ELECTROMAGNETIC THEORY NOTES

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Electromagnetic Theory Notes, First Edition

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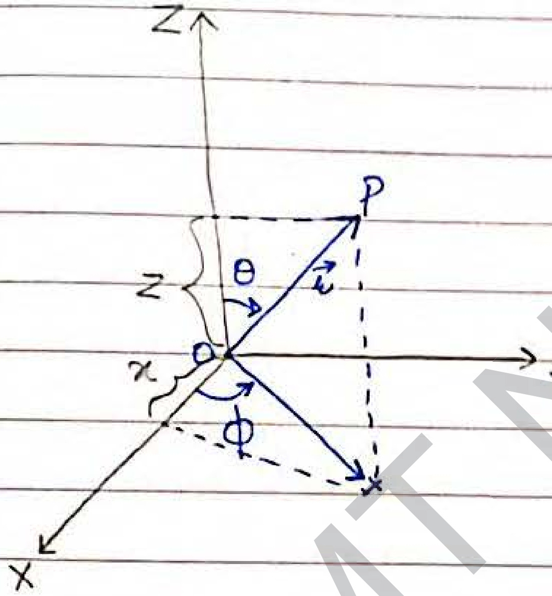
Puffin
Date 03-09-2017
Page

Vectors

☆ SPHERICAL POLAR COORDINATE (r, θ, ϕ)



$z = r\cos\theta$
$x = r\sin\theta\cos\phi$.

$\vec{dl} = dl_r\hat{r} + dl_\theta\hat{\theta} + dl_\phi\hat{\phi}$

$\Rightarrow \vec{dl} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

(||ly. like ~~dV~~ $dl = dx + dy + dz$)

$d\tau = (dr)(r\,d\theta)(r\sin\theta\,d\phi)$

(||ly like $d\tau = (dx)(dy)(dz)$)

$V = \int d\tau = \int r^2 \sin\theta\,dr\,d\theta\,d\phi$

$= \int_0^R r^2 dr \int_0^\pi \sin\theta\,d\theta \int_0^{2\pi} d\phi$

$V = \left(\frac{R^3}{3}\right)(2)(2\pi) = \frac{4\pi}{3}R^3$.

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Date ____
Page ____

Q. SA of sphere

$$dA = (r\sin\theta\, d\phi)(r\, d\theta)$$

$$A = \int dA$$

$$= R^2 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$A = 4\pi R^2$$

Q. Area of circle

$$dA = \int_{\phi=0}^{2\pi} r\sin\theta\, d\phi \int_0^{R} dr$$

$$= \left(\frac{R^2}{2}\right)\left(\sin\frac{\pi}{2}\right)\int_0^{2\pi} d\phi$$

$$= \frac{R^2}{2}(1)(2\pi)$$

$$= \pi R^2$$

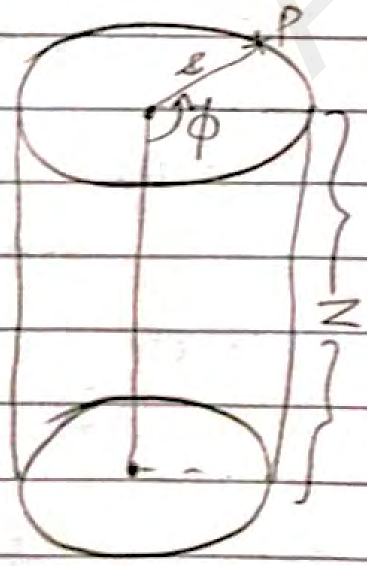
☆ CYLINDRICAL COORDINATE SYS

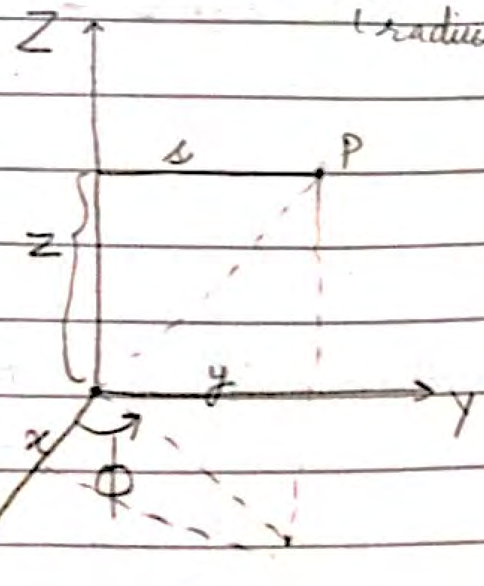
(S, Z, ϕ)

