ELECTICAL MACHINES NOTES

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Electrical Machines Notes, First Edition

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TRANSFORMERS

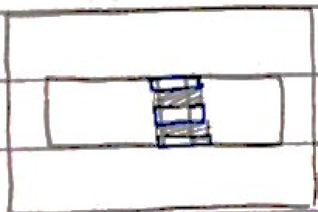
* Flux :
  - Mutual : Flux linked to both windings of transformer
  - Leakage : Flux linked to only one windings of transformer.

- Limbs of a transformer : The 2 sides where coil is wound.

- LV & HV winding : Low Voltage & High Voltage winding

why ? - ½ of LV winding close to the core & ~~so~~ ½ of HV windings ~~too~~ above LV. (all insulated). This is CORE TYPE (To maximise mutual flux & reduce Leakage flux).

- SHELL TYPE (Sandwich winding) : 3 limbs ∃. Winding along central limb. LV & HV are cut & placed alternatively.



*• In terms of ⅁ of operⁿ :-
1. Iron core : 25 - 400 Hz.
2. Air core : High ⅁ operⁿ.

* In terms of cooling :
1. Naturally cooled (small rated transformers)
2. Forced cooling transformer (big transformer in cooling oil)

* Transformer tanks (with cooling oil) are SEALED.
Why? : To prevent moisture ingress.
* Cooling oil : prevents heat (due to resistance)
protect insulⁿ.
* Transformer is a static device
↳ Its efficiency will be higher than rotating machines. (friction, air drag).
* Primary & Secondary windings are isolated from each other & the core.
• CORE TYPE : concentric coils
• SHELL TYPE : sandwich or pancake coils.
* Windings are of Solid/ Stranded Cu or Al strip conductors.

Imp. Insulation class : A, B, C, F, H.
↳ Highly inflammable industry : class H insulⁿ.
• Class 'A' can withstand upto 105°C
• " H " " 180°C

☆ Natural Cooling
• Small size transformers - Natural circulⁿ
- because of large coeff. of expansion of oil
- Heat removed from walls of the tank by radiation or air convention.
- Cooling area of tank increased by cooling fins or tubes — Medium size transformers.

☆ Forced Cooling
• For size > 5 mVA, supplementing tank surface by a separate radiator in which oil is circulated

by a pump.
- Better cooling options – oil to air heat exchanger, oil to water exchanger.
- Form^n of hot spots in conditioner damages core or insul^n property – ducts inside core/winding to remove heat are cut.

* Transformer oil : highly refined mineral oil
  : canola oil can be used.
* Oil : cools transformer
  : provides insul^n.
* Superconducting windings : Removes Cu loss
* When transformer is on NO-LOAD, its open circuited. So, no current should flow in secondary. Only current (I_0) flows in the primary called as EXCITING CURRENT or NO-LOAD CURRENT

* EMF eq^n of a transformer :-

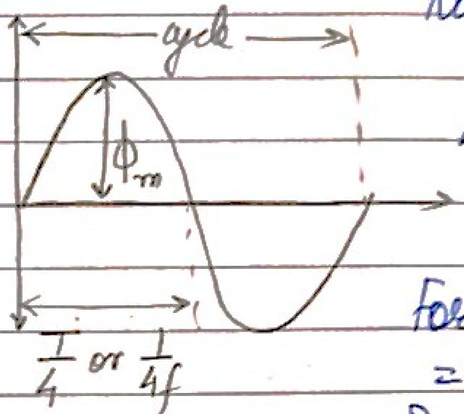
Deriv^n

N_1 : no. of turns of primary
N_2 : " " " secondary
ϕ_m : max. flux in core (Wb)
or f : frequency of A.C i/p (Hz)

Avg rate of flux $= \dfrac{\phi_m}{\frac{1}{4f}} = 4f\phi_m$ Wb/s.

PTO

★ $\phi_{max} = \dfrac{V}{\sqrt{2}\,\pi f N}$ Wb → Max core flux in ideal transformer under no load



Rate of change of flux/turn
= induced emf

Avg. induced emf/turn
$= 4 f \phi_m$ volt

For sinusoidal flux, rms value
= Avg value × Form factor

RMS value of emf/turn
$= 1.11 \times 4 f \phi_m$ ~~$\times N_1$~~ $= 4.44 f \phi_m$ volt

RMS value of emf in whole of primary $(E_1) = 4.44 f \phi_m \times N_1$

Assuming no leakage flux & no winding resistance
$E_1 \simeq V_1$ & $e_2 = N_2 \dfrac{d\phi}{dt}$.

with secondary open, $V_2 = e_2 \Rightarrow \dfrac{e_1}{e_2} = \dfrac{N_1}{N_2} = a$.

$\left[\dfrac{E_1}{E_2} = \dfrac{N_1}{N_2} = a\right.$ (ratio of transformation$\left.\right]$

or Transfer Ratio
or Voltage Ratio

★ For a transformer
$v_1 i_1 = v_2 i_2$
$\Rightarrow \dfrac{v_1}{v_2} = \dfrac{i_2}{i_1} = \dfrac{a_T}{4} = \dfrac{1}{4f}$

✯ Ideal Transformer :-

Assumptions :-

1. Primary & Secondary have no winding resistance. (no loss in R)
2. No leakage flux (all flux confined to core & link both windings)
3. Core has infinite permeability (μ) so that no I_m (magnetising current) is needed for given ϕ_{max} ($\phi_m = \frac{F}{l/\mu A}$) → MMF → Permeability
4. $I_i = 0$ i.e. core loss $= 0$. ← iron loss

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = a.$$

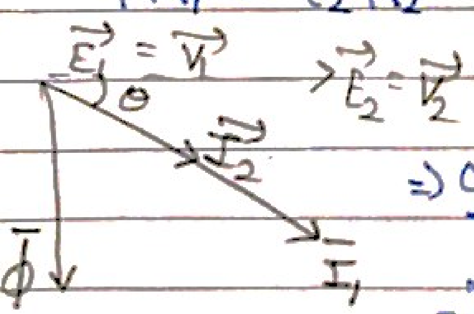
✯ IDEAL TRANSFORMER ON LOAD

$i_2 \neq 0$; secondary mmf $\mathcal{F}_2 = i_2 N_2$ opposes core flux (Lenz Law)

To keep $\phi =$ constt., i_1 from primary flows
($\because$ $\text{Resistance}_{\text{winding}} = 0$)

To mmf. balance :

$$i_1 N_1 = i_2 N_2 \Rightarrow \frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}.$$



$P_1 = v_1 i_1 = v_2 i_2 = P_2$

$\Rightarrow$ Instt. Power of Primary = Instt. Power of Secondary

$\Rightarrow$ no loss in ideal transformer.

☆ Impedance : Resistance offered in AC circuit.

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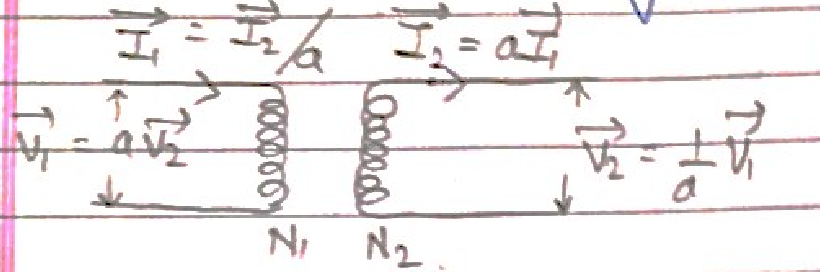
- • equivalent circuit diagram for ideal transformer:



$\frac{V_2}{I_2} = Z_2$

$\Rightarrow \frac{(N_2/N_1)V_1}{(N_1/N_2)I_1} = Z_2$

$\Rightarrow \frac{V_1}{I_1} = \left(\frac{N_1}{N_2}\right)^2 Z_2 = Z_2'$

$\supset a^2$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \& \quad \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$\Rightarrow \frac{V_1/V_2}{N_1/N_2} = \frac{I_1/I_2}{N_2/N_1} \Rightarrow \frac{V_1}{I_1} = \left(\frac{N_1}{N_2}\right)^2 \& \frac{V_2}{I_2} = \left(\frac{N_1}{N_2}\right)^2.$$

$$= Z_1$$

$$\Rightarrow Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2 = a^2 Z_2 = Z_2'$$

= Impedance of secondary ~~to~~ referred to the primary (as seen by)

So, $Z_1' = \frac{1}{a^2} Z_1$ (||ly)

$Y_2' = \frac{1}{a^2} Y_2$ (admittance)

☆ RULES

1. Secondary impedance referred to primary (as seen by)
   $= a^2 Z_2$
   ; a = ratio of transformⁿ.

* $\frac{1}{\text{Resistance}(R)\,\Omega}$ = Conductance (G) ℧
* $\frac{1}{\text{Reactance}(X)\,\Omega}$ = Susceptance (B) ℧
* $\frac{1}{\text{Impedance}(Z)\,\Omega}$ = Admittance (Y) ℧

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2. Primary impedance referred to secondary
$$= \frac{1}{a^2} Z_1$$

3. Secondary admittance referred to primary
$$Y = \frac{1}{a^2} Y_2$$

4. Primary admittance referred to secondary
$$= a^2 Y_1$$

* a. Transferring resistance from secondary to primary
$$R_2' = a^2 R_2.$$

(b) ~~Secon~~ Primary resistance referred to secondary
$$R_1' = \frac{1}{a^2} R_1$$

* Leakage flux is modelled as Reactance (X)
Reactance comes through Leakage flux

* R (Resistance), X (Reactance), B_m (magnetic ~~susceptance~~ susceptance), G_i (Iron loss conductance).

All in one circuit

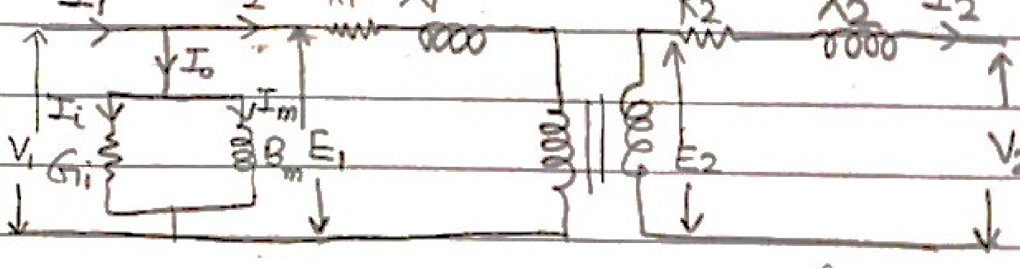
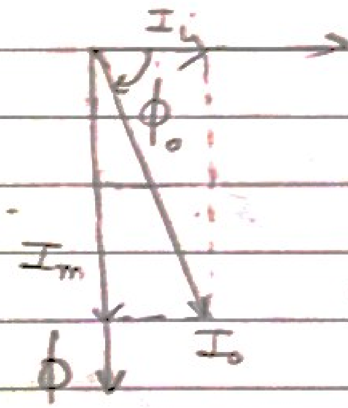


fig : 3.14

* Secondary voltage: also called Terminal voltage.

* No ~~no~~ load Phasor diagram:



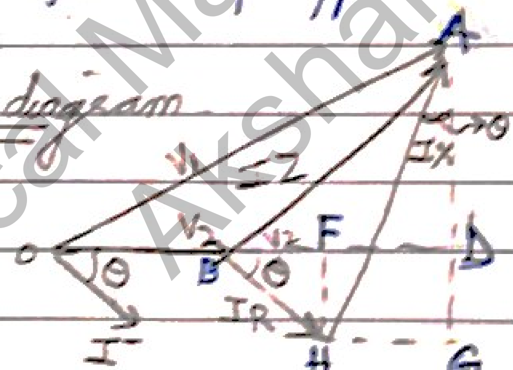
I_0 : exciting/no load current
I_m : magnetising current (wattless)
I_i = Iron loss current (Watt)

- $\phi_0 \simeq 90°$
- Since $I_m >> I_i$
- $\cos\phi_0$ = no load power factor $\simeq \cos 90° =$ less

$I_m = I_0 \sin\phi_0$
$I_i = I_0 \cos\phi_0$

* Power transform^n ~~for~~ approximate form of T circuit model.

Phasor diagram



(fig 1)
(used later)

non loading test

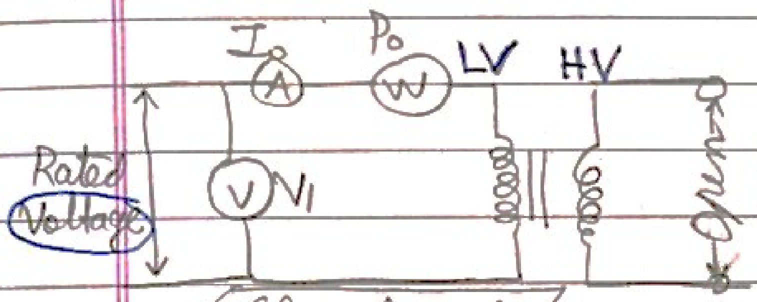
* TESTING OF TRANSFORMER

OC, SC test

Sumpner's test

R, I, S → P ×a?
25 S → P ×/a²

OC test



Shunt branch parameters:

Core loss $= P_0$

w.r.t LV side

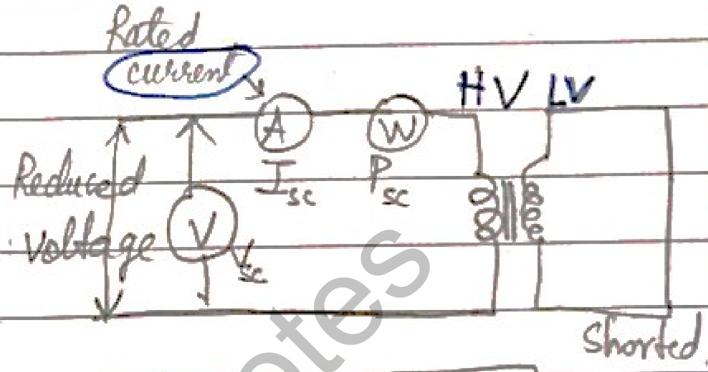
$$Y_0 = \frac{I_0}{V_1}$$

$$G_i = \frac{P_0}{V_1^2}$$

$$B_m = \sqrt{Y_0^2 - G_i^2}$$

- In OC test, wattmeter is a measure of iron core loss.

SC test



Series Parameters:

$$Z = \frac{V_{sc}}{I_{sc}}$$

$$R = \frac{P_{sc}}{I_{sc}^2}$$

$$X = \sqrt{Z^2 - R^2}$$

w.r.t HV side. (So we can change from R to R' & all)

$\boxed{\text{Full load copper loss} = P_{sc}}$

★ Wattmeter in SC test is a measure of full load Copper loss (Propo^nal to I^2)

Copper loss
↳ 6 full load = 1000 W, say
↳ ½ load
$= \left(\frac{1}{2}\right)^2 \times 1000.$

(not $\frac{1}{2} \times 1000$)

Q Obtain the approximate equivalent circuit of a given LV 200/2000 HV V, 30 KVA single phase transformer having the following test results:

(See fig on previous page)

OC Test : $V_1 = 200\,V$, $I_0 = 6.2\,A$, $P_0 = 360\,W$ } on LV side

SC Test : $V_{sc} = 75\,V$, $I_{sc} = 18\,A$, $P_{sc} = 600\,W$ } on HV side

Ans :- OC Test (LV side)

$$Y_0 = \frac{6.2}{200} = 31\times10^{-3}\ \mho$$

$$G_i = \frac{360}{4\times10^4} = 9\times10^{-3}\ \mho$$

$$B_m = \sqrt{(31)^2 - 9^2}\ (\times10^{-3})$$

$$= \sqrt{880}\times10^{-3}$$

$$B_m = 29.664\times10^{-3}\ \mho$$

31
900
961
−81
880

SC Test (HV side)

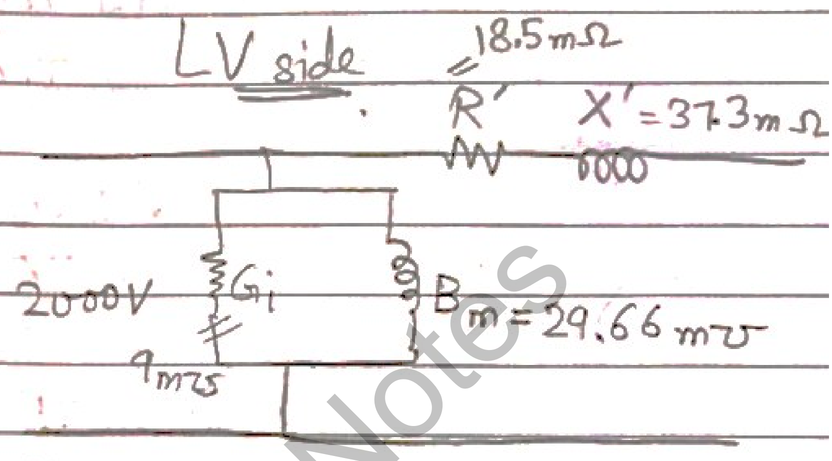
$$Z = \frac{75}{18} = 4.166\ \Omega$$

$$R = \frac{600}{18^2} = 1.85185\ \Omega$$

$$X = \sqrt{(4.166)^2 - (1.85)^2} = 3.731783\ \Omega$$

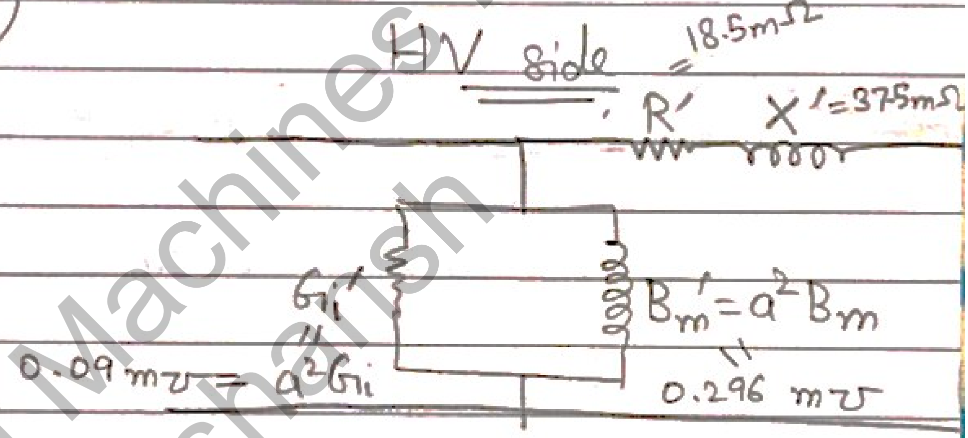
Now, transform R & X to LV side (as we got them in HV)

$$R' = a^2 R$$
$$X' = a^2 X$$



$$a = \frac{V_1}{V_2} = \frac{200}{2000}$$

$$\Rightarrow a = \frac{1}{10}$$



* Efficiency (η) = $\frac{O/p}{I/p}$.
  - → max. efficiency when Cu Loss = Fe loss
  - → o/p corresponding to η_{max}, $I_2 = \sqrt{\frac{W_i}{R_{02}}}$

- R_{01} : equivalent resistance in the primary
  $= R_1 + R_2'$
  $= R_1 + a^2 R_2$; $a = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{i_2}{i_1}$

$V_2 I_2 \cos\theta_2$

- $\eta = \frac{\cancel{Ou}\ O/p}{O/p + \text{Losses}} = \frac{I/p - \text{losses}}{I/p}$.

