accl^n).

✓ Speed, KE = constt

✲ Radius of curvature due to magnetic force

$$R = \frac{mv}{qB} = \frac{P}{qB} = \frac{v}{\alpha B}$$

P → momentum; α → q/m.

$$TP = \frac{2\pi m}{qB}, \quad \omega = \frac{2\pi}{TP} = 2\pi\nu$$

✲ Max speed with which a charge q must be flicked from axis of solenoid s.t it doesn't strike the solenoid :-

$$\left(\frac{R}{2}\right) = \frac{m v_m}{qB} \Rightarrow v_m = \frac{qB \cdot R}{2m}$$

R → Radius of solenoid

✓ Result is independent of dir^n of projection

* If a charged particle is projected in a uniform magnetic field with a vel. at a general angle θ with uniform magnetic field $\vec{B}$ $(\theta \neq 0^\circ, \pi, \frac{\pi}{2})$

<u>HELIX</u> Pitch $= V_{Parallel} \times TP = V_{Parallel} \times \frac{2\pi m}{qB}$.

Radius $= \frac{m V_\perp}{qB}$

* For a particle to move on an undeviated path in the presence of electric field $(\vec{E})$ & magnetic field $(\vec{B})$

$\vec{E} = \vec{B} \times \vec{vel}$ i.e. $\vec{E} \perp \vec{B}$, $\vec{E} \perp \vec{vel}$

* $vel = \frac{E}{B \sin\theta}$

* <u>UNIFORM CYCLOID</u> : If a particle is released from rest in a region of uniform ⊥ electric & magnetic fields

$\vec{B} = -B_0 \hat{j}$, $\vec{E} = E_0 \hat{k}$: Motion in X-Z plane

$V_x = \left(\frac{q B_0}{m}\right) Z$; $V_z = \sqrt{\frac{2qE_0 Z}{m} - \left(\frac{q B_0 Z}{m}\right)^2}$

vs distance $\quad \omega$

$V_{nett} = \sqrt{V_x^2 + V_z^2} = \sqrt{\frac{2qE_0 Z}{m}}$

$V_{Z(t)} = \frac{E_0}{B_0} \sin \omega t$; $\omega = \frac{q B_0}{m}$.

vs time

* when $V_z = 0$ then, $Z = \frac{2mE_0}{qB_0^2}$ = <u>Width of cycloid</u>

* Radius of curvature in terms of x & y for finding any path is given as

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}.$$

* $d\vec{F}_m = i[d\vec{l} \times \vec{B}]$

$\rightarrow \vec{F}_m = i[\vec{l} \times \vec{B}]$: for thin, rigid, current carrying wire, B is uniform

* If a wire's magnetic force balances gravit^nal force

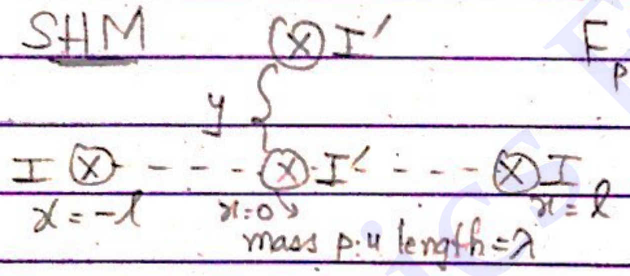
$i(lB\sin\theta) = mg \Rightarrow B = \frac{mg}{il\sin\theta}$

* Force of interaction b/w 2 wires (in their plane) having currents I_1 & I_2 & distance d apart

$\frac{dF}{dl}$ = Force p.u. length = $\frac{\mu_0}{4\pi}\left(\frac{2I_1I_2}{d}\right)$; see dir^n of current

* SHM

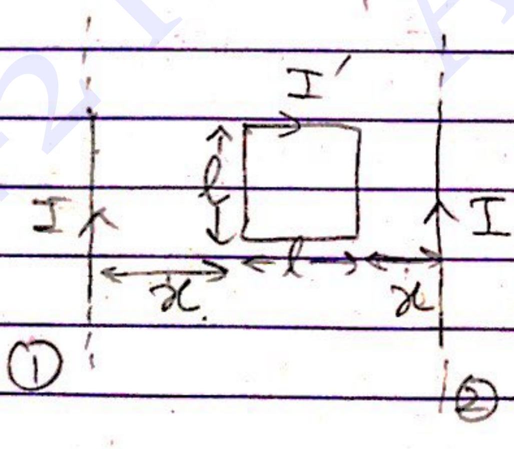
$F_{p.u\ length} = \left(\frac{\mu_0}{4\pi}\frac{4II'}{l^2}\right)y \quad (F \propto y)$



$TP = 2\pi\sqrt{\frac{m_{p.u.length}}{k_{p.u\ length}}}$

$= 2\pi\sqrt{\frac{\lambda\pi l^2}{\mu_0 II'}}$

* WD_ag in carrying an infinite wire from position ① to position ② as shown:-



$F_{ag} = \frac{\mu_0}{4\pi}(2II'l)\left[\frac{1}{x} - \frac{1}{x+l}\right]$