S → distance from z axis; ρ or s → 0 to R (radius)

Z → height

ϕ → 0 to 2π





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$z = z$

$x = s\cos\phi$

$y = s\sin\phi$.

$$\vec{dl} = ds\,\hat{s} + dz\,\hat{z} + s\,d\phi\,\hat{\phi}$$

$d\phi = \frac{l}{s}$

$\Rightarrow l = s\,d\phi$

Q Vol. of cylinder

$$V = \int_0^R ds \int_0^H dz \int_0^{2\pi} s\,d\phi$$

$$= \frac{R^2}{\cancel{2}}(H)(\cancel{2}\pi)$$

$$V = \pi R^2 H$$

* $\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$: operator or vector operator

hence,

$\nabla \cdot \vec{V}$ = Divergence of $\vec{V}$ ($\vec{V}$ any vector)

$\nabla \times \vec{V}$ = curl of $\vec{V}$.

* DIVERGENCE

Consider a vector $\vec{V} = V_x\hat{x} + V_y\hat{y} + V_z\hat{z}$

$$\nabla \cdot \vec{V} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\left(V_x\hat{x} + V_y\hat{y} + V_z\hat{z}\right)$$

$$= \frac{\partial}{\partial x}V_x + \frac{\partial}{\partial y}V_y + \frac{\partial}{\partial z}V_z \quad \text{(scalar)}$$

* Divergence is a scalar.

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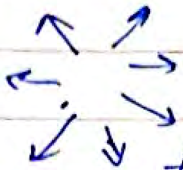
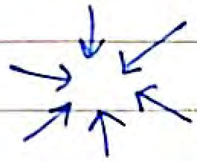
* Divergence is a measure of how much vector ($\vec{V}$) spreads out (diverges) from the pt.

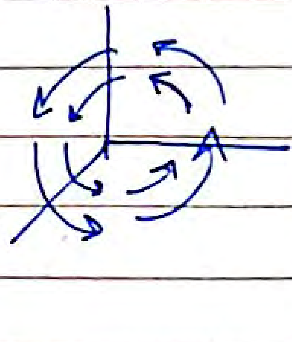

+ve divergence


-ve divergence

* CURL

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} \quad \text{(Vector)}$$

* Curl is a measure of how much vector ($\vec{V}$) curls around the point.
ex: whirlpool.



* LINE INTEGRAL

$$\int_a^b \vec{V} \cdot \vec{dl}$$

↳ If $a = b$, its a closed loop integral

$$\oint \vec{V} \cdot \vec{dl}$$

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☆ SURFACE INTEGRAL

$$\int_S \vec{V}\cdot d\vec{A} = \text{Flux}$$

↳ For closed surface

$$\oint \vec{V}\cdot d\vec{A}$$

☆ VOLUME INTEGRAL

$$\int_{volume} T \cdot d\tau$$

↳ $d\tau = dx\,dy\,dz \Rightarrow$ Infinitesimal vol. ~~extent~~ elemt.

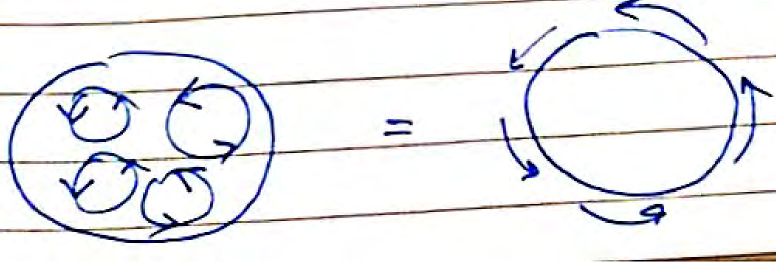
☆ DIVERGENCE THM.

$$\int_{vol.} (\nabla\cdot\vec{V})\,d\tau = \oint_S \vec{V}\cdot d\vec{A}$$

eg: water coming out of a tank

☆ CURL THM.

$$\int_S (\nabla\times\vec{V})\cdot da = \oint \vec{V}\cdot d\vec{l}$$



(∵ all inner loops cancel each other)

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Puffin Date Page

☆ Curl of electric field = 0. ($\oint \vec{E} \cdot \vec{dl} = 0$)
↳ In case of electroSTATICS.