→ $I^2 R + P_i$

* Full load Cu loss $= I_2^2 R_{02}$

$$= I_1^2 R_{01}$$

$$= I_1^2 R_1 + I_2^2 R_2$$

Primary i/p $= V_1 I_1 \cos\phi_1$

$$\eta = \frac{V_1 I_1 \cos\phi_1 - \text{Losses}}{V_1 I_1 \cos\phi_1}$$

$$= \frac{V_1 I_1 \cos\phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos\phi_1}$$

$$= 1 - \frac{I_1 R_{01}}{V_1 \cos\phi_1} - \frac{W_i}{V_1 I_1 \cos\phi_1}$$

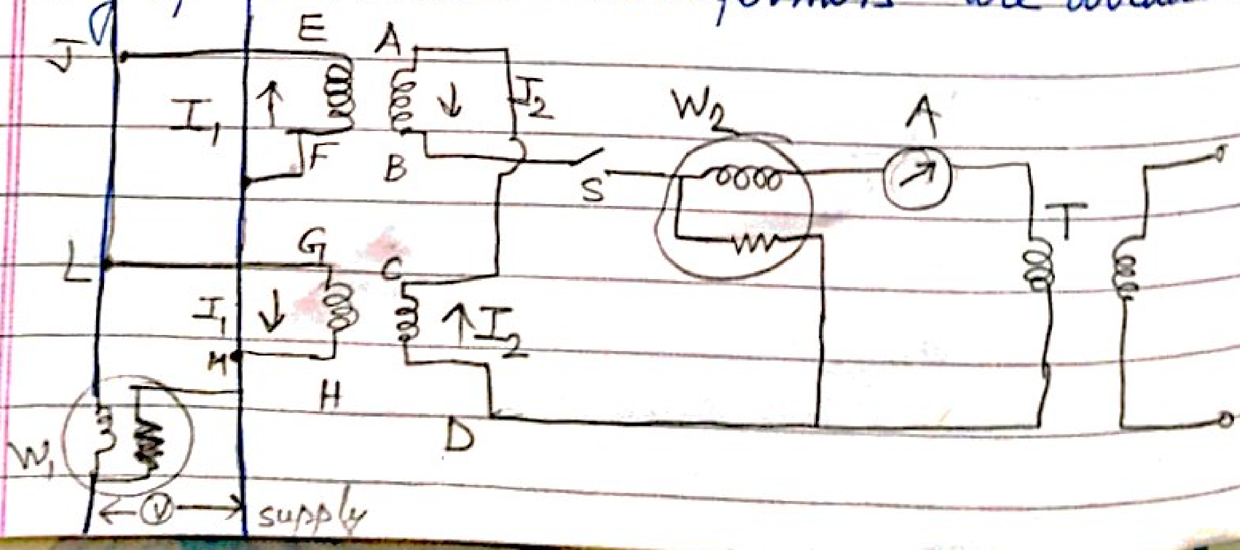
- Differentiating w.r.t I_1, $\frac{d\eta}{dI_1} = 0 \ni W_i = I_1^2 R_{01}$.

So, for max η, Cu loss = Iron loss

↳ o/p current corresponding to η_{max}, $I_2 = \sqrt{\frac{W_i}{R_{02}}}$

* SUMPNER'S TEST or Back to Back Test

- Sumpner's test for regul^n, η, heating under load only if 2 identical transformers are available.



- Primaries of transformers are connected in parallel across the same AC supply.
- With switch S open, W_1 = core loss for 2 transformers

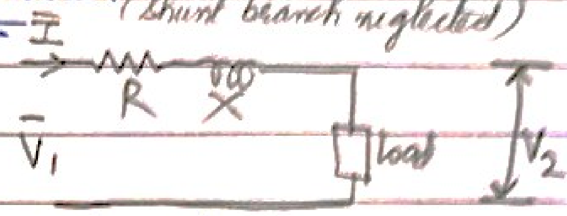
Secondaries of 2 transformers so connected that their potentials are in ϕ (→phase) oppos^n to each other (i.e., If $V_{AB} = V_{CD}$ & A is joined to C while B is to D.

- T is auxiliary low voltage transformer, which is used to circulate full load current (or any desired current), in secondary circuit after closing switch S.
- I_2 flows from D to C & then from A to B.
- W_2 reads full load Cu loss (or Cu loss at I_2)
- Net power i/p = $2P_0 + 2P_c$ = only losses.

✡ VOLTAGE REGULATION

(Shunt branch neglected)

(Fig 1 : used previously)



Approx. voltage drop ; $V_1 = OA = OD$

$$V_1 - V_2 = BD = BF + FD = BF + HG$$
$$= IR\cos\theta + IX\sin\theta$$
$$= I(R\cos\theta + X\sin\theta) \; ; \theta \text{ lagging}$$
$$= I(R\cos\theta - X\sin\theta) \; ; \theta \text{ leading}$$

$\Rightarrow$ Voltage regul^n $= \dfrac{I(R\cos\theta \pm X\sin\theta)}{V_2} \times 100$.

$$\left(= \frac{\text{Secondary vol. on no load} - \text{Secondary vol. on full load}}{\text{Secondary vol. on full load}} \times 100 \right)$$

- For max. regul^n; $\frac{d}{d\theta}(\text{Reg}) = 0 = -R\sin\theta + X\cos\theta$

So, $\tan\theta = \frac{X}{R}$.

Power factor

$Pf = \cos\theta = \frac{R}{\sqrt{R^2+X^2}}$; lagging

IIly, $Pf = \cos\theta = \frac{X}{\sqrt{R^2+X^2}}$; leading.

- All inductive loads : lagging power factor
  capacitive loads : leading power factor.
  resistive loads : Unity powerfactor (UPF).

Q. A 50 kVA, 1100/220 V, 50 Hz, 1 phase (ϕ) transformer has $R_1 = 0.125\,\Omega$, $X_1 = 0.625\,\Omega$, $R_2 = 0.005\,\Omega$, $X_2 = 0.025\,\Omega$.

Find equivalent circuit w.r.t HV side
LV sides

Core loss = 580 W

Find η at (i) full load unity power factor
(ii) $\frac{3}{4}$th full load, 0.8 power factor lagging

$a = \frac{1100}{220} = 5$

w.r.t HV

R_{01} = Equivalent resistance at primary $= R_1 + R_2'$
$= R_1 + a^2 R_2$
$= 0.25$

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w.r.t HV $\{ X_{01} = X_1 + a^2 X_2 = 1.25$

w.r.t LV $\begin{cases} R_{02} = R_2 + \frac{1}{a^2} R_1 = 0.01 \\ X_{02} = X_2 + \frac{1}{a^2} X_1 = 0.05 \end{cases}$

(i) Full load current (Secondary) $I_2 = \frac{50000 \text{ VA}}{220 \text{ V}}$

$= 227.27 \text{ A}$

Full load Cu loss $= I_2^2 R_{02}$

$= 516.516529$

Total loss = Iron loss + Full load Cu loss

Full load o/p = Full load KVA × power factor

$= 50,000 \times 1$

$= 50,000 \text{ W}$

(a) $\eta = \frac{o/p}{o/p + \text{losses}}$

$= \frac{50,000}{50,000 + 580 + 516.5} = 0.9785402$

$= 97.854\%$

(b) f.l. o/p = f.l kVA × pf

$= \frac{3}{4}(50,000) \times 0.8$

$= (40,000)\frac{3}{4} = 30000$

516.5 × x =

$\eta = \frac{30000}{30000 + 580 + \boxed{516.5 \times \left(\frac{3}{4}\right)^2}} \times 100$

$= 0.971800$

$= 97.18\%$

→ Cu loss changes

→ Iron loss doesn't change

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Q, The above transformer is fully loaded on secondary side at (a) 0.8 power factor lagging
(b) 0.8 pf leading
(c) Unity pf
while it is fed on primary side at 1100 V
Calculate %age regulⁿ. & voltage at secondary terminals for above condⁿ

(a) Total voltage drop at secondary at fl at 0.8 pf lag

$$= I_2 (R_{02} \cos\phi + X_{02} \sin\phi)$$

$$= 227.27[(0.01 \times 0.8) + (0.05 \times 0.6)]$$

$$= 8.636 \text{ V}$$

$$\text{\%age regul}^n = \frac{8.63}{220} \times 100 = 3.925\%$$

Secondary voltage at fl at 0.8 pf lag
= 211.37

(b) Leading

$$\text{Total} = I_2 (R_{02} \cos\phi - X_{02} \sin\phi)$$

$$= -4.99994 \text{ V}$$

$$\text{\%age regul}^n = -2.2727\%$$

Secondary voltage at f.l at 0.8 pf lead
$= 220 - (-4.99)$
$= 224.9999$ V

(c) Unity

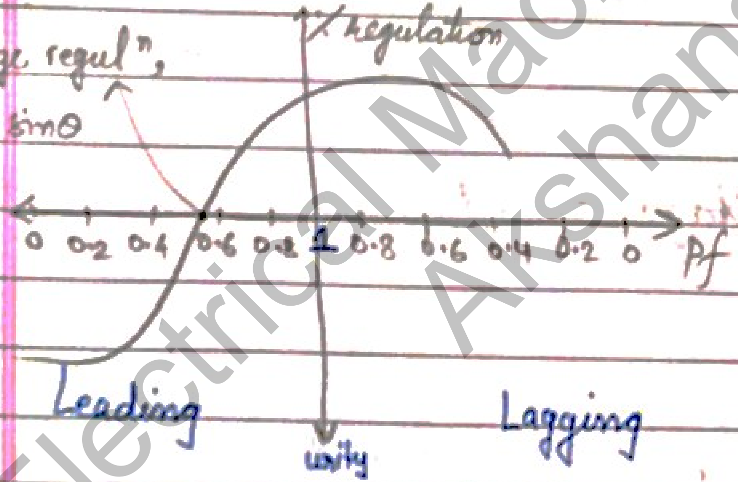
$$\text{Total} = I_2 (R_{02}(1) + X_{02}(0))$$
$$= 2.2727$$

%age regul^n = 1.03%

Secondary Voltage at fl. at unity pf
$$= 220 - 2.2727$$
$$= 222.2727 \text{ V} \checkmark$$

Pf	% regulation	
Lagging	+ve	✓ always true
Leading	−ve	* may/may not be true
upf	+ve	✓ always true

Leading
For 0 voltage regul^n,
$IR\cos\theta = IX\sin\theta$
$\Rightarrow \tan\theta = \frac{R}{X}$



Q. 9.11 The following test results were obtained on 20 kVA, 2200/220 V transformer

OC Test (LV) : 220 V, 1.1 A, 125 W → W_i

SC Test (HV) : 52.7 V, 8.4 A, 287 W

(a) upf load on secondary, with $V_2 = 220$ V, η_{max} = ? load

(b) Transformer is fully loaded. Determine pf for zero voltage regul^n. → leading $\tan^{-1}\left(\frac{R}{X}\right) = \theta$

* OC Test gives Iron loss.
* SC Test gives full load Cu loss.

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$a = \frac{2200}{220} = 10$

$R_{01} = 287/(8.4)^2 = 4.0674\ \Omega$

$X_{01} = \sqrt{Z^2 - R^2} = 4.775645\ \Omega$

$Z_{01} = \frac{52.7}{8.4} = 6.273\ \Omega$

For max η; $I_1^2 R_{01}$ = iron loss = 125 W (W_i)

$\Rightarrow I_1 = \sqrt{\frac{125}{4.0674}}$

$\Rightarrow I_1 = 5.5436$ A

Current corresponding to η_{max}

O/P $= VI\cos\phi$

$= (2200)(5.5436)(1)$

$= 12188$ W

$\eta = \frac{O/P}{O/P + \text{losses}}$

losses → Core loss + Iron loss (125, 125) — Same as Iron loss

$=$

$= 0.9799$

$\Rightarrow \eta_{max} = 97.99\%$

SC Test Rated current: $\frac{20KVA}{2200} = \frac{20{,}000}{2200} = 9.09$ A ($\neq 8.4$ A)

8.4 A → 287

9.09 → ?

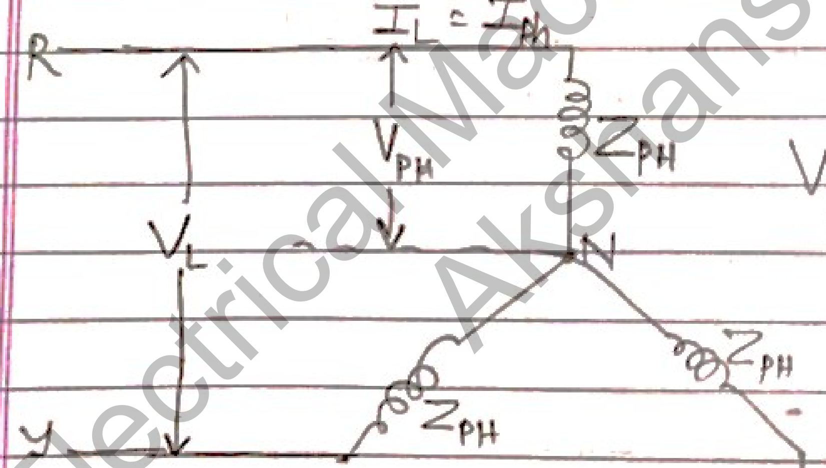
Full load Cu loss = $\frac{287}{(8.4)^2} \times (9.09)^2$ ($P \propto I^2$)

THREE-PHASE TRANSFORMERS

- Y : Wye, star
- Δ : Delta.

3 ϕ (phase) transformer : A set of 3 similar single phase transformers may be connected to form a 3 phase transformer.

- In a Balanced 3 phase, * $\boxed{\text{star connected system}}$, phase Voltage $(V_{ph}) = \dfrac{\text{line voltage } (V_L)}{\sqrt{3}}$

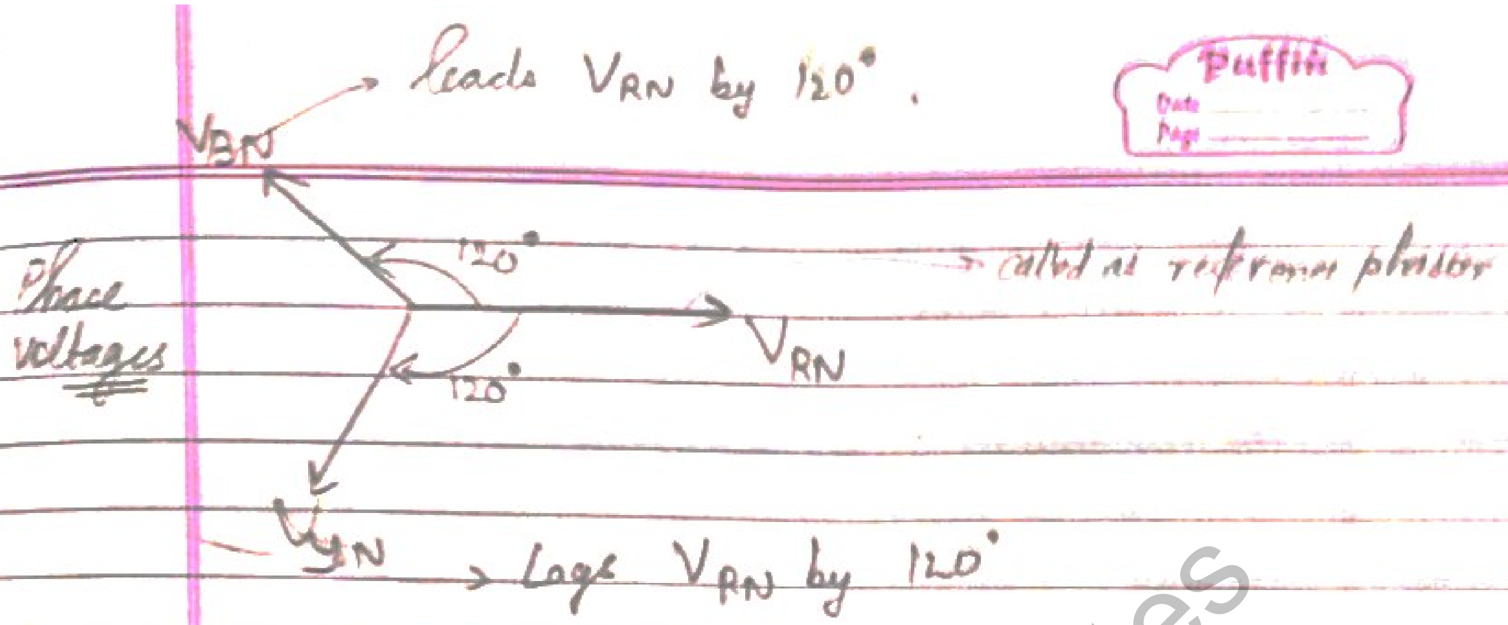


$$V_{Ph} = \frac{V_L}{\sqrt{3}}$$

And line current (I_L) = Phase current (I_{PH})

- Above, ∃ 3 line voltages $V_{(R \text{ to } Y)}, V_{(Y \text{ to } B)}, V_{(R \text{ to } B)}$
- Line voltage : Also called Line to line voltage
- Phase impedance : Also called per phase impedance.
- Above, ∃ 3 phase voltages: $\underbrace{V_{RN}, V_{YN}, V_{BN}}_{\text{or, can be said}}$

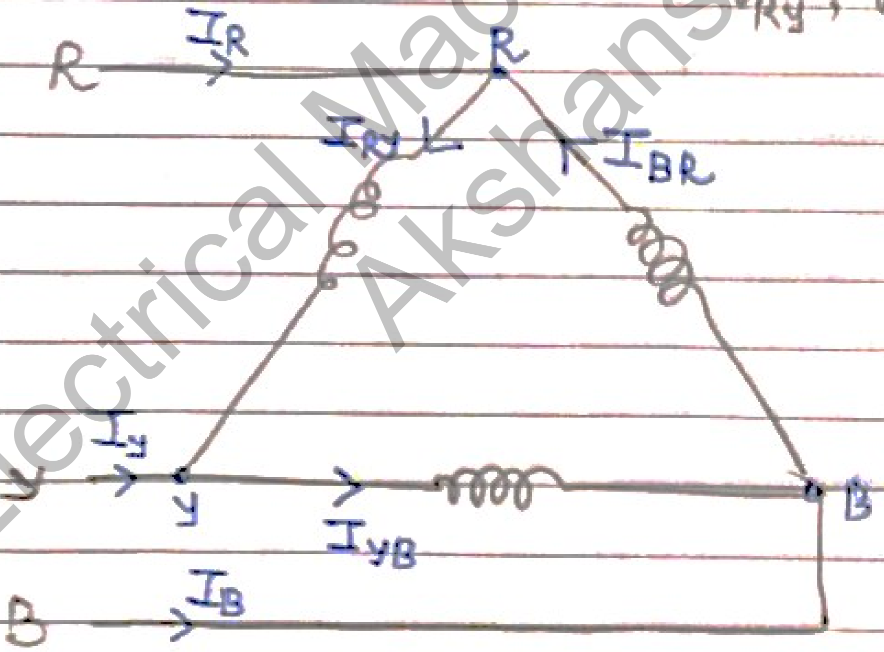
V_R, V_y, V_B



* Line voltages also have a phase shift of 120° between them.

☆ Delta Connected Sys.

∴ ∃ 3 line voltages V_{RY}, V_{YB}, V_{BR}



For delta connection :- $V_L = V_{Ph}$.

$$I_{Ph} = \frac{1}{\sqrt{3}} I_L$$

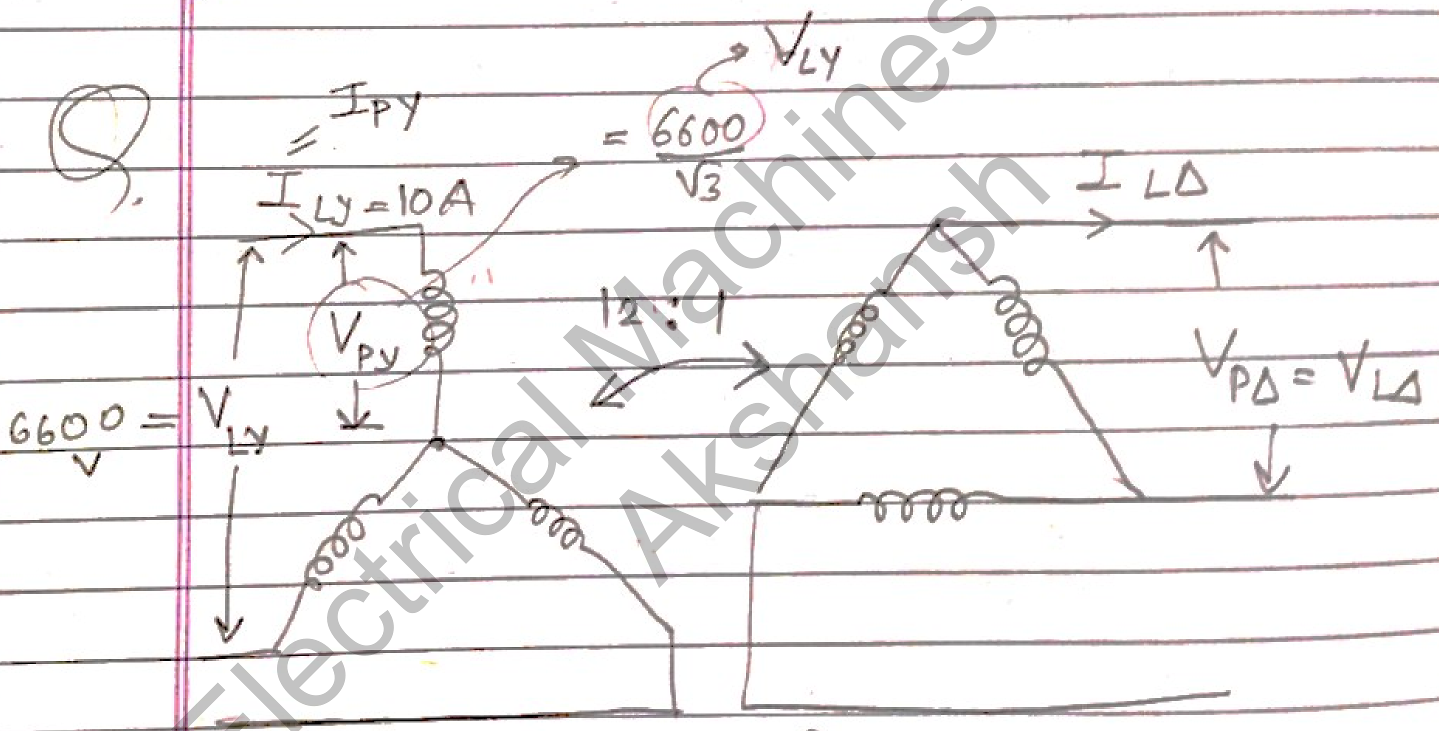
☆ Notation :- Upper case : PRIMARY ; lower case : SECONDAR

- $+30^\circ$ connection :- Wye-Delta connected transformer. Phase shift of 30° b/w V_{AN} & V_{ab}

- -30° connection :- Delta - Wye connected transformer. Phase shift of -30° b/w V_{AB} & V_{an}.

Imp ☆ Transfer ratio / transformⁿ ratio is the ratio b/w the phase values.

Q.



Find $V_{L\Delta} = ?$ $I_{L\Delta} = ?$

o/p KVA = ?

$$\left(\begin{aligned} &= \sqrt{3}\, V_{L\Delta} I_{L\Delta} \\ &= 3\, V_{P\Delta} I_{P\Delta} \end{aligned} \ \text{: formula} \right)$$

P: Phase
L: line
Y: Wye = Star
Δ: Delta.

Transformation Ratio is :-

$$\frac{\text{Primary} \leftarrow V_{pY}}{\text{Secondary} \leftarrow V_{p\Delta}} = \frac{I_{p\Delta}}{I_{pY}}$$

Here, $\frac{V_{pY}}{V_{p\Delta}} = 12$; $\frac{I_{p\Delta}}{I_{pY}(=I_{LY})} = 12$

Also, $V_{pY} = \frac{V_{LY}}{\sqrt{3}} = \frac{6600}{\sqrt{3}} = 3810.62$ V

So, $V_{p\Delta} = V_{L\Delta}$, $I_{p\Delta} = \frac{I_{L\Delta}}{\sqrt{3}}$

$12 \times I_{pY} = 12 \times 10$

$\Rightarrow I_{L\Delta} = \sqrt{3} \times 10 \times 12$ A $= 207.84$

$\frac{V_{pY}}{V_{L\Delta}} = 12 \Rightarrow V_{L\Delta} = \frac{V_{pY}}{12} = \frac{V_{LY}}{\sqrt{3} \times 12} = \frac{6600}{12 \times \sqrt{3}}$

$= 317.55$ V

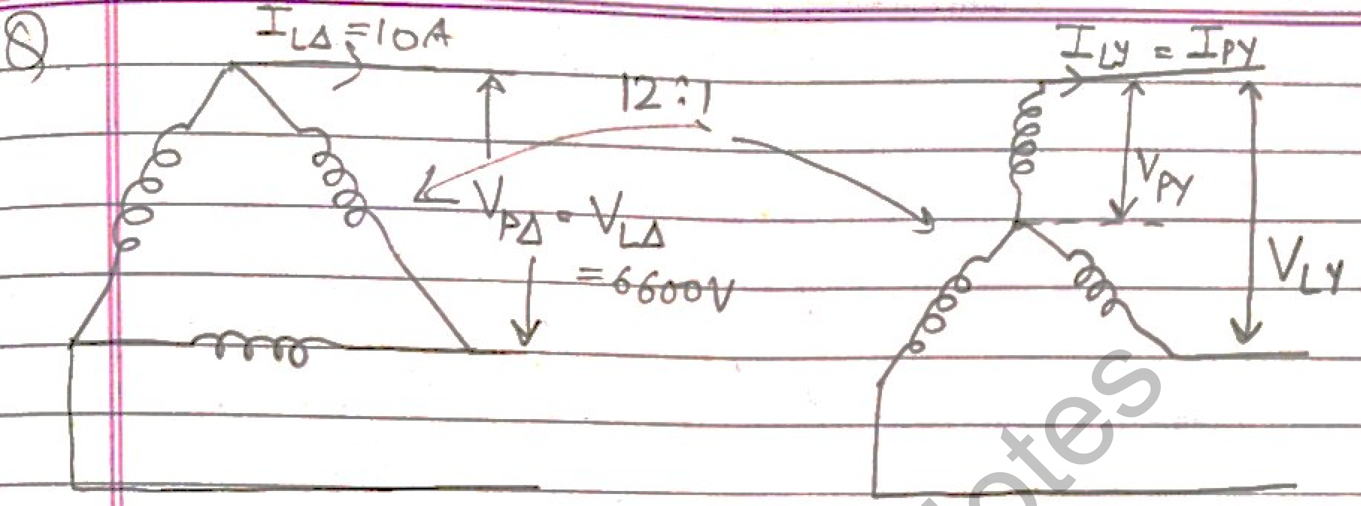
O/P KVA $= \sqrt{3} \times \frac{6600}{\cancel{12} \times \cancel{\sqrt{3}}} \times \cancel{\sqrt{3}} \times 10 \times \cancel{12}$ KVA

$= 114315.35$ VA

$= 114.315$ KVA

Ans

8)



Find :- $V_{LY} = ?$, $I_{LY} = ?$, o/p KVA = ?

$\left(= \sqrt{3}\, V_{L\Delta} I_{L\Delta} = 3\, V_{P\Delta} I_{P\Delta} \right)$

Transform^n ratio $= 12 = \dfrac{V_{P\Delta}}{V_{PY}} = \dfrac{I_{PY}}{I_{P\Delta}}$

$$V_{PY} = \frac{V_{P\Delta}}{12} = \frac{6600}{12}$$

✓ $V_{LY} = \sqrt{3}\, V_{PY} = \dfrac{\sqrt{3}}{12} \times 6600 = 952.62\,V$

$$12 = \frac{I_{PY}}{I_{P\Delta}} = \frac{I_{LY}}{I_{P\Delta}}$$

$\Rightarrow I_{LY} = 12 \times I_{P\Delta}$

$I_{L\Delta} = \sqrt{3}\, I_{P\Delta}$

$\Rightarrow I_{P\Delta} = \dfrac{10}{\sqrt{3}}$

✓ $\Rightarrow I_{LY} = \dfrac{12 \times 10}{\sqrt{3}} = 69.284\ A$

✓ o/p KVA $= \sqrt{3} \times 6600 \times 10 = 114312\ VA$

$= 114.312\ KVA$

Q. The HV terminals of a 3 phase bank of 3 single phase transformers are connected to a 3 wire, 3 phase, 11 KV line-to-line sys.

The LV terminals are connected to a 3 wire 3 phase load, rated of 1000 KVA & 2200 V line-to-line. Specify the voltage, current & KVA ratings of each transformer (both HV & LV windings for following cond^ns :-

(a) $HV \to Y$; $LV \to \Delta$

(b) $HV \to \Delta$; $LV \to Y$

(c) $HV \to Y$; $LV \to Y$

(d) $HV \to \Delta$; $LV \to \Delta$

Given :-

$V_{HV} = 11\,KV$, $V_{LV} = 2200\,V$,

For Y side :- $\frac{V_{LY}}{\sqrt{3}} = V_{PY}$.

For Δ side :- $V_{L\Delta} = V_{P\Delta}$

(a) Transform^n ratio $= \frac{11/\sqrt{3}}{2.2} \left(\frac{V_{PY}}{V_{P\Delta}} \right) = \frac{I_{P\Delta}}{I_{PY}}$

$V_{PY} = 11/\sqrt{3}\,KV$	$V_{P\Delta} = 2.2\,KV$
$V_{LY} = 11\,KV$	$V_{L\Delta} = 2.2\,KV$
$I_{PY} =$ } equal	$I_{P\Delta} = 454.54$
$I_{LY} =$ } 157.454	$I_{L\Delta} = 262.439$

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Transfer ratio $= \frac{I_{P\Delta}}{I_{PY}} = \frac{11}{\sqrt{3}\times 2.2}$

O/P KVA $= \sqrt{3} \times V_{L\Delta} \times I_{L\Delta}$

$\Rightarrow 1000 = \sqrt{3} \times 2.2 \times I_{L\Delta}$

$\Rightarrow I_{L\Delta} = \frac{1000}{\sqrt{3}\times 2.2} = 262.439$ A

$I_{P\Delta} = \sqrt{3}\, I_{L\Delta} = \frac{1000}{2.2}$ A

$\Rightarrow I_{PY} = \frac{\sqrt{3}\times 2.2}{11} \times I_{P\Delta}$

$\Rightarrow I_{PY} = \frac{\sqrt{3}\times \cancel{2.2}}{11} \times \frac{1000}{\cancel{2.2}} =$

(b) Transform ratio $= \frac{V_{P\Delta}}{V_{PY}} = \frac{I_{PY}}{I_{P\Delta}}$

$= \frac{11}{2.2/\sqrt{3}} = \frac{I_{PY}}{I_{P\Delta}} \rightarrow (1)$.

V_{PY}	$V_{P\Delta}$
V_{LY}	$V_{L\Delta}$
I_{PY}	$I_{P\Delta}$
I_{LY}	$I_{L\Delta}$

TOPICS OF SELF-STUDY

- Parallel operation of transformer
  - ↳ The 4 cond^{ns} to be satisfied.
- SCOTT connection — 3 phase to 2 phase conversion.

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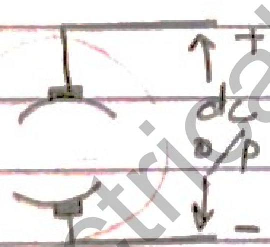
Chapter - 7

DC Machines

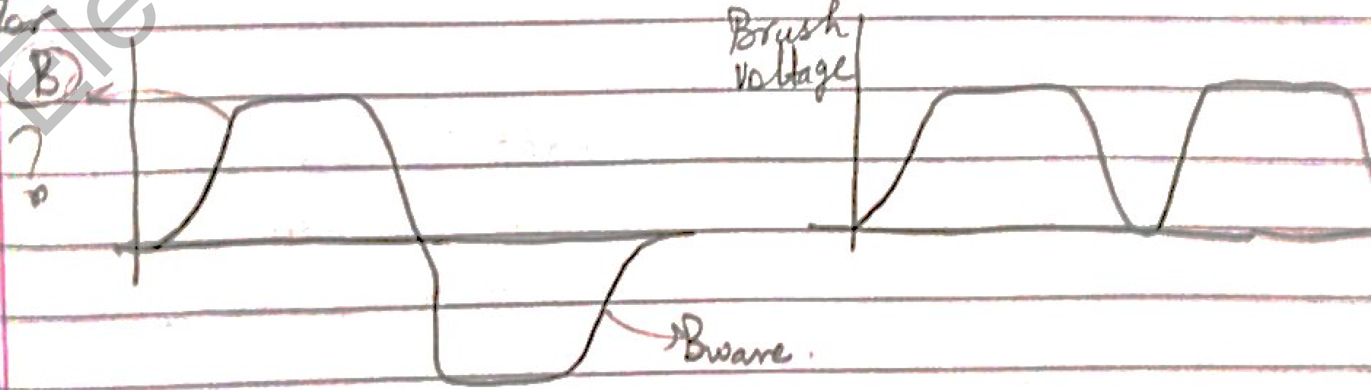
* Even in DC machine, whatever is there inside a conductor, is alternating in nature. (→ flux)
* Electric traction = electric train.

* DC machine symbol:-



• voltage/current becomes unidirectional as the rings of commutator get exchanged b/w brushes while motion.

Commutator



Date 30.9.12

Chapter - 10

★ EMEC : Electromechanical Energy Conversion.

★ Armature windings

Lap winding

No. of || paths (A) = no. of poles (P)

(armature winding is divided into n || paths)

Wave winding

No. of || paths (A) = 2 (always)

★ Conductor current $(I_c) = \frac{I_a}{A}$ → armature

★ Armature emf $(E_a) = \frac{\phi \omega_m Z}{2\pi} \left(\frac{P}{A}\right)$

$= k_a \phi \omega_m$.

ϕ : flux per pole (Wb)

ω_m : armature speed (rad/s)

$= \frac{2\pi n}{60}$, where n is speed in rpm.

Z : total armature conductors.

A : no. of || paths.

$K_a = \frac{ZP}{2\pi A}$.

or, $\boxed{E_a = \frac{\phi n Z}{60}\left(\frac{P}{A}\right)}$: EMF eq^n

$\rightarrow E_a \propto \phi\,\omega_m$
$\propto \phi\, n.$

Torque eq^n : $\boxed{\tau = \frac{1}{2\pi}\phi I_a Z\left(\frac{P}{P}\right)}$ N-m

$= \boxed{k_a \phi I_a}$

$\rightarrow \tau \propto \phi I_a.$

ays)

☆ Power balance:-

Mechanical power $\tau\omega_m = \underbrace{k_a \phi \omega_m}I_a$

$= E_a I_a$

$=$ Electrical Power.

☆ Energy conserv^n principle :-

Electrical & mechanical ~~pow~~ powers must balance in a machine.

$$\tau = \left(\frac{1}{\omega_m}\right)E_a I_a \text{ N-m.}$$

☆ Linear Magnetisation

If magnetic circuit of machine is assumed linear, $\phi \propto I_f \Rightarrow \phi = k_f I_f$; I_f : field current

$$E_a = k_a\phi\omega_m = k_a k_f I_f \omega_m = k_a k_f I_f\left(\frac{2\pi n}{60}\right)$$
$$= k_e I_f n \;;\text{ where } k_e = k_a k_f\left(\frac{2\pi}{60}\right)$$

specifically meant for motors.

Also, $\tau = k_a \phi I_a$

$= k_a k_f I_f I_a = k_t I_f I_a$

$\hookrightarrow k_t = k_a k_f$

Q. A 6 pole DC machine armature has 36 slots, 2 coil - sides / slot, 8 turns / coil & is lap wound. Flux/pole = 0.01 Wb.

Find: Gross torque & mechanical power o/p

Given: when machine is operating as a motor at 1200 rpm with $I_a = 10$ A.

$P = 6$ given

$\Rightarrow A = 6$ (lap wound)

$\Rightarrow \frac{P}{A} = 1$

No. of armature conductor, $Z = 36 \text{ slots} \times \frac{2 \text{ coil sides}}{\text{slot}} \times \frac{8 \text{ turns}}{\text{coil}}$

$= 576$

E_a, induced emf $= \frac{\phi n Z}{60}\left(\frac{P}{A}\right)$

$= \frac{0.01 \times 1200 \times 576}{60}$

$\Rightarrow E_a = 115.2$ V

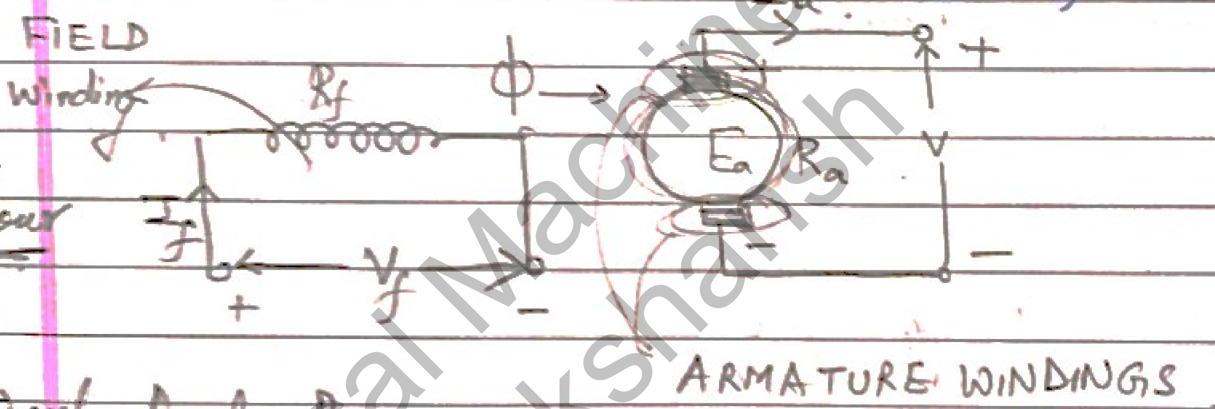
Gross Mechanical power $= E_a I_a$
$= 1152\ W$
$= 1.15\ KW$

Gross Torque (N-m) $= \dfrac{\cancel{1152}^{\cancel{576}\ 288} \times \cancel{60}}{2\pi \times \cancel{1200}_{\cancel{20}} \times 10}$

$\Rightarrow T = 9.1719$ N-m

* Gross Mechanical Power(W) = Gross Torque (N-m) $\times \dfrac{2\pi n}{60}$

DC behaviour



→ only R_f & R_a are shown. No role of X_L ($\because X_L = \omega L$
$= 2\pi f L$
$\Rightarrow X_L = 0$; $f = 0$ for DC)

* Generating mode:
The machine operates in generating mode (puts out electrical power) when I_a is in the dir^n of induced emf E_a ~~as in fig. 7.6(a)~~. For the armature circuit,

V (terminal voltage) $= E_a - I_a R_a$; $E_a > V$.