☆ $\nabla \times E \neq 0$
↳ In case of electroDYNAMICS ($\vec{B}$ changes).

$\to E_{induced}$

Q. Find Divergence & curl of any vector in cylindrical coordinate & polar coordinate sys.

Q. Find $\nabla \cdot (\nabla \times \vec{A}) = ?$

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}.$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left[\hat{x}\left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y\right) - \hat{y}\left(\frac{\partial}{\partial x} A_z - \frac{\partial}{\partial z} A_x\right) + \hat{z}\left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x\right) \right] \cdot \underbrace{\left[\frac{\partial}{\partial x}\hat{x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right]}_{\nabla}$$

$$= \frac{\partial}{\partial x}\left[\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y\right] - \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} A_z - \frac{\partial}{\partial z} A_x\right) + \frac{\partial}{\partial z}\left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x\right)$$

Puffin
Date
Page

$$= \frac{\partial^2}{\partial x \partial y} A_z - \frac{\partial^2}{\partial y \partial x} A_z + \frac{\partial^2}{\partial x \partial z} A_y - \frac{\partial^2}{\partial z \partial x} A_y$$

$$\frac{\partial^2}{\partial y \partial z} A_x - \frac{\partial^2}{\partial z \partial y} A_x$$

$$= 0 \quad \left(\because \frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x} \right)$$

We know, $\nabla \cdot \bar{B} = 0$ (Magnetic field doesn't diverge from one pt.)

So, $\boxed{\vec{B} = \nabla \times \vec{A}}$.

Q Find $\nabla \times (\nabla \cdot T)$ $\rightarrow \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$

T: scalar

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}.$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial T}{\partial x} & \frac{\partial T}{\partial y} & \frac{\partial T}{\partial z} \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial^2 T}{\partial y \partial z} - \frac{\partial^2 T}{\partial y \partial z} \right) - \hat{y}\left(\frac{\partial^2 T}{\partial x \partial z} - \frac{\partial^2 T}{\partial x \partial z} \right)$$

$$+ \hat{z}\left(\frac{\partial^2 T}{\partial x \partial y} - \frac{\partial^2 T}{\partial x \partial y} \right)$$

$$= 0$$

We know, $\nabla \times \vec{E} = 0$ (Electric field doesn't curl)

So, $E = -\nabla V$ i.e, $\vec{E}$ can be written in terms of a scalar.

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✯ POLAR COORDINATE SYS. (r, θ, ϕ)

Divergence $\nabla \cdot \vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, V_\theta\right) + \frac{1}{r\sin\theta}\left(\frac{\partial V_\phi}{\partial\phi}\right)$

$\vec{V} \to V_r, V_\theta, V_\phi$

$V_r \to$ radial component of $\vec{V}$

Curl: $\nabla \times \vec{V} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, V_\phi) - \frac{\partial V_\theta}{\partial\phi}\right]\hat{r}$

$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial V_r}{\partial\phi} - \frac{\partial}{\partial r}(r V_\phi)\right]\hat{\theta}$

$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(r V_\theta) - \frac{\partial V_r}{\partial\theta}\right]\hat{\phi}$

Q. $\vec{V} = (r\cos\theta)\hat{r} + (r\sin\theta)\hat{\theta} + (r\sin\theta\cos\phi)\hat{\phi}$
Calculate divergence & curl

$\nabla\cdot\vec{V} = \frac{1}{\cancel{r^2}}\cos\theta\,(3\cancel{r^2}) + \frac{\cancel{r}}{\cancel{r}\cancel{\sin\theta}}(2\cancel{\sin\theta}\cos\theta) + \frac{\cancel{r\sin\theta}}{\cancel{r\sin\theta}}(-\sin\phi)$

$= 3\cos\theta + 2\cos\theta - \sin\phi$

$\nabla\cdot\vec{V} = 5\cos\theta - \sin\phi$.