- The mechanical power converted to electrical form is :

$$P_{mech}(in)\Big|_{net} = E_a I_a = P_{elec}(out)\Big|_{gross}$$

- The net electrical power output is

$$P_o = V I_a$$

$V \to$ terminal voltage

Also, $E_a I_a - V I_a = I_a^2 R_a$ = Armature copper loss.

& $P_{mech}(in)\Big|_{gross}$ = shaft power = $P_{mech}(in)\Big|_{net}$ + Rotational loss

☆ Motoring Mode:

- V (terminal voltage) $= E_a + I_a R_a$; $V > E_a$.
- Electrical Power converted to mechanical form is

$$P_{elec}(in)\Big|_{net} = E_a I_a = P_{mech}(out)\Big|_{gross}.$$

- Electrical power i/p is $P_i = V I_a$.

Also, $V I_a - E_a I_a = I_a^2 R_a$ = armature copper loss

& $P_{mech}(out)\Big|_{net}$ = shaft power = $P_{mech}(out)\Big|_{gross}$ − Rotational loss.

* electrical load → eg: resistance.

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- In this mode, torque (τ) of electromagnetic origin is in the dir^n of armature rot^n ie mechanical power is put out and is absorbed by load (mechanical) → eg: a generator, steam turbine

(Q. 7.2) $E_a \propto n\phi$

or $n \propto \frac{E_a}{\phi}$

$I_a = \frac{P}{V}$

$(E_a)_{gen} = V + I_a R_a$

$(E_a)_{motor} = V - I_a R_a$

$\phi_{motor} = \left(1 + \frac{10}{100}\right)\phi_{generator}$.

$$\frac{n_{gen}}{n_{motor}} = \frac{(E_a)_{gen}}{(E_a)_{motor}} \times \frac{\phi_{motor}}{\phi_{gen}}.$$

* PM : Prime mover

Ex A DC shunt generator driven by a belt from an engine runs at 750 rpm while feeding 100 kW of electric power into 230 V mains. When the belt breaks, it continues to run as a motor drawing 9 kW from the mains. At what speed would it run?
Given - Armature resistance is 0.08 Ω & field resistance is 115 Ω.

Generator

$I_f = \frac{V}{R}$ ($V \to 230$, $R \to 115$) : Remains const. in oper^n change-over → has to be assumed in any problem.

$I_L = \frac{P}{V}$ ($P \to 10$ kW, $V \to 230$)

$I_a = I_L + I_f$ (KCL)

$E_a(g) = V + I_a R_a$ ✓ ($R_a \to 0.08$)

$n_a(g) = 750$ rpm → Given.

Motor

$I_L = \frac{P}{V}$ ($P \to 9$ kW, $V \to 230$).

I_f (same as above $= \frac{V}{R}$)

$I_a = I_L - I_f$ (KCL)

$E_a(m) = V - I_a R_a$.

Note :- Field current (I_f) didn't change during this oper^n (of both kinds). So,
E_a (emf induced) $\propto$ n_a (armature speed).
(Actually, $E_a \propto n\phi$ But ϕ is const ($\because I_f$ is const) So, in all, $E_a \propto n_a$)

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So, $\frac{E_a(g)}{E_a(m)} = \frac{n_a(g)}{n_a(m)}$

$\Rightarrow n_a(m) = [n_a(g)] \frac{E_a(m)}{E_a(g)}$

$= 750 \times \frac{229.3}{237.8}$

$n_a(m) = \cancel{73}\ 723.1917$ rpm

★ The axis in which brushes are located is called Quadrature axis (q-axis)
axis ⊥ to brushes, its called Direct axis (d-axis)

★ MNA : Magnetic Neutral Axis (Also called zero flux pt.)
★ Compensating winding (kept at ~~wave pole~~ brush axis) are put to prevent armature rxn.

Q. A separately excited DC generator has $R_a = 0.1\Omega$, total brush drop = 2V. $N_1 = 1000$ rpm $I_{L1} = 100A$ at 250V. If $N_2 = 700$ rpm, with I_f unchanged find current delivered to load.

$R_L = \frac{V}{I}$, $E_g = V + I_{a_1}R_a$ + Brush drop

$E_{g_1} \propto I_f N_1 \Rightarrow \frac{E_{g_1}}{E_{g_2}} = \frac{N_1}{N_2} \rightarrow E_{g_2}$ ✓

$E_{g_2} = V + I_{a_2} R_a$ + Brush drop $\rightarrow I_{a_2}$?
?

Q. DC shunt motor
$V = 220\,V$
$N_1 = 500$ rpm , $R_a = 0.2\,\Omega$
$I_{a_1} = 50A$.
T is doubled.
Torque
Find : new N_2.

$$T \propto \phi I_a$$
$$\propto I_a \quad (\phi : const)$$
$$\Rightarrow \frac{T_1}{T_2} = \frac{I_{a1}}{I_{a2}}$$
$2T_1$
$$\Rightarrow \frac{1}{2} = \frac{I_{a_1}}{I_{a_2}} \Rightarrow I_{a_2} = 2I_{a_1} = 100A$$

Now, $E_a \propto N$
$$\Rightarrow \frac{E_{a_1}}{E_{a_2}} = \frac{N_1 \rightarrow 500}{N_2 \rightarrow ?}$$
$V - I_{a_1} R_a$
$V - I_{a_2} R_a$

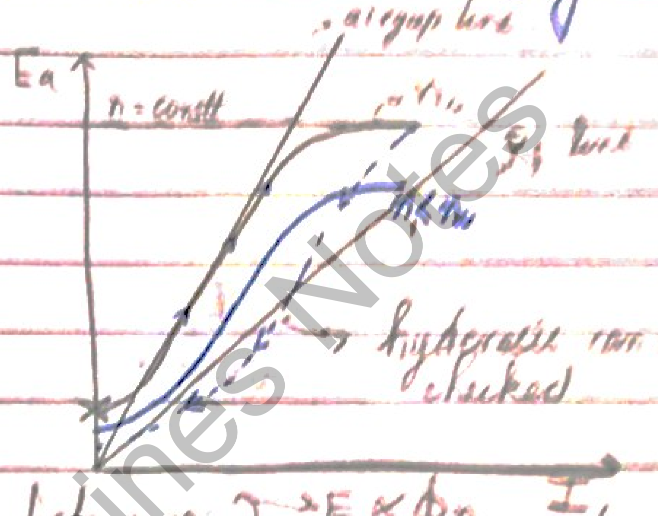
Ans

Q. A shunt generator delivers 50 kW, 250 V, 400 rpm
$R_a = 0.02\,\Omega$, $R_f = 50\,\Omega$
What is speed of machine if it runs as a shunt motor, taking 50 kW i/p at 250 V.
$I_L \,(P/V)$

Generator : $I_a = I_L + I_f$; $E_a = V + I_a R_a$
Motor : $I_a = I_L - I_f$; $E_a = V - I_a R_a$
$E_a \propto I_f N$

★ NO-LOAD Saturation Characteristics or Open Circuit Characteristics (OCC) or Magnetiz^n Characteristics — of DC generator

* $E_a \propto n\phi$



n_1 has lesser value of n (comes below n_2) $\rightarrow E \propto \phi n$

We can reduce n_2 only till the airgap line doesn't coincide with R_f line. So, the min. value of n_2 that can come is : CRITICAL SPEED

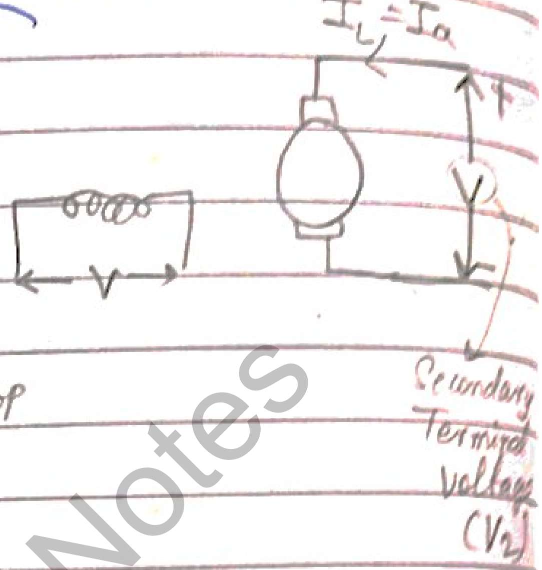
★ Cond^{ns} for build-up of voltage of generator :

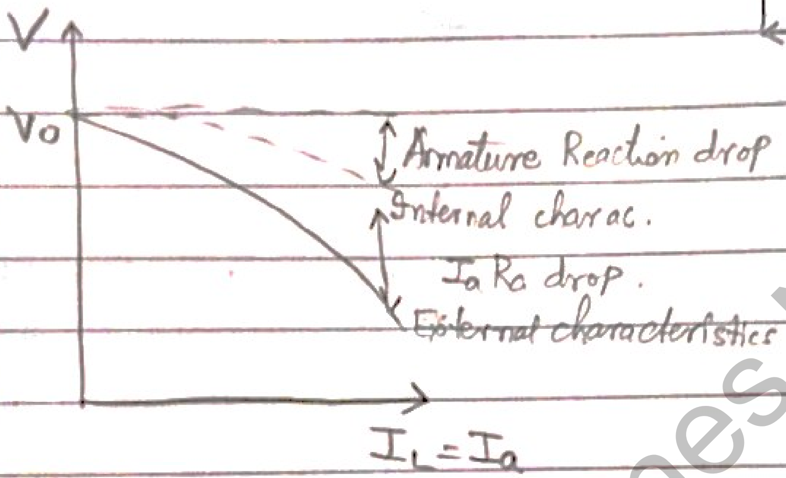
1. Residual magnetism must be present in the poles.
2. For given direction of rotation, shunt field coils should be so connected that E_a reinforces emf produced initially due to residual magnetism.
3. R_f < Critical resistance.

Imp example 7.8

Characteristics of DC Generator

Separately excited

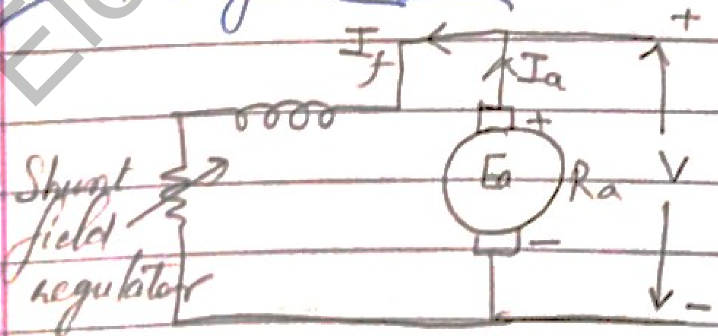




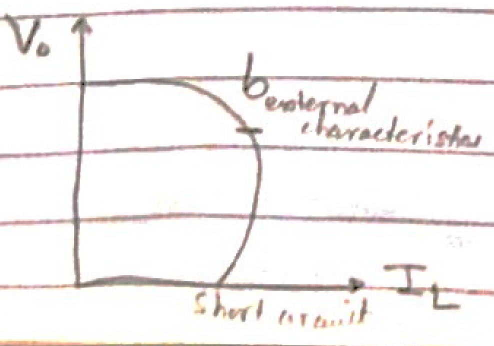
* Rated speed – I_f adjusted to give rated voltage at no load (V_0).

$V = E_a - I_a R_a$ — with no load, demagnetising effect of the armature ↓ ϕ/pole due to which E_a ↓.

Shunt generator (Self excited generator)



Due to $I_a R_a$ drop & armature reaction, V↓ more rapidly because V↓, I_f↓ & ϕ↓, E_a↓.

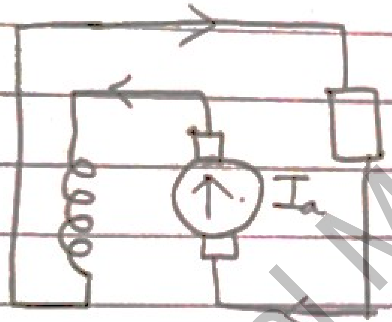


b : breakdown pt.

* I_L : Load current : in generator
Line current : in motor

For currents >>> rated current, beyond pt. b — due to severe armature rxn & $\uparrow_{ed}$ $I_a R_a$ drop drastic $\downarrow$ in V than $\uparrow$ in I_L
As I_L $\uparrow$ further, generator is short circuit (V=0), though there is some E_a due to residual magnetism.

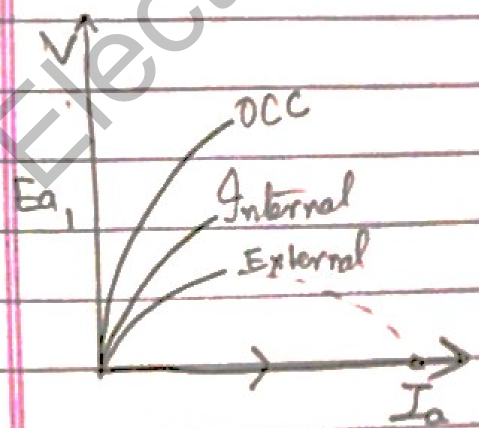
✯ SERIES generator



Field winding in series with armature & field carries I_a.
As load $\uparrow$, $I_a \uparrow$, $\phi \uparrow$, $E_a \uparrow$.

Rising voltage charac.
– At high loads, armature rxn & $I_a R_a$ drop high, term. vol $\downarrow$

$I_L = I_a = I_f$



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Characteristics of DC Motor

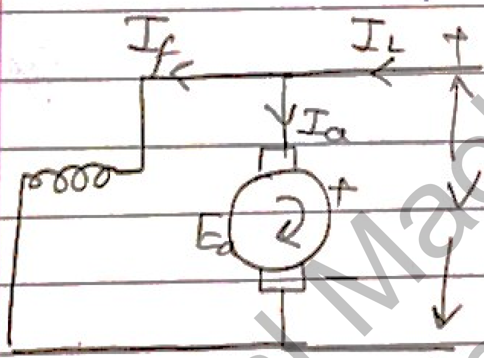
Motor : Imp. parameters : τ (torque), speed (n)
Generator : " : o/p power & Terminal voltage

$\tau = K_a \phi I_a$: Generator
$\tau = K_T \phi I_a$: Motor.

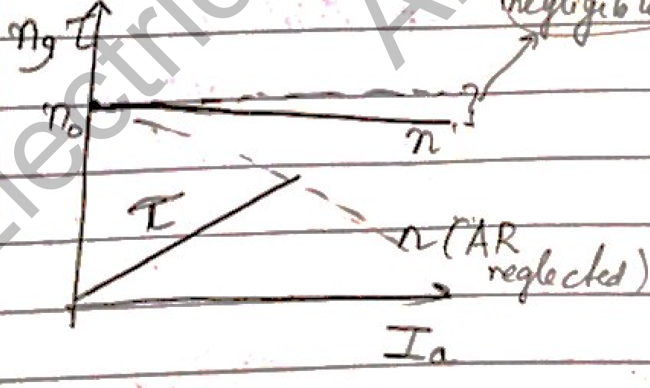
- $E_a = k_a \phi \omega$.
- $n = k_N \frac{E_a}{\phi}$.

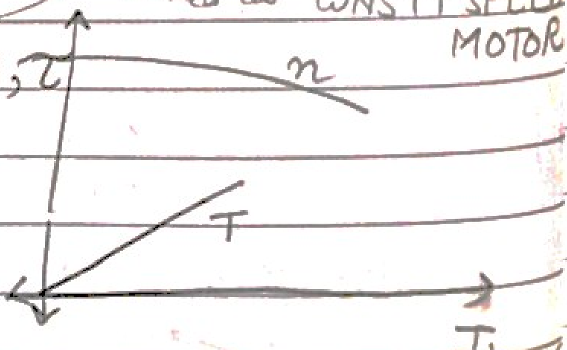
~~Shu~~ Shunt motor

- $E_a = V - I_a R_a$
- $n = k_N \frac{(V - I_a R_a)}{\phi}$
- $\tau = k_T \phi I_a$



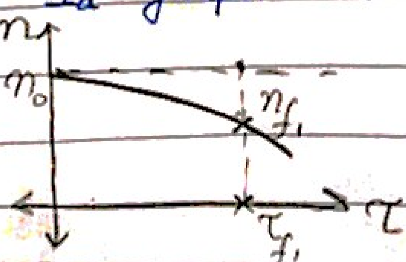
decrease very negligible ⇒ Taken as having constt speed ↓ called as CONSTT SPEED MOTOR





- Decrease in ϕ due to armature reaction in Shunt motor (constt I_f)
  ↳ compensates for ↓ in n with E_a (graph).

$\tau - I_a$ graph : st. line without AR (Armature rxn)



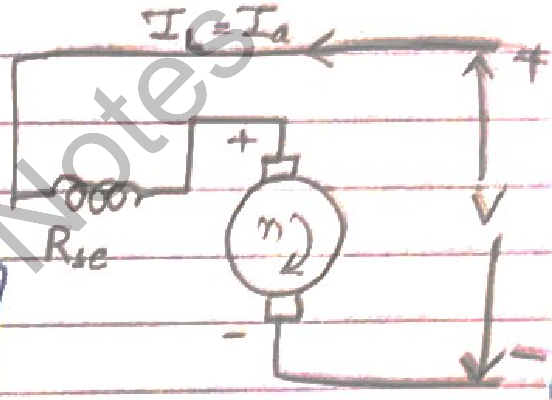
- Constt. speed characteristics.

$$n = \underbrace{\frac{k_N}{\phi}}_{n_0} - \left[\frac{K_n R_a}{k_T \phi^2}\right] \tau ; \quad \phi \propto I_f$$

§ Series motor

$$n = k_N \left(\frac{V - I_a (R_a + R_{se})}{\phi} \right) \longrightarrow (1)$$

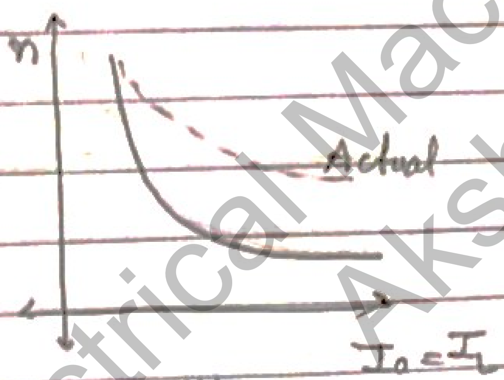
$$T = k_T \phi I_a$$

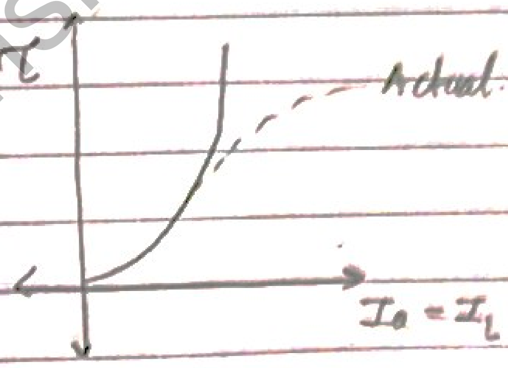


$$\phi = k_f I_f = k_f I_a$$

$$n = \frac{k_N}{k_f} \left(\frac{V}{I_a} - (R_a + R_{se}) \right)$$

(from (1))





$\Downarrow$

$n = \infty$ when $I_L = 0$,

So, for no load, $n = \infty$

$\Downarrow$

don't run series motor at no load ($I_L = 0$)

$\because$ it can be damaged.

In this case

$\phi \propto I_a$

& $T \propto \phi I_a$

$\Rightarrow \boxed{T \propto I_a^2}$

$\Downarrow$

- suitable for trains etc.
- for starting a large accelerating torque.

- Railway / Traction motor : applicⁿ of DC series motor

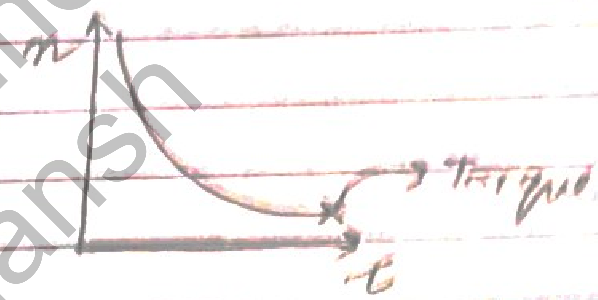
- From $n - I_a$ eq^n, $I_a \to 0$, $n \to \infty$.
  At no load, speed $\to \infty$.
  So, series motor should <u>never</u> be run on NO LOAD.

$$n = \frac{k_N}{k_f}\left[\frac{V}{I_a} - (R_a + R_{se})\right]$$

↳ $\tau = k_T k_f I_a^2$

$$\Rightarrow n = \frac{k_N}{k_f}\left[\frac{V\sqrt{k_T k_f}}{\sqrt{\tau}} - (R_a + R_{se})\right]$$

$$\Rightarrow n \propto \frac{1}{\sqrt{\tau}}$$



✯ DC MACHINE
2 STD. FORMULAS

1) emf eq^n

$$E_a = \left(\frac{\underbrace{\phi}_{\text{flux}}\ \underbrace{n}_{\text{speed}}\ \underbrace{Z}_{\text{no. of armature conductors}}}{60}\right)\left(\frac{\underbrace{P}_{\text{no. of poles}}}{\underbrace{A}_{\text{no. of || paths}}}\right)$$

↳ Lap winding ; $P = A$
↳ Wave winding ; $A = 2$

$E_a = V + I_a R_a$ = Generator
$E_a = V - I_a R_a$ = Motor

$$\Rightarrow E_a = \phi \underbrace{\left(\frac{2\pi n}{60}\right)}_{\omega} \underbrace{\left(\frac{ZP}{2\pi A}\right)}_{\text{fixed for a machine} = K_a \text{ (constt)}}$$

ω → angular speed in radians/s

$$\Rightarrow \boxed{E_a = K_a \phi \omega}$$

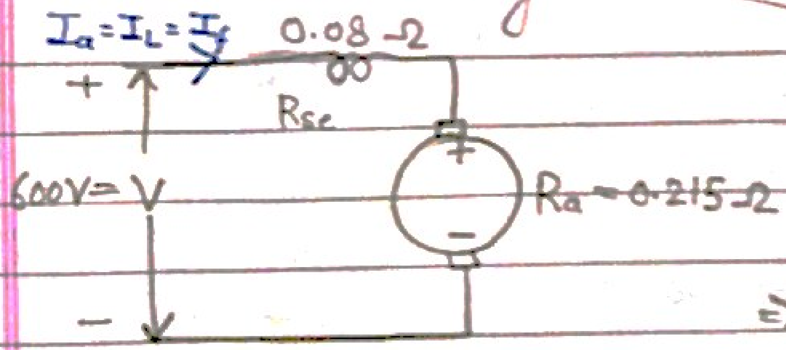
2) Torque eq^n

In any electromechanical device
Mechanical power = Electrical power.

$\omega \times T$ = $E_a I_a$

$$\Rightarrow \boxed{T = K_a \phi I_a} \ \star$$

Q A 60 kW, 600 V series railway motor has $R_a = 0.215\,\Omega$, $R_f = 0.08\,\Omega$. At rated voltage & at a current of 80 A, the speed is 750 rpm. Find the speed & torque when current = 95 A. Assume linear magnetisation characteristics.



$\Rightarrow \phi \propto I_f$

$\Rightarrow \phi = K_f I_f$

$E_a = K_a (K_f I_f) I_a$

$\Rightarrow E_a = K_a K_f I_a I_a$

At rated voltage, $i_a = 80\,A$

$$\boxed{E_{a_1} = V - I_{a_1}(R_a + R_{se})}$$

this additional factor comes in case of DC series motor

$$= 600 - 80(0.08 + 0.215)$$
$$= 600 - 8(2.95)$$
$$\Rightarrow E_{a_1} = 576.4\ V$$

Now,

$$E_a = K_a \phi \omega$$
$$= K_a (K_f I_f) \omega$$

$$\Rightarrow E_{a_1} = (K_a K_f) I_{a_1} \left(\frac{2\pi n_1}{60} \right) \longrightarrow (1)$$

($I_{a_1} = 80$, $n_1 = 750$)

$$\Rightarrow (K_a K_f) = \frac{(576.4) \times 60}{80 \times 2\pi (750)} = 0.09178$$

K_a', say

Now, $I_{a_2} = 95\,A$.

$$E_{a_2} = V - I_{a_2}(R_a + R_{se})$$

600, 95, 0.08, 0.215

$$= 600 - 9.5(2.95)$$
$$\Rightarrow E_{a_2} = 571.975$$
$$\Rightarrow E_{a_2} = (K_a K_f) I_{a_2} \left(\frac{2\pi n_2}{60} \right) \longrightarrow (2)$$

$$\Rightarrow \frac{E_{a_1}}{E_{a_2}} = \frac{I_{a_1}}{I_{a_2}} \left(\frac{n_1}{n_2} \right) \longrightarrow (1) \div (2)$$

$$\Rightarrow n_2 = \left(\frac{I_{a1}}{I_{a2}}\right)\left(\frac{E_{a2}}{E_{a1}}\right)(n_1)$$

$$= \left(\frac{80}{95}\right)\left(\frac{571.975}{576.4}\right)(750)$$

$$\Rightarrow n_2 = 626.73 \text{ rpm.}$$

<u>Also</u>

$$\tau = K_a \phi I_a$$

$$\Rightarrow \tau_1 = K_a(K_f I_f) I_a$$

$$= K_a K_f I_a \cdot I_a$$

$$= (K_a K_f)(I_a)^2_{95}$$

$$= 828.345 \text{ Nm.}$$

0.09178

$$(\tau \propto I_a^2)$$

<u>Aliter</u>

$$\omega \tau = E_a I_a$$

$$\omega = \frac{2\pi n}{60}$$

Q. A 250 V DC series motor has linear OCC with a slope of 12 V / field ampere at 1200 rpm. Find the speed at which the motor will run when developing τ of 40 Nm. What current would it draw from the mains?

Given: $R_a + R_{se} = 0.6\,\Omega$.

$$E_{a_1} = V - I_{a_1}(R_a + R_{se})$$

$$\underset{12\text{ V}}{E_a} = (K_a K_f)\, \underset{1}{I_{a_1}} \, \omega$$

per field ampere

$$\Rightarrow 12 = K_a'(1)\left(2\pi \times \frac{1200}{60}\right) \Rightarrow K_a' = 0.0955$$

$T = K_a \phi I_a$

$= K_a K_f I_a I_a$

$\Rightarrow I_a^2 = \frac{T}{K_a'}$

$\Rightarrow I_a = \sqrt{\frac{40}{0.0955}}$

$\Rightarrow I_a = 20.465$

$\omega = ?$

↳ Idea :- $E_{a_1} = V - I_{a_1}(R_a + R_{se})$

& $E_{a_1} = K_a' I_{a_1} \omega$

$E_{a_1} = 250 - 20.465(0.6)$

$\Rightarrow E_{a_1} = 237.721$

$\Rightarrow \omega = \frac{237.721}{(0.0955)(20.465)}$

$\omega = 121.633$ rad/s

$\omega = \frac{2\pi n}{60}$

$\Rightarrow n = \frac{\omega \times 60}{2\pi}$

$\Rightarrow n = 1162.101$ rpm

Ans

Q. A DC <u>shunt</u> motor runs at 1200 rpm on no load drawing 5A current from 220 V mains.
When loaded, the motor draws 62A ($I_{L_1} = 62$ A)
What would be speed? ($n_1 = ?$)
Assume: armature rxn demagnetises the field by 5%.
i.e $\phi_1 = (0.95)\phi_0$.
Also calculate internal torque developed at no load & load condns. What is the shaft torque available at load condns specified ($T_{shaft} = \tau_1 - \tau_0$)
Given :- $R_a = 0.25\,\Omega$
$R_{SH} = R_f = 110\,\Omega$

no load

$$I_{a0} = I_{L0} - I_f = 5 - \frac{\cancel{220}^{\,2}}{\cancel{110}} = 3\text{ A}.$$

$$E_{a0} = V - I_{a0}R_a = 220 - 3(0.25) = 219.25$$
$$= k_a \phi_0 \omega_0$$
$$= k_a \left(\frac{2\pi}{60}\right)\phi_0 n_0 = k_a k_f \left(\frac{2\pi}{60}\right) \underbrace{I_f}_{2} (\underbrace{n_0}_{1200})$$

$$\tau_0 = k_a \phi_0 I_{a0} = k_a \phi_0 (3)$$
$$= k_a k_f \underbrace{I_f}_{2} (3)$$

$$E_{a_0} = 219.25 = k_a k_f \left(\frac{2\pi}{\cancel{60}}\right)(2)(\cancel{1200}^{\,20})$$

$$\Rightarrow k_a k_f = \left(\frac{219.25}{80\pi}\right) = 0.8728$$

$$\tau_0 = \left(\frac{219.25}{80\pi}\right) \times 6. \qquad \text{Ans}$$
$$= 5.23 \text{ Nm}$$

Load $I_{L_1} = 62\,A$

$$\phi_1 = (0.95)\phi_0$$

$$I_{a_1} = I_{L_1} - I_f$$
$$= 62 - 2 = 60\,A$$
$$E_{a_1} = V - I_{a_1} R_a$$
$$= 220 - (60)(0.25)$$
$$E_{a_1} = 205\ V$$
$$= k_a \phi_1 \left(\frac{2\pi n_1}{60}\right)$$
$$= k_a (0.95)\phi_0 \left(\frac{2\pi n_1}{60}\right)$$

Now

$$E_{a_1} = (k_a \phi_0)\left(\frac{0.95 \times 2\pi n_1}{60}\right) = 205$$

$$\Rightarrow k_a \phi_0 = \frac{205 \times 60}{2\pi n_1 \times 0.95} \longrightarrow (1)$$

& $$E_{a_0} = (k_a \phi_0)\left(\frac{2\pi n_0}{60}\right) = 219.25$$

$$\Rightarrow k_a \phi_0 = \frac{219.25 \times 60}{2\pi n_0} \longrightarrow (2)$$

(1) & (2)

$$\Rightarrow \frac{205 \times \cancel{60}}{\cancel{2\pi} n_1 \times 0.95} = \frac{219.25 \times \cancel{60}}{\cancel{2\pi} n_0}$$

$$\Rightarrow n_1 = \frac{205 \times n_0}{219.25 \times 0.95} = \frac{205 \times 1200}{219.25 \times 0.95}$$

$$\Rightarrow n_1 = 1181.059$$

$$\tau_1 = k_a \phi_1 I_{a_1}$$

$$= k_a \phi_0 (0.95) I_{a_1}$$

$$= \left(\frac{219.25 \times 60}{2\pi \times 1200}\right)(0.95)(60)$$

$$\Rightarrow \tau_1 = 995.003 \text{ Nm}$$

$$\tau_{sh} = \tau_1 - \tau_0$$

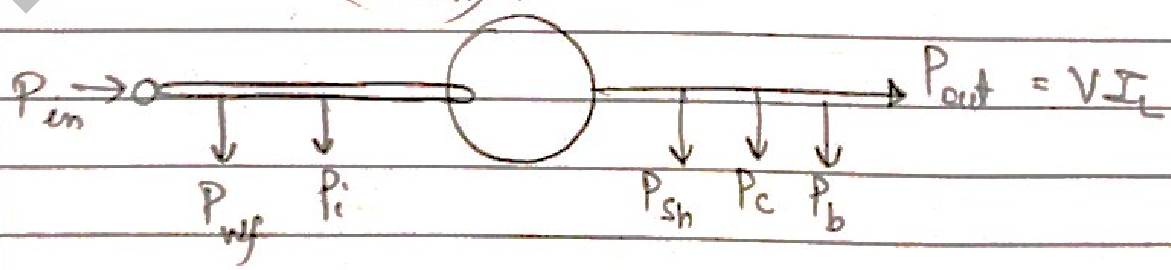
$$\Rightarrow \tau_{sh} = 989.77 \text{ Nm}$$

✯ EFFICIENCY OF DC MACHINE

* Power flow diagram for

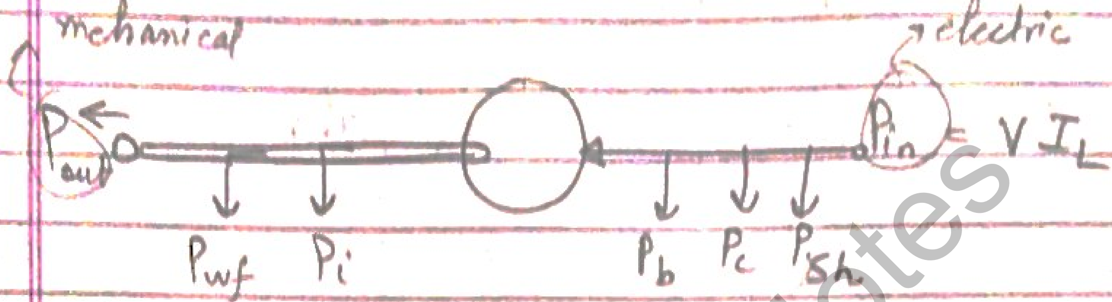
GENERATING Mode :

$$(P_{mech})_{in} = E_a I_a$$



MOTORING Mode

$$(P_{mech})_{out} = E_a I_a$$



P_{wf} : friction & windage loss (or mechanical loss)

P_i : total core loss (made of: C_1 : no load iron loss, C_2 : stray load iron loss)

P_{sh} : shunt field loss (in shunt machine) (or field copper loss)

P_c : armature copper loss (including loss in series winding)

P_b : brush contact loss.

—o—

- Alternative terms :

P_{wf} + no load iron loss = rot^{nal} losses (P_{rot})

P_{wf} : mechanical losses.

Q. A 600 V DC shunt motor drives a 60 kW load at 900 rpm.

$R_f = R_{sh} = 100\,\Omega$ $\qquad P_{out} = 60\ kW$.

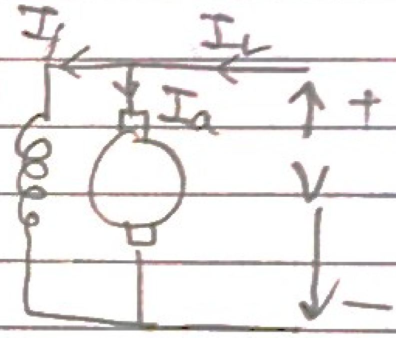
$R_a = 0.16\,\Omega$

Motor $\eta = 0.85 = 85\%$

Find (a) Rot^{nal} loss (P_{rot})
(b) no load armature current (I_{ao})
(c) Speed (n)
(d) Speed regulation
(e) armature current for electromagnetic torque of 600 Nm.

$$\eta = \frac{P_{out}}{P_{in}} \Rightarrow P_{in} = \frac{60 \times 10^3}{0.85}$$

$$\Rightarrow P_{in} = 70.5 \; kW$$



Now, $P_{in} = V \times I_L$

$\Rightarrow 70.5 \times 10^3 = 600 \times I_L$

$\Rightarrow I_L = 117.65 \, A$

$$I_f = \frac{V}{R_f} = \frac{600}{100} = 6 \, A.$$

$$I_a = I_L - I_f$$

$\Rightarrow I_a = 111.65 A$

Now, $E_a = V - I_a R_a$

$= 600 - (111.65)(0.16)$

$\Rightarrow E_a = 582.14 \; V$

Now,

$$E_a = k_a \phi \omega = k_a \times \phi \left(2\pi \times \frac{900}{60}\right)$$

$$\Rightarrow k_a \phi = \frac{582.14 \times 60}{2\pi \times 900} = 6.1798$$

P_L = Total losses
= armature cu loss + P_{sh} + P_{rot}
$= I_a^2 R_a + V I_f + P_{Rot}$

$= (111.65)^2 \times 0.16 + (600)(6) + P_{rot}$

$P_L = 5594.5156 + P_{rot}$

Now
$P_{in} = P_{out} + P_L$

$\Rightarrow P_L = P_{in} - P_{out}$
$= 70.5 - 60$ kW
$\Rightarrow P_L = 10.5$ kW

$\Rightarrow P_{rot} = P_L - 5.594$ kW

$\Rightarrow P_{rot} = 4.906$ kW

(b) <u>No Load Condition</u>

$V I_{a0} = P_{rot} + (I_{a0})^2 R_a$

$\Rightarrow 600 \times I_{a0} = 4906 + (I_{a0})^2 (0.16)$
$\Rightarrow (I_{a0})^2 - \frac{60,000}{16} I_{a0} + \frac{490600}{16} = 0$
$\Rightarrow (I_{a0})^2 - 3750 I_{a0} + 30662.5 = 0$

In No load condⁿ, o/p is negligible.

So whatever i/p comes to motor, it totally gets used in losses.

So, $V I_{ao} = P_{rot} + 0$

$\Rightarrow I_{ao} = \frac{P_{rot}}{V}$

$= \frac{4906}{600}$

$I_{ao} = 8.176 \text{ A}$. ignoring the drop.