$\nabla\times\vec{V} = \frac{1}{r\sin\theta}\left[2r\cos\phi\sin\theta\cos\theta - 0\right] + \hat{r}$

$+ \frac{1}{r}\left[\frac{1}{\sin\theta}(0) - 2r\sin\theta\cos\phi\right]\hat{\theta}$

$+ \frac{1}{r}\left[2r\sin\theta + r\sin\theta\right]\hat{\phi}$

$$= (2\cos\theta\cos\phi)\hat{r} - (2\sin\theta\cos\phi)\hat{\theta} + (2\sin\theta + \sin\theta)\hat{\phi}$$

✱ CYLINDRICAL COORDINATE : (s, z, ϕ)

- $\nabla \cdot \vec{V} = \frac{1}{s}\frac{\partial}{\partial s}(sV_s) + \frac{1}{s}\frac{\partial}{\partial \phi}V_\phi + \frac{\partial}{\partial z}V_z$

  $\to V_s, V_\phi, V_z$

- $\nabla \times \vec{V} = \left(\frac{1}{s}\frac{\partial}{\partial \phi}V_z - \frac{\partial}{\partial z}V_\phi\right)\hat{s} + \left(\frac{\partial}{\partial z}V_s - \frac{\partial}{\partial s}V_z\right)\hat{\phi}$

  $+ \frac{1}{s}\left(\frac{\partial}{\partial s}(sV_\phi) - \frac{\partial}{\partial \phi}V_s\right)\hat{z}$

Q. $\vec{V} = s(2+\sin^2\phi)\hat{s} + s\sin\phi\cos\phi\,\hat{\phi} + 3z\,\hat{z}$

$\nabla \cdot \vec{V} = \frac{1}{s}(4s + 2s\sin^2\phi) + \frac{1}{s}(s)\left[-\sin^2\phi + \cos^2\phi\right]$

$+ 3$

$= 4 + 2\sin^2\phi - \sin^2\phi + \cos^2\phi + 3$

$= 8$

$\nabla \times \vec{V} = \left(\frac{1}{s}(0) - 0\right) + (0 - 0) + \frac{1}{s}\left(\underbrace{2s}_{\sin\phi\cos\phi} - s(\sin 2\phi)\right)$

$= 2\sin\phi\cos\phi - \sin 2\phi$

$= 0$

Q. $\vec{V} = \frac{1}{r^2}\hat{r}$.

✓ Find $\nabla\cdot\vec{V}$.

$\nabla\cdot\vec{V} = \frac{1}{r^2}(0) = 0 \rightarrow (1)$

✓ Find $\oint_S \vec{V}\cdot da$ for a sphere of radius R.

$= \oint_S \frac{1}{R^2}\cdot(R\,d\theta)(R\sin\theta\,d\phi)$

$r = R$ on surface

$= \int_0^{\pi}\sin\theta\,d\theta\int_0^{2\pi}d\phi$

$= 2(2\pi)$

$= 4\pi \rightarrow (2)$

By divergence thm.,

$$\int(\nabla\cdot V)\,d\tau = \oint_S V\cdot da$$

$\int(\nabla\cdot V)\,d\tau \rightarrow 0$, $\oint_S V\cdot da \rightarrow 4\pi$

$$0 = 4\pi \;?$$

Reason :- $\int v\cdot dA$ doesn't obey the property of a f^n. It's a DIRAC-DELTA f^n.

We are leaving the pt. $r=0$ while finding divergence. So, make a f^n giving 4π at $r=0$ & 0 at other values of r.

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✲ DIRAC-DELTA F^n :-

1D : $\delta(x)$

$\delta(x)$: an infinitely high, infinitesimally narrow spike with area unity (1)

- $\delta(x) = \begin{cases} 0 & , x \neq 0 \\ \infty & , x = 0 \end{cases}$, practically, a very large value.

(Fig 1)

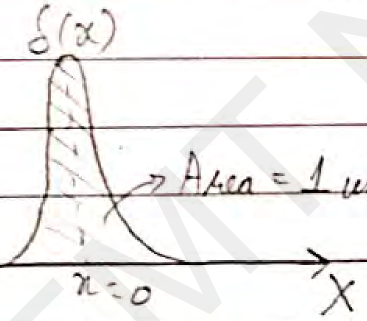

Fig 1

Let $f(x)$ = some arbitrary f^n

$$f(x) \cdot \delta(x) = f(0)\,\delta(x).$$

$$\int_{-\infty}^{\infty} f(x)\,\delta(x)\,dx = f(0) \int_{-\infty}^{\infty} \delta(x)\,dx$$

(Area = 1)

$$= f(0)$$

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\,\delta(x)\,dx = f(0)$$

(Any f^n multiplied by Delta f^n, we get value only at $x=0$)

- $\delta(x-a) = \begin{cases} 0 & x \neq a \\ \infty & x = a \end{cases}$ with $\int_{-\infty}^{\infty} \delta(x-a)\,dx = 1$