(c) Now, $E_{ao} = V - I_{ao} R_a \simeq V$ (at no load)

$= 600 - \underbrace{(8.176)(0.18)}_{\text{neglect}}$

$= 600 \text{ V}$

Now,

$E_{ao} = (k_a \phi)\left(\frac{2\pi n_o}{60}\right)$

$\Rightarrow 600 = (6.1798)\left(\frac{2\pi}{60}\right) \times n_o$

$\Rightarrow n_o = \frac{600 \times 60}{2\pi \times 6.1798}$

$\Rightarrow n_o = 927.616 \text{ rpm}$

(d) Speed regulation $= \frac{\text{no load speed} - \text{load speed}}{\text{load speed}} \times 100$

$= \frac{927.616 - 900}{900} \times 100$

$= 3.068 \%$

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(e) Torque

$T = k_a \phi I_a$ → not I_{a0}

$\Rightarrow 600 = 6.178 \, (I_a)$

$\Rightarrow I_a = 97.118 \text{ A}$

Ans

TESTING OF DC MACHINES

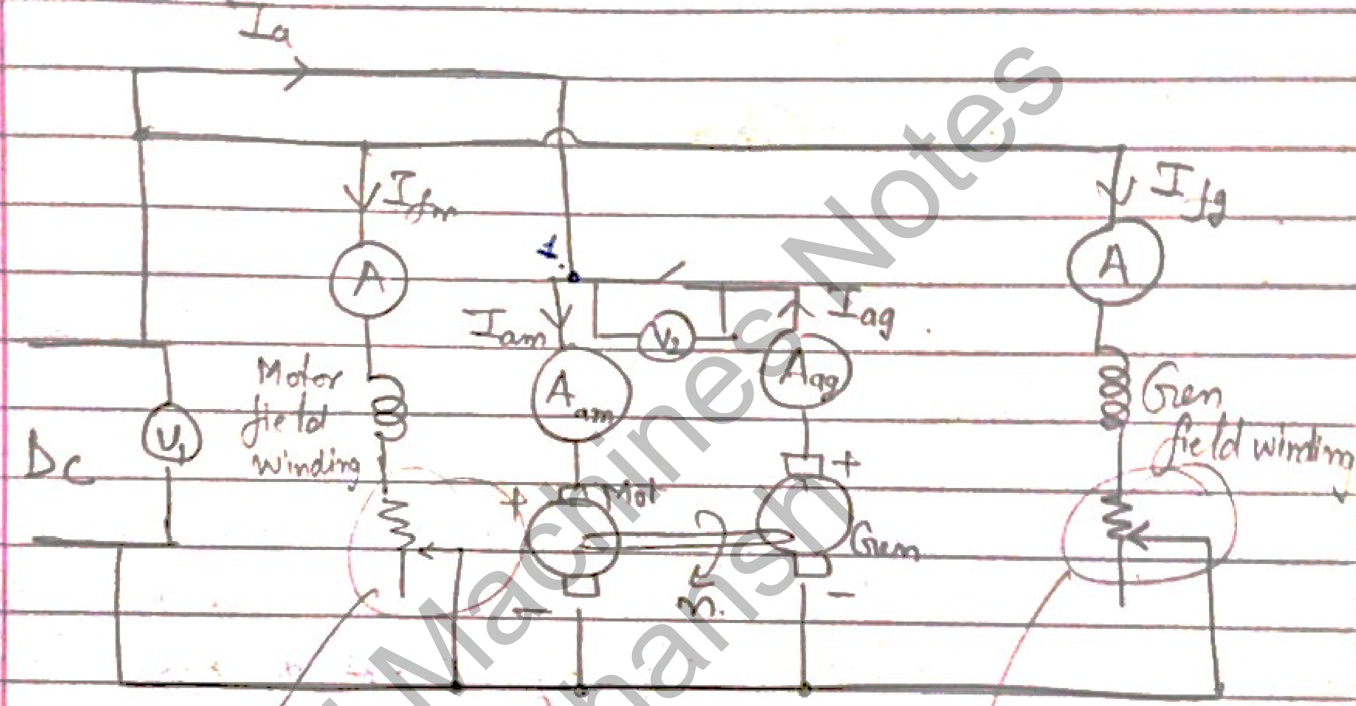
- Swinburne's test } non loading tests.
- Hopkinson's test }
  → measuring losses & finding efficiency
  → mostly conducted on shunt motor : for no loads in series current is very high & speed → ∞

HOPKINSON'S Test

Regenerative test in which two IDENTICAL DC shunt machines are mechanically coupled.

- One as DC generator, other as motor.
- The set of both draws power (only that much) that it can makeup for ~~losses~~ losses from mains.
- (akin to Sumpner's test of transformer).

★ Why not direct loading tests ?
- eg: suppose 6000 DC machines → test each one with load → lot of E loss.
- load might not be available.



∀ experiments
vary field rheostat of motor is used to set motor to rated speed.

field rheostat of generator is used to set generator to rated voltage

S1) Motor made to run at rated speed.
S2) Switch is closed after checking V_2 (reads zero or negligible) & similar polarities of machine are connected across switch S.

I_a = current drawn from supply.
$= I_{am} - I_{ag}$.

KCL at node 1,
$I_a + I_{ag} = \cancel{I_{fm}}\ I_{am}$.

Total i/p to armature circuit = $V I_a$
= Total armature loss
($\because$ set o/p $\rightarrow$ 0)

$V I_a$ = armature Cu loss (generator) + armature Cu loss (motor) + Total stray loss

(generator) → $I_{ag}^2 R_{ag}$ (motor) → $I_{am}^2 R_{am}$

stray loss → Winding frictional, rotational + other losses

$\Rightarrow$ Total stray loss = $V I_a - I_{am}^2 R_{am} - I_{ag}^2 R_{ag}$

$= \left((P_{wf} + P_{io}) + \text{Stray load loss} \right)$ → some loss additional

Winding → P_{wf}; P_{io} → iron loss / core

$= \left(P_{rot} + \text{Stray load loss} \right)$

P_{rot} → rotational

P_{stray} (each machine) $= \frac{1}{2}\left[V I_a - I_{am}^2 R_{am} - I_{ag}^2 R_{ag} \right]$

assumed

Q Following results were obtained while Hopkinson's test was performed on 2 similar DC shunt machines:-

Supply voltage = 250 V

Field current of motor = 2 A = I_{fm}

Field current of generator = 2.5 A = I_{fg}

I_{ag} = 60 A

I_a = 15 A (Current taken from supply)

$R_{am} = R_{ag} = 0.2\ \Omega$.

Calculate η of motor & generator under these cond^{ns} of load.

$\boxed{\text{Motor}}$

$\because\ I_a + I_{ag} = I_{am}$

$\Rightarrow\ 15 + 60 = I_{am}$.

$$P_{stray} = \frac{1}{2}\left[V I_a - I_{am}^2 R_{am} - I_{ag}^2 R_{ag}\right].$$

motor I/P. $P_{in,\ motor} = V I_{am} + V I_{fm}$.

Loss of motor $= P_{Lm} = \left(P_{stray} + V I_{fm}\right) + I_{am}^2 R_{am}$.

Field Cu loss

Arm. Cu loss.

$$\eta_{motor} = \frac{P_{in,\ motor} - P_{Lm}}{P_{in,\ m}} = 86.6\ \%.$$

Generator $\quad P_{Lg} = P_{stray} + V I_{fg} + I_{ag}^2 R_{ag}$.

$P_{out,\ g} = V I_{ag}$.

$$\eta_G = \frac{P_{out,\ g}}{P_{out,g} + P_{Lg}} = 86.7\ \%.$$

✲ Self Study

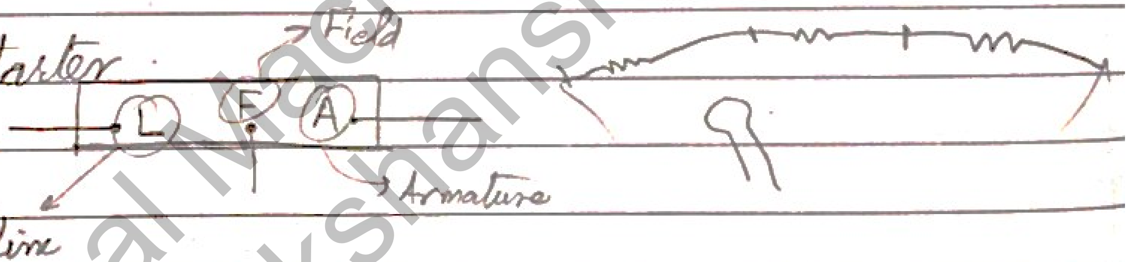
Q. What are the advantages of Hopkinson's test over Swinburne's test? What are its limitations?

Q. Which type of losses cannot be determined by Swinburne's test and how are these estimated for calculating η?

—x—

§ Starting of DC MOTOR

- 3-pt. starter



- DC motors are inherently self starting (∵ it'll start even if 3 pt. starter isn't there)

At start,

$E_a = 0$ (back emf = 0).

$V - I_a R_a = 0$

$I_{a(start)} = \frac{V}{R_a} = \frac{220}{0.5} = 440\text{ A}$ → very high

= INRUSH CURRENT

≃ 6 – 8 times ~~too~~ rated normal current

So, 3-pt. starter used.

* $T_{starting} \propto$ resistance of armature circuit

  ↳ Higher the starting torque, it'll be able to lift heavy loads at start.

* Inf Inrush current left unhandled

(i) heavy spark at brushes destroying commutators.

(ii) $I_{a(start)}$ gives rise to high starting torque causing mechanical shock, ↓ machine life

(iii) Supply fluctuation due to 'inrush' current.

- Max allowable starting current ~~$\propto$~~ 1.5 to 2 times rated value

- Starter: at start, resistance fully included, gradually cut out as motor picks up speed to give a good torque.

* Protections in a Starter:

1. NVC (No Volt Coil): Failure of I_f (due to accidental or open circuit of field), this control releases handle (held electromagnetically) & goes back to OFF.

2. OL (over load) release: relay at $> I_a$ (overload or short circuit) brings handle to OFF.

☆ Self Study

Speed Control

- For shunt motor
  (i) Field control (constt power control)
  (ii) Armature control (constt. torque control)

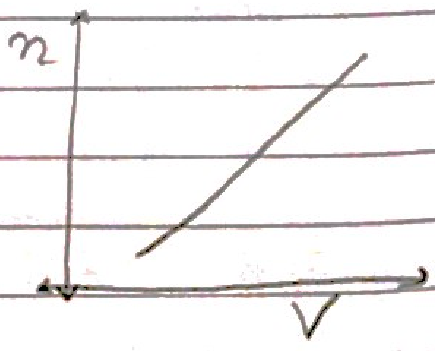
—x—

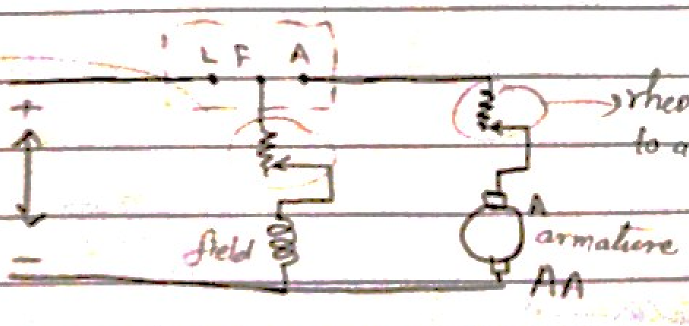
☆ $n \propto \frac{E_b}{\phi}$

$\propto \frac{V - I_a R_a}{I_f}$

Armature control ($V - I_a R_a$)
field control (I_f)

Field control	Armature control.
speed ≃ 1500–1800 rpm	
$n > n_{rated}$	$n < n_{rated}$.
V is constt (constt power) "field weakening".	I_f is constt (constt torque)





☆ Self Test (4th Edition)

* Ex 7.6, 7.7, 7.8, 7.9, 7.32, 7.33, 7.34, 7.45, 7.58, 7.59, 7.62.

% Imp.

Q. 7.10, 7.12, 7.14, 7.17, 7.19, 7.26, 7.28, 7.30, 7.32, 7.35, 7.36, 7.40, 7.46, 7.47, 7.48, 7.50.

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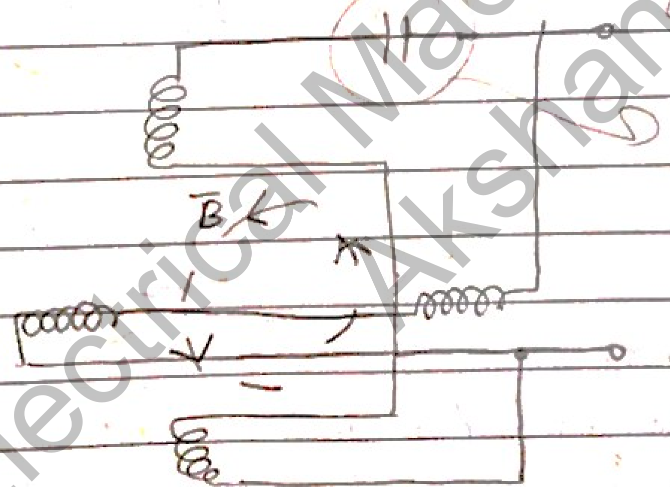
☆ Ch - 3 ϕ Induction motor

☆ For creating rotating magnetic field, consider 2 ϕ rotating magnetic field.

windings $\rightarrow$ 90° displacement in space.

their currents $\rightarrow$ 90° displacement in phase.

Phase difference of 90° caused using a capacitor



should have very large value for it to work

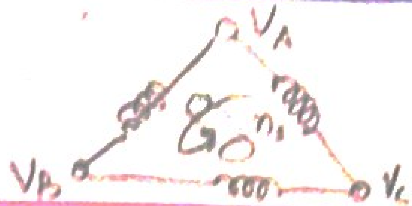
$\Downarrow$

Sol^n

Use 3 ϕ induction motor

☆ The field of the stator rotates at synchronous speed (n_s) $\left(f = \frac{n_s P}{120}\right)$, where f is supply frequency, P: no. of poles.

☆ The stator winding is wound for 3 ϕ displaced in space by 120° & whose currents have a phase shift of 120°, creates a rotating magnetic

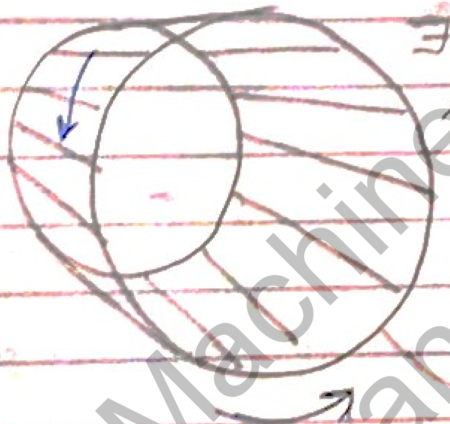


field which revolves at n_s.

The most popular type of rotor of a 3 ϕ induction motor is squirrel cage rotor which has rotor leads made of Cu / Al rather than winding.

Singly phase motor :- Supply is given only to stator.

Squirrel cage rotor :



90 % of motors of that type.

$\exists$: conductors (not wires) – heavy bars of Cu, Al or alloys – kept in slots with short-circuitry end rings

→ has lower starting torque

* WRIM : Wound Rotor Ind^n motor : other type of Ind^n motor (not that popular)

→ also called : Slip ring Ind^n motor.

→ has as many poles as stator poles.

→ has 3 leads brought out of machine via slip rings that are tapped by Cu-carbon brushes. Ext. resistance can be added in the rotor circuit. Hence, ↓ starting current ⟶ ↑ starting torque.

* Advantage of WRIM over squirrel cage

Add ext. resistance to motor : WRIM ✓

Squirrel cage ✗

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☆ Rotating magnetic field

If motor speed = motor field speed = n_s → synchronous speed.

↳ then, no relative speed ⇒ no emf ⇒ no current
no torque to sustain rotation

So, rotor runs at speed, $n < n_s$

- Slip, $S = \frac{n_s - n}{n_s} \times 100$ (in %)

; $n_s - n$: slip speed.

- Slip ↑ with load, ∵ $n \downarrow$. Full load slip ≈ 3-10%
- Here, singly fed machine is stator fed unlike sync. m/c
- These machines dont run at n_s ∵ τ is not possible at n_s.

So, they are called asynchronous machines.

☆ Circuit model

Generalised transformer in which rotor emf & frequency both depend on rotor speed.

↳ conversion of energy takes place

↳ V_1 : applied voltage / phase
↳ K_{w1} : stator winding factor
↳ N_{ph1} : stator series turns / phase

- $\frac{E_1}{E_2} = \frac{K_{w1} N_{ph1}}{K_{w2} N_{ph2}} = a$ (turns ratio)
- Rotor frequency at any speed n, $f_2 = sf$ (s → slip; f → stator frequency)
- I_0 (exciting current or no load current) : 40% of FL current.

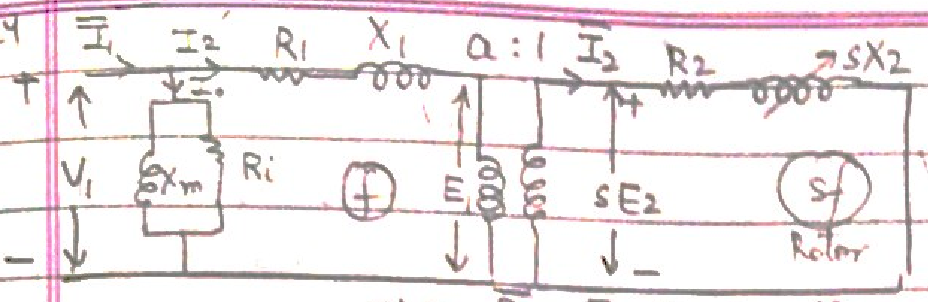
In a transfor, %age of FL current taken by no load current

Q. Why Im shouldn't be operated at no/light loads for long periods of time?

- Rotor impedance, $Z_2 = R_2 + j s X_2$

Rotor resistance; standstill rotor leakage reactance.

fig 12.24 (TB)



Stator $\frac{P_G}{3}$ ↻ $\frac{P_m}{3}$ (mech)

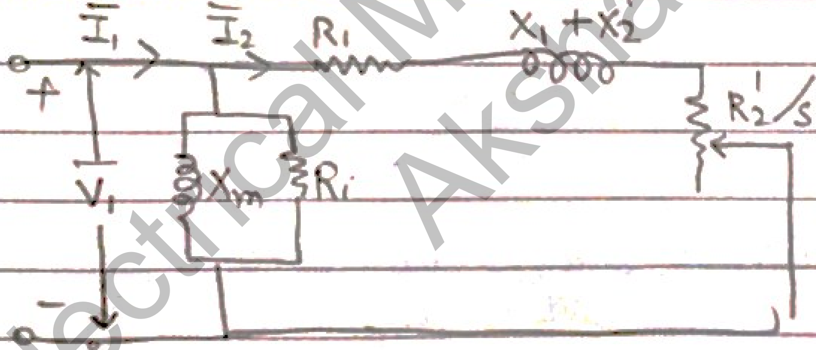
\# Supply frequency $= f \longrightarrow$ Rotor side $= sf$ → slip

$$\bar{I}_2 = \frac{s\bar{E}_1}{R_2' + js X_2'} = \frac{E_1'}{\frac{R_2'}{s} + jX_2'}$$

$$\frac{P_m}{3} = \frac{(I_2')^2 R_2'}{s} - (I_2')^2 R_2' = (I_2')^2\left(\frac{1}{s}-1\right)R_2'$$

$$R_L = R_2'\left(\frac{1}{s}-1\right)$$

* Generalised trf in which frequency is also transformed in proportion of slip (s) s.t. rotor emf $= sE_2$ & rotor reactance $= sX_2$



: Approx. model.
: Shunt branch shifted to front.

• Power across air gap

$$P_G = \frac{3(I_2')^2 R_2'}{s} = \frac{3I_2^2 R_2}{s}$$

$$= \frac{\text{rotor Cu loss}}{\text{slip}}$$

$\Rightarrow$ Rotor Cu loss $=$ Slip $\times P_G$.

☆ Gross mech. power o/p $= P_m = P_G -$ electrical loss in rotor resistance

$$= P_G - 3(I_2')^2 R_2'$$

$$= 3(I_2')^2 R_2'(1/s - 1)$$

$$\Rightarrow P_m = (1-s)\,P_G \quad ☆$$

- Electromagnetic torque $(T) = \dfrac{P_m}{W}$ — rotor speed in rad/s

or

(Shaft torque) $= \dfrac{3(I_2')^2 R_2 (1/s - 1)}{(1-s) W_s}$ = (1-s) Ws

or

(useful torque) $= \dfrac{3(I_2')^2 R_2/s}{W_s} = \dfrac{P_G}{W_s}$ N-m

$$\Rightarrow P_G = T W_s$$

= Torque in sync. watts

note
- When IM runs, it runs less than rated speed.
- ∴ slip is always +ve for IM.
- Subsynchronous speed IG runs at $n > n_s$: slip -ve
- Super synchronous speed slip will never be 0.

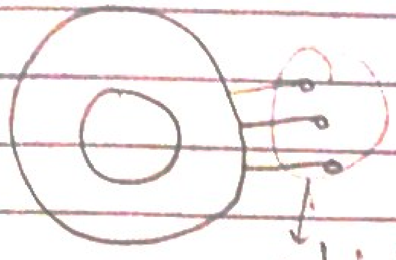
So, Net mech. power o/p = P_m − (windage, friction, stray load losses)

Summary :- $P_m = WT$: shaft power o/p.
$P_G = W_s T$

Power Stages in IM.

- Stator Cu, Iron loss.
- Rotor i/p (P_G) (or) Power across air gap:
  - Rotor Cu loss
  - Mech. power o/p → Gross (P_m)
    - Friction, windage loss
    - Net Power o/p or rotor o/p.

* Torque-Speed characteristics of Induction motor.



n_s : synchronous speed
: Stator FIELD rotates at this speed

* Rotor rotates at speed $< n_s$. (n)

* $n_s = \frac{120 f}{P}$ (f → stator frequency; P → no. of poles.)

* $\boxed{\text{Slip}, s\ (\%) = \frac{n_s - n}{n_s} \times 100}$.
  - At start, $n = 0$. So, $s = 1$ or 100%
  - s is always +ve for indn motor ($\because n < n_s$).

* Induction generator
  Slip, s is −ve ($\because n > n_s$)

Indn generator: (n) Speed higher than n_s : super synchronous speed
Indn motor : n lower than n_s : subsynchronous speed.

* Rotor can never run at n_s.
  So, Slip $\neq 0$. (never = 0)

Q Find n_s; rotor speed, n; f of rotor current of a 3ϕ ind^n motor with stator poles $(P) = 4$ & a slip of (s) 0.03%.

f : stator frequency = 50 Hz (assume)

(a) $n_s = \frac{120f}{P} = \frac{\cancel{120}^{30} \times 50}{\cancel{4}} = 1500$ rpm.

(b) frequency of rotor current = slip x supply frequ...

$\Rightarrow \boxed{f_2 = s \times f}$

Now

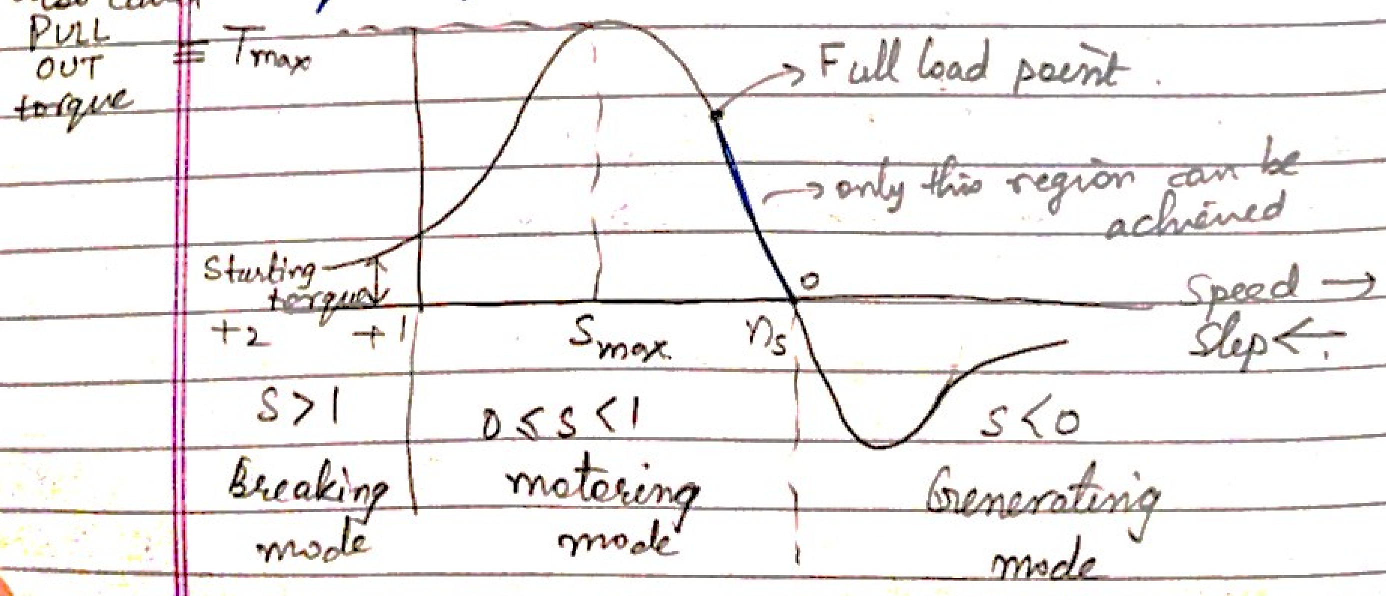
$0.03 = \frac{1500 - n}{1500}$ $\Rightarrow 45 = 1500 - n$

$\Rightarrow n = 1455$ rpm

$\Rightarrow f_2 = 0.03 \times 50 = 4.5$ Hz

★ Note :- NO SLIP ≡ NO INDUCTION MOTOR.

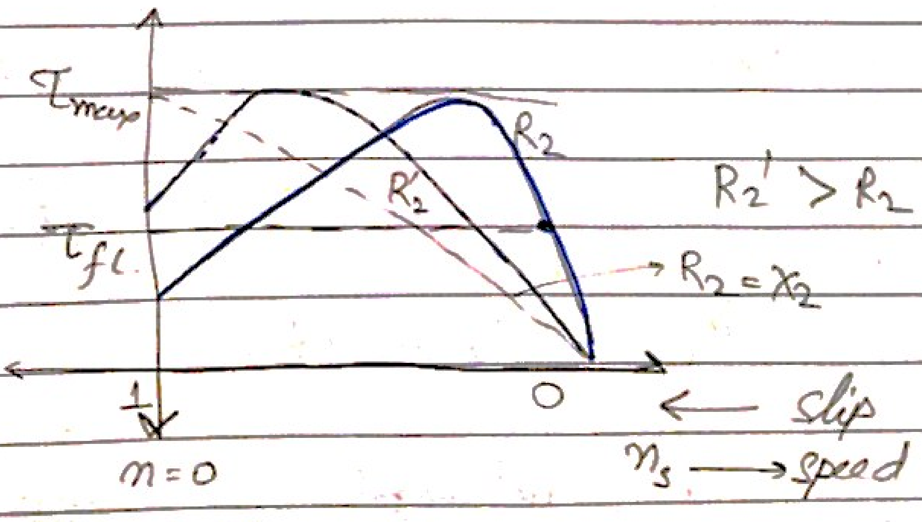
★ Torque Speed Characteristics.



* Motoring mode
$0 < S \leq 1$: Subsynchronous speed
* Breaking mode
$S \geq 1$: motor runs in opp. dir^n to field of stator.
* Generating mode:
$S < 0$: supersynchronous speed in dir^n of rotating field.

* <u>WRIM</u>

External resistance can be added to rotor - only possible in slip ring rotor. As R_2 is added,
(i) Breakdown torque (max torque) remains unchanged.
(ii) Slip at breakdown torque ↑
(iii) τ_{start} becomes max.



⇒ As R ↑, S at τ_{max} ↑.

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Q. A 6-pole, 50 Hz, 3ϕ IM running on f.l develops a useful torque of 160 N-m when rotor emf makes 120 complete ~~as~~ cycles/min. Calculate shaft power o/p. If mech. torque lost in friction & that for core loss is 10 N-m, find

(a) Cu loss in motor (b) I/p ~~to~~ motor (c) η

Ans Here, Total stator loss = 800 W.

frequency of rotor emf, $f_2 = \frac{120}{60} = 2 Hz$ (given) $= sf$

$Slip = \frac{2}{50} = 0.04 = 4\%$

$n_s = \frac{120 \times 50}{6} = 1000$ rpm

n = rotor speed $= (1-s) \times n_s = (1-0.04) \times 1000$
$= 960$ rpm

$\omega = \frac{2\pi n}{60} = 2\pi \times \frac{960}{60} = 100.53$ rad/sec

Shaft power o/p $= T.\omega = 160 \times 100.53 = 16.085$ k

Mechanical power developed } P_m = (Useful T + T in rot^{nal} loss) $\times \omega$
$= (160 + 10) \times 100.53$
$= 17.09$ kW.

(a) $P_m = 3(I_2')^2 R_2' \left(\frac{1}{s} - 1\right)$

Rotor Cu loss $= 3(I_2')^2 R_2' = P_m \left(\frac{s}{1-s}\right) = 712$ W

Net salary = Gross Salary + Deduction
|||
Loss

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(b) Motor i/p = P_m + Rotor Cu loss + Total stator loss
$= 17.09 + 0.712 + 0.8 = 18.602$ kW.

(c) $\eta = \frac{\text{Shaft Power of}}{\text{motor i/p}}$
$= \frac{16.085}{18.602} = 86\%$

Q A 40 kW, 440 V, 3φ, 50 Hz, 8 pole squirrel cage rotor IM has a slip of 0.03 when operated at rated voltage & frequency. If full load line current is 68.9 A & efficiency 89.6%, find
(a) Shaft T delivered to load.
(b) pf at which motor operates.

$\eta = \frac{o/p}{i/p} = 89.6 \Rightarrow o/p = \frac{89.6 \times 40}{100} = 35.84$

i/p → 40 kW

Now, o/p power $= \tau_{sh} \times \omega$

$\Rightarrow \tau_{sh} = \frac{35.84 \times 60}{2\pi n} \rightarrow (1)$

Now, $n_s = \frac{120 f}{P} = \frac{\cancel{120}^{\cancel{30}\,15} \times 50}{\cancel{8}\,\cancel{2}} = 750$ rpm

Now, $s = 0.03 = \frac{n_s - n}{n_s}$

$\Rightarrow n = n_s(1 - 0.03)$
$\Rightarrow n = 750(0.97) = 727.5$ rpm

* Note :- find slip (s) or n_2 (rotor speed) in any problem of IM

$$\Rightarrow T_{sh} = \frac{3584}{2\pi \times 727.5} \times 60$$

$$= 0.47 \text{ k Nm}$$

$$\Rightarrow T_{sh} = 470 \text{ Nm.}$$

(b) Motor i/p $= \sqrt{3}\, V_L I_L \cos\phi$

$$= \sqrt{3} \times 440 \times 68.9 \times \cos\phi$$

40 kW $\Rightarrow$ pf $= 0.7617$

* Normally, IM has a lagging pf.

Imp. Learn ★ **Torque & Slip characteristics.**

$$T \propto E_r I_r \cos\phi_2 \quad \text{or} \quad T \propto \phi I_r \cos\phi_2$$

for rotor, for standstill (since $E_r \propto \phi$)

E_r : rotor emf / phase under running cond^n

I_r : rotor current / phase " " "

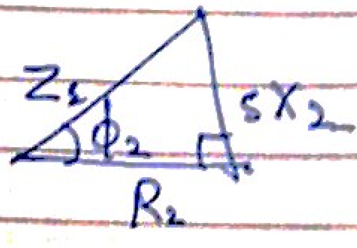
$E_r = s E_2$ where E_2 : standstill rotor induced emf/ϕ.

$$I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

R_2 : rotor resistance/ϕ

X_2 : standstill rotor reactance/ϕ.

$$\cos\phi_2 = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$



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$\cong E_r \qquad I_r \qquad \cos\phi_2$

$$T \propto \phi \cdot \frac{sE_2}{\sqrt{R_2^2+(sX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2+(sX_2)^2}} = \frac{k\phi\, sE_2 R_2}{R_2^2+(sX_2)^2} \rightarrow (A)$$

$$T = \frac{k_1 s E_2^2 R_2}{R_2^2+(sX_2)^2}, \text{ since } E_2 \propto \phi.$$

For T_s, put $s=1$

At standstill, $s=1$, $T_{starting} = T_s = \dfrac{k_1 E_2^2 R_2}{R_2^2+X_2^2}$

If V is constt, $\dfrac{dT_s}{dR_2} = 0$.

$$\Rightarrow \frac{dT_s}{dR_2} = \underbrace{k_2}_{= E_2^2 k_1}\left[\frac{1}{R_2^2+X_2^2} - \frac{R_2(2R_2)}{(R_2^2+X_2^2)^2}\right] = 0.$$

$\Rightarrow R_2 = X_2$: Cond^n for max starting torque.

- Starting torque of squirrel cage motor:
  - rotor resistance can't be changed

Starting torque $= 1.5 \times T_{f.l}$

Starting current $= 5$ to $7 \times I_{fl}$

$$\frac{dT_s}{dR_2} = 0$$

Proof $\Rightarrow$
$$\frac{(R_2^2+X_2^2)(k_1E_2^2) - k_1E_2^2R_2(2R_2)}{(R_2^2+X_2^2)^2}$$

$$= k_1\left[\frac{E_2^2}{R_2^2+X_2^2} - \frac{2E_2^2R_2^2}{(R_2^2+X_2^2)^2}\right] = 0$$

$$\Rightarrow R_2^2+X_2^2 = 2R_2^2$$

$$\Rightarrow R_2 = X_2,$$

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- Cond^n for max. torque → differentiating τ expression w.r.t s & equate to zero.

From (A)

$$\frac{d\tau}{ds} = \frac{[R_2^2+(sX_2)^2][E_2R_2k\phi] - (k\phi E_2R_2s)(2sX_2^2)}{(R_2^2+(sX_2)^2)^2}$$

$= 0$

$\Rightarrow 2s^2X_2^2 = R_2^2 + s^2X_2^2$

$\Rightarrow \boxed{R_2 = sX_2}$

For max. τ : Substitute $R_2 = sX_2$ in (A).

$$\Rightarrow \tau_{max} = \frac{k\phi s E_2 (sX_2)}{(sX_2)^2+(sX_2)^2}$$

$$= \frac{k\phi s E_2}{2sX_2}$$

$$\Rightarrow \boxed{\tau_{max} = \frac{k\phi E_2}{2X_2}}$$

★ Effect of change in supply voltage V on torque:
$E_2 \propto \phi \propto V$.

So, $\tau = \dfrac{k\phi s E_2 R_2}{R_2^2 + (sX_2)^2} \propto V^2$.

So, T is very sensitive to change in V.

* Full load T & T_{max} :- $\frac{T_f}{T_{max}} = \frac{2 a s_f}{a^2 + s_f^2} \longrightarrow (1)$

f.l.

s_f : ~~slip~~ slip corresponding to f.l. torque.

* Starting torque & T_{max} $\frac{T_s}{T_{max}} = \frac{2a}{1+a^2} \longrightarrow (2)$

starting

$a = \frac{R_2}{X_2}$.

Proof of eq^n (1)

$$T_f = \frac{k \phi s_f E_2 R_2}{R_2^2 + (s_f X_2)^2}, \quad T_{max} = \frac{k \phi s E_2 (s X_2)}{(s X_2)^2 + (s X_2)^2}$$

s_f

$$T_{max} = \frac{k \phi E_2}{2 X_2}$$

$$\Rightarrow \frac{T_f}{T_{max}} = \frac{\cancel{k\phi} s_f \cancel{E_2} R_2}{R_2^2 + (s_f X_2)^2} \times \frac{2 X_2}{\cancel{k \phi E_2}}$$

$$\Rightarrow \frac{2 s_f \cancel{X_2} R_2}{\left[\left(\frac{R_2}{s_f X_2}\right)^2 + 1\right] (s_f X_2)^{\cancel{2}}}$$

$$= \frac{2 R_2}{\left(\frac{a^2}{s_f^2} + 1\right) s_f X_2}$$

$$= \frac{2 s_f^{\cancel{2}} R_2}{(a^2 + s_f^2) \cancel{s_f} X_2} = \frac{2 a s_f}{(a^2 + s_f^2)}$$

Q A 4 pole 50 Hz 3ϕ IM has rotor resistance per phase $(R_2/\phi) = 0.03\,\Omega$, $X_{2/\phi} = 0.12\,\Omega/\phi$.

Rotor standstill reactance

✓ What is speed at T_{max}?

✓ Find amt. of $R_{external}/\phi$ to be inserted to get 75% of T_{max} at start?

S1) Find synchronous speed

needed to find rotor speed

$$n_s = \frac{120 f}{P} = \frac{\cancel{120} \times 50}{\cancel{4}} = 1500 \text{ rpm}$$

Note :- For T_{max} :- $R_2 = s X_2$

$$\Rightarrow \left(R_2/\phi\right) = s\left(X_2/\phi\right)$$

$$\Rightarrow s = \frac{0.03}{0.12}$$

$$\Rightarrow s = 0.25$$

Now, n_s, s ✓

$\Rightarrow (n)$ ✓

rotor speed

$$0.25 = \frac{n_s - n}{n_s}$$

$$\Rightarrow n = (1-s)n_s$$

$$= 1125 \text{ rpm}$$

$$T_{\substack{reqd \\ at\ start}} = (0.75)\, T_{max}$$

$$\Rightarrow \frac{T_s}{T_{max}} = 0.75 = \frac{2a}{1+a^2}.$$

$$\cancel{\begin{aligned} &0.75 + 0.75a^2 = 2a \\ &\Rightarrow 75 + 75a^2 = 200a \\ &\Rightarrow 3 + 3a^2 = 4a \\ &\Rightarrow 3a^2 - 4a + 3 = 0 \\ &\Rightarrow a = +4 \pm \sqrt{16-} \end{aligned}}$$

$$a = \frac{R_2}{X_2} = \frac{R_2}{0.12}.$$

$$\Rightarrow 0.75 = \frac{2\left(\frac{R_2}{0.12}\right)}{1 + \left(\frac{R_2}{0.12}\right)^2}$$

time consuming

$$\Rightarrow 0.75 \times 0.12 = \frac{2R_2}{(0.12)^2 + R_2^2}$$

$$\Rightarrow (0.75 \times 0.12 \times 0.0144) + \frac{900}{10000} R_2^2 = 2R_2$$

$$\Rightarrow \frac{1296}{1000000} + \frac{900 R_2^2}{10000} = 2R_2$$

$$0.75 = \frac{2a}{1+a^2} \longrightarrow \Rightarrow a = 0.451\,\Omega$$

(large value neglected)

$$\text{Now, } a = \frac{R_2 + r}{X_2}$$

what is asked (external R/ϕ to be added to rotor)

$$\Rightarrow r = 0.024\,\Omega.$$

Can be used. * T_{max} : also called PULL OUT torque or Breakdown torque.

Q. A 50Hz 8 pole IM has $S_f = 0.04$, $R_2 = 0.001$ per phase & standstill reactance = $0.005\ \Omega/\phi$.

(I) Find T_{max}/T_{fl} & speed at which T_{max} occurs.

(II) A 3ϕ IM has starting $T_s = 100\%$ of $T_{f.l}$ & $T_{max} = 200\%$ of $T_{f.l}$.

Find (a) Slip at T_{max}

(b) f.l. slip.

(I) Just like previous ques.

(II) $$\frac{T_f}{T_{max}} = \frac{2aS_f}{a^2 + S_f^2} \longrightarrow (1)$$

$$\frac{T_s}{T_{max}} = \frac{2a}{1+a^2} \longrightarrow (2)$$

Now, $T_s = T_f$ & $T_{max} = 2T_f$.

$$\Rightarrow \frac{T_{max}}{T_f} = 2 \longrightarrow (3)$$

$$\text{IIly}, \frac{T_s}{T_{max}} = \frac{1}{2} \quad (\because T_f = T_s)$$

$\Rightarrow$ $S_{\text{at } T_{max}} = 0.04$ at start

(a) Solve for s_f using (1), (2) & (3).

(b) We know $a = \frac{R_2}{X_2}$. At T_{max} $R_2 = s X_2$

$$\Rightarrow \underbrace{s}_{\text{at } T_{max}} = \frac{R_2}{X_2}$$

☆ Max. torque can also be measured in a unit Synchronous Watts.

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Q 9.2 & 9.5 P_G : motor i/p − stator Cu loss

A 3ϕ 440V, 50 Hz 4 pole Y connected IM has $R_2 = 0.1\,\Omega/Ph$ & $X_2 = 0.9\,\Omega/\phi$. Ratio of stator to rotor turns is 3.5. Calculate :-

(a) Gross o/p at a slip of 5%

(b) Max. torque in $\underbrace{\text{syn. watts}}_{\text{a unit}}$ & corresponding slip.

E_1 = Stator phase voltage $= \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}}$.

$a = \frac{\text{Stator turns}}{\text{Rotor Turns}} = 3.5$ (given)

$\frac{E_1}{E_2} = \frac{n_1}{n_2}$ $(E \propto n)$

$\Rightarrow a = \frac{E_1}{E_2} = 3.5$

$\Rightarrow E_2$: Standstill rotor Emf /ϕ $= \frac{E_1}{3.5} = 72.6$

Transform^n req^d

E_r = Rotor emf $= sE_2 = 0.05 \times 72.6 = 3.63V$

$Z_2 = \sqrt{R^2 + (SX_2)^2} = 0.109\,\Omega$

$I_2 = \frac{E_r}{Z_2} = 33.1\ A$

Rotor Cu Loss $= 3I_2^2R_2$ (derived before)

$= 3 \times (33.1)^2 0.1$

$\simeq 330W$

We know,

Power across air gap, $P_G = \frac{\text{Rotor Cu Loss}}{\text{Slip}}$

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& P_m = Gross o/p = $\frac{(1-s)\,P_G}{1}$

$= \frac{1-s}{s} \times$ Rotor Cu loss

$= \frac{1-0.05}{0.05} \times 330$

$= 19 \times 330$

$= 6270$ W.

(b) At T_{max}

$S = \frac{R_2}{X_2} = \frac{0.1}{0.9} = 0.11$

$\Rightarrow E_r = SE_2 = 0.11 \times 726 = 8.07$ V
(transformed again)
(due to slip change

Now, $Z_2 = \sqrt{R_2^2 + (S X_2)^2} = 0.1407\ \Omega$

$I_2 = \frac{E_r}{Z_2} = 57.1$ A

Rotor Cu Loss $= 3 I_2^2 R_2 = 978.123$

★ $P_G = \frac{\text{Rotor Cu Loss}}{S_m} = T_{max}$ in syn. watts

$= 8892.02 = T_{max}$ in syn. watts.

★ T_G = Gross torque $= \frac{P_m}{2\pi N/60}$; $N = N_s(1-S)$

★ Speed at $T_{max} = (1 - S_m) N_s$; $S_m = R_2/X_2$

★ Gross torque $= \frac{P_m}{2\pi N/60} = \frac{P_G}{2\pi N_s/60}$

★ For induction machine, Induction motor is always used & Indⁿ generator is seldom used.

Ch- Synchronous Machines

basic structure

★ Stator is similar to induction machine, wound for 3ϕ. In sync. machine, rotor is excited with DC winding.

Synchronous machine

Sync. Generator (or Alternator)

Sync. Motor (Seldom used)

Diffᵗ rotor types :- • Salient pole or Projecting pole type rotor

Concentrated field winding

• Non-salient pole type rotor (or cylindrical rotor)

field windings are distributed

★ Indⁿ machine advantage: It is singly fed.

★ Sync. machine — 2 supplies :- AC to stator & DC to rotor → to create permanent magnet

★ Rotor Types

Cylindrical Rotor (Non-salient pole)

• Cylindrical form with DC field winding distributed

Projecting Pole type (Salient pole)

• Field winding concentrated

* Ind^n motor — rotor speed < n_s (sync.)
* Ind^n generator — rotor speed > n_s

Non Salient	Salient
• high speed turbo alternators (≃ 20 MW power to ~~be~~ generated)	• low speed generators & sync. motors.
• Long rotor with small diameter.	• Large diameter with small length rotor.
• 2 or 4 pole machine types.	• Large no. of poles.
• Has uniform air gap.

* Synchronous machine ⇒ rotor runs at n_s $\left(=\frac{120f}{P}\right)$ for production of torque - spring like synchronous link 1:1 stator & rotor field.

* rotor field & ~~the~~ ~~stor~~ stator field run at n_s. So, they are stationary relative to each other.

* Circuit models of synchronous machines
  ✓ with ~~X_l (leakage reactance)~~ R_a neglected.
  ✓ including X_l.

* Due to armature rxn, in

DC machines	Synchronous machines.
cross magnetising	cross magnetising
demagnetising	demagnetising
	magnetising.

* X_s = synchronous reactance
$= X_a + X_l$
armature, leakage

* E_r : emf under loaded cond^n
→ called as : AIR GAP Emf.

* |||ly as in DC machine, here, we have

Generating mode | Motoring Mode

$V_t = \bar{E}_f - jX_s\bar{I}_a$ | $\bar{V}_t = E_f + jX_s\bar{I}_a$.

(I_a flows into machine)

* Z_s : synchronous impedance
$=$ Total impedance/phase
$= \sqrt{R_a^2 + X_s^2}$

* Determination of X_s.
→ OC & SC Tests.

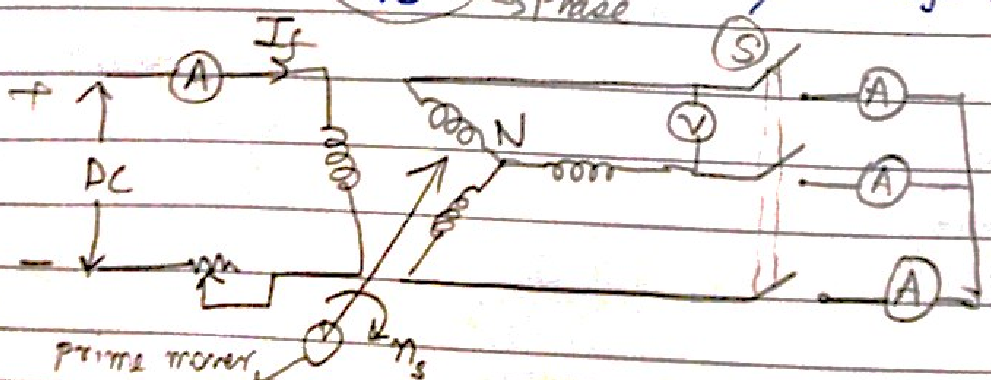
* OC

$I_a = 0$

$V_t = E_f = \dfrac{V_{oc}}{\sqrt{3}}$

V_{oc} → Line value.
$\dfrac{V_{oc}}{\sqrt{3}}$ → Phase

TPSTS : Triple Pole Single Throw Switch



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OCC : Open circuit characteristics :- V_{oc} vs I_f.

* SC   S : switch is closed.
Low I_f circulated.

$V_t = 0$

$\Rightarrow X_S \text{(unsaturated)} = \left. \dfrac{V_{oc}/\sqrt{3}}{I_{sc}} \right|_{I_f \text{ : const.}}$

SCC : I_{sc} vs I_f : linear : for low I_f
: unsaturated magnetic circuit.
: one pt. enough to plot graph

* OCC graph.
DC machine : has residual voltage
Synchronous : no residual voltage.

* Under Loading Cond^ns :-
$X_s \text{(adjusted)} < X_s \text{(unsaturated)}$
(for a saturated magnetic circuit)

* $X_s \text{(adjusted)} = \left. \dfrac{V_{oc}/\sqrt{3}}{I_{sc}} \right|$ I_f corresponding to V_t(rated) on OCC.

* Voltage regulation $= \left. \dfrac{V_{t\,(\text{no load})}\big|_{I_f \text{ same as at full load}} - V_{t\,(\text{rated})}}{V_{t\,(\text{rated})}} \right|_{\text{at specified pf}}$

☆ On air gap line → unsaturated
On OCC line → adjusted.

Q. Given :- OCC & SCC data.

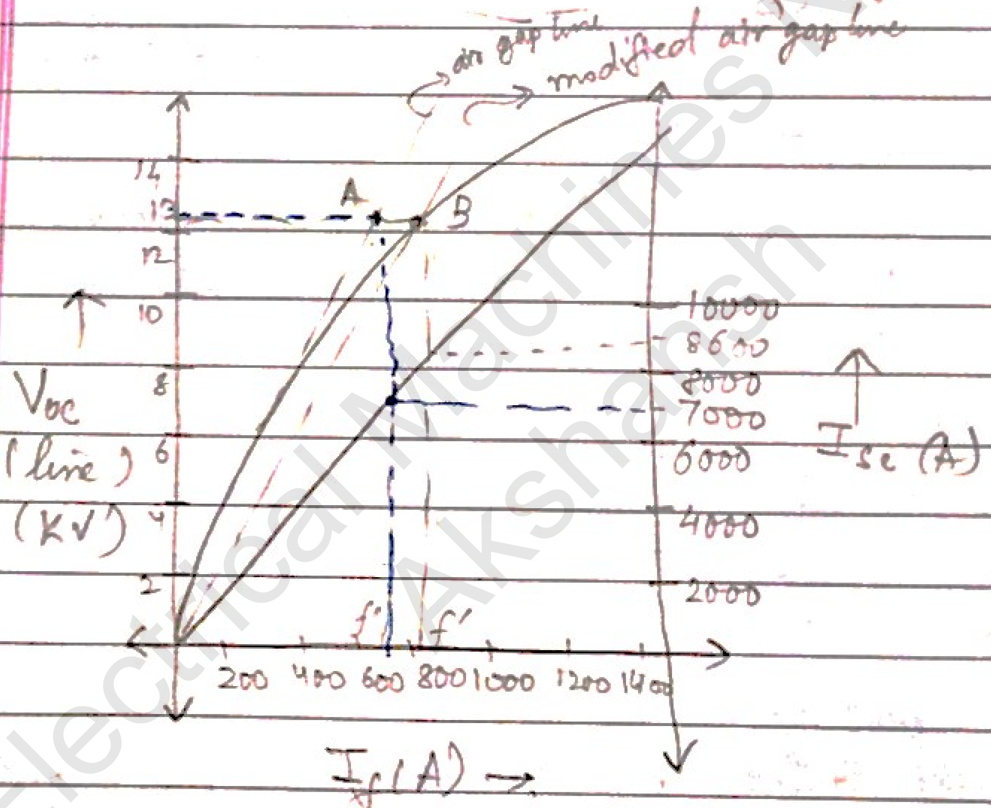
General format

Find :- (a) unsaturated X_s

(b) adjusted X_s

(c) excit^n voltage needed to give rated voltage at full load, 0.8 pf lagging. Use unsaturated X_s.

(d) Voltage regulation above load



(a) pt. A

X_s (unsaturated) $= \dfrac{(13 \times 10^3)/\sqrt{3}}{7000}$ $\left[= \dfrac{V_{oc}/\sqrt{3}}{I_{sc}}\right]$ $V_{oc\,phase}$

$= 1.072\ \Omega$.

(b) pt. B

X_s (adjusted) $= \dfrac{(13 \times 10^3)/\sqrt{3}}{8600.} = 0.873\ \Omega$

Clearly, $X_{s(adj)} < X_{s(unsat.)}$

☆ Synchronous generator :- Also called Alternator.

(c) Excit^n voltage to be found
→ Always we require 2 things
✓ load
✓ pf

$$V_t = \frac{(13 \times 10^3)}{\sqrt{3}} = 7505\ V \text{ (phase value)}$$

$$V_t = 7505 \angle 0°\ V.$$

$\cos\phi = 0.8$, $\Rightarrow \phi = 36.90$

$$I_a(fL) = \frac{\text{Power (rated)} \rightarrow \text{'s VA}}{\sqrt{3}\ \text{Voltage} \rightarrow 13 \times 10^3}$$

$$\left(\because \text{Power} = \sqrt{3} V_L I_L \text{ or } 3 V_{ph} I_{ph} \right)$$

$\Rightarrow I_a = 7837\ A$

$\Rightarrow I_a = 7837 \angle -36.9°.$

$= 7837 (0.8 - j0.6)$

For generating oper^n :-

$$\bar{E}_f = \bar{V}_t + j I_a X_s.$$

Case(I) :- (Phase value)

$X_s(\text{unsat}) = 1.072\ \Omega$

$E_f = 7505 + j(7837)(0.8 - j0.6) \times 1.072$

$= 12546 + j6721$

$E_f = 14233\ V$ or $24.65\ kV$ (line) $\rightarrow 14233 \times \sqrt{3}$

$= V_t(OC)$ (linear mode)

★ Diff. b/w Watt & VA (Volt Ampere)

(VA) Power $= \sqrt{3} V_L I_L = 3 V_{ph} I_{ph}$

(W) Power $= \sqrt{3} V_L I_L \cos\phi = 3 V_{ph} I_{ph} \cos\phi$

$\Rightarrow$ Pf comes into picture when P given in watts

(d) Regul^n :- $\dfrac{V_{t(OC)} - V_t}{V_t} \times 100$

$= \dfrac{14233 - 7505}{7505} \times 100 = 89.64\%$

Note :- Voltage regul^n values :-

1-2 % : Transformer

>60 % : Alternator

★ Purpose of sync. generator :- To generate a 3ϕ AC voltage

★ Bulb experiment in lab

Bus Bar :- Sth with which we connect alternator (generating voltage)

↓

Feeds power to city → constt f, V

★ Synchroniz^n :- Process of above connection.

also called Paralleling

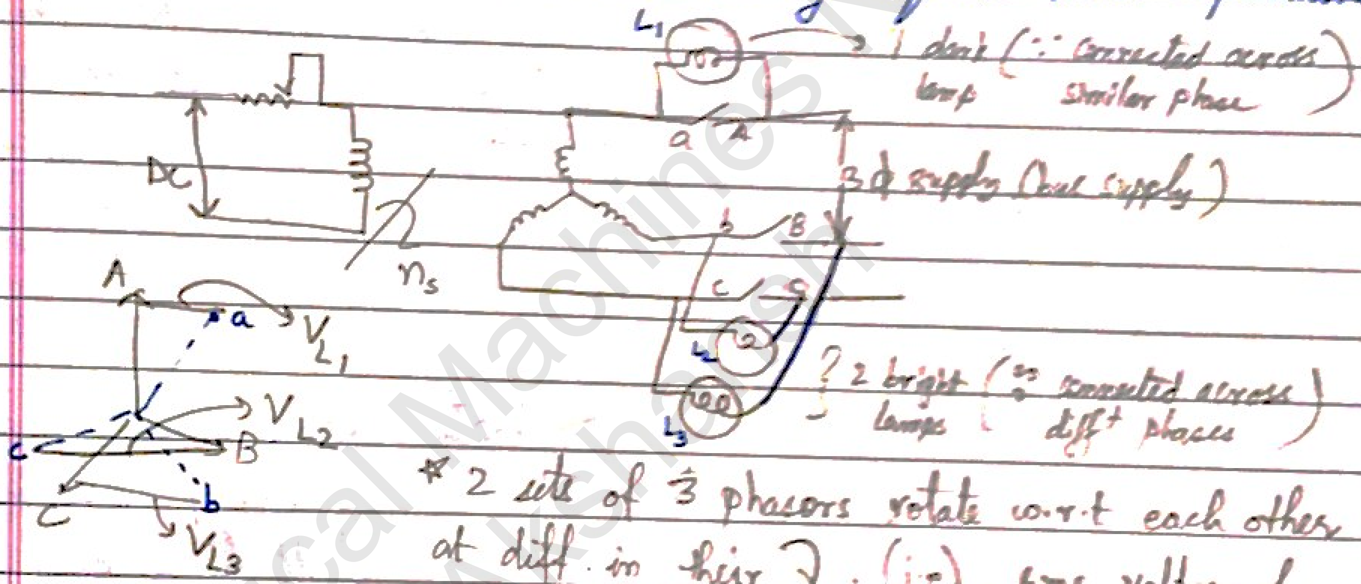
also called Bus bar

Synchronising to mains

- machine runs as generator - same phase sequence as mains

Its speed & field current are adjusted s.t

- machine V_t nearly equals that of mains.
- machine ω nearly equal that of mains.



☆ 2 sets of 3 phasors rotate w.r.t each other at diff. in their ω. (i.e). rms voltage of 3 lamps rotate at diff^t ω

- At instant of sync- action, 2 sets of phasors co-phasal

$V_{L1} = 0$, $V_{L2} = V_{L3}$ bright machine is switched on the mains
dark

- Instead of lamps, synchroscope is used.
- Acceptable φ diff in phasor set is $\simeq 5°$.
- DAMPER winding (or AMORTESSUR winding)
  - → Oscill^{ns} due to synchronism
  - → hunting - tackled by SC Cu bars in rotor pole faces
  - → Ind^n start synch. motor ———— LEARN

end of syllabus