(we can shift the spike from $x=0$ to some other pt., $x=a$)

PTO (Fig 2)

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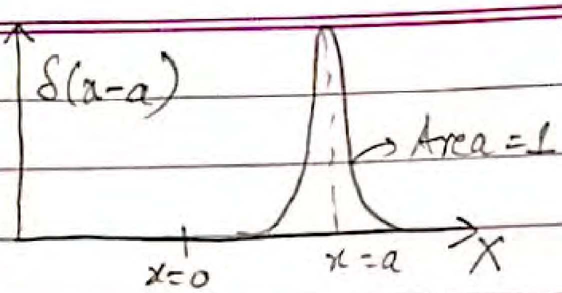

Fig 2

So, $f(x)\,\delta(x-a) = f(a)\,\delta(x-a)$

$$\int_{-\infty}^{\infty} f(x)\,\delta(x-a)\,dx = f(a)$$ ★

Q. 1.43(a) $\int_{2}^{6} (3x^2-2x-1)\,\delta(x-3)\,dx$

$\to f(x)$, put $x=3$ in $f(x)$

$= 3(3)^2 - 2(3) - 1 = 27 - 7 = 20$ Ans

(b) $\int_{0}^{5} \cos x\,\delta(x-\pi)\,dx = -1$ $(\cos\pi)$.

$\to f(x)$, put $x=\pi$

(c) $\int_{-\infty}^{\infty} \log(x+3)\,\delta(x+2)\,dx = \log(-2+3) = 0$.

★ Property :- $\delta(kx) = \frac{1}{|k|}\delta(x)$

Let $\int f(x)\,\delta(kx)\,dx = \,?\ \to (1)$

Let $y = kx$, $x = y/k \Rightarrow dx = \left(\frac{1}{k}\right)dy$.

So, $\int f(y/k)\,\delta(y)\,\frac{dy}{k} = \boxed{\frac{f(0)}{k}}$ ★

$= \frac{1}{k}\int f(x)\,\delta(x)\,dx \to (2)$

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Comparing ① & ②

$$\delta(kx) = \frac{1}{k}\delta(x)$$

Q.1.44 (a) $\int_{-2}^{2} (2x+3)\,\delta(3x)\,dx$

$= \frac{1}{3}(2(0)+3) = 1$ Ans

$$\left[\begin{array}{l} \delta(3x) = \frac{1}{3}\delta(x) \\ \frac{1}{3}\int (2x+3)\,\delta(x)\,dx \Rightarrow f = \frac{1}{3}(2.0+3) = 1 \end{array}\right]$$

(b) $\int (x^3+3x+2)\,\delta(1-x)\,dx$

$\delta(1-x) \Rightarrow \delta(x-1)$

$= 1+3+2 = 6$

☆ Property :- $\delta(-x) = \delta(x)$

(c) $\int 9x^2\,\delta(3x+1)\,dx$

$= \frac{1}{3}\int 9x^2\,\delta\left(x+\frac{1}{3}\right)dx$

$= \frac{1}{3}\,\cancel{9}\left(+\frac{1}{\cancel{9}}\right) = \frac{1}{3}$ Ans

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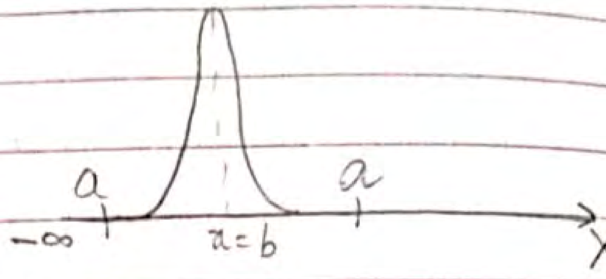
(d) $\int_{-\infty}^{a} \delta(x-b)\,dx$

$$= \begin{cases} 1 & a > b \\ 0 & a < b \end{cases}$$



☆ 3D : DELTA f^n

$$\delta^3(\vec{r}) = \delta(x)\,\delta(y)\,\delta(z)$$
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

- $\int \delta^3(r)\,d\tau = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x)\,\delta(y)\,\delta(z)\,dx\,dy\,dz = 1$

- $\int f(r)\,\delta^3(r-a)\,d\tau = f(a)$

  $\hookrightarrow$ eg :- $f = \dfrac{\hat{r}}{r^2}$

$$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\, \underline{\delta^3(r)}$$

multiply the previous value by Delta f^n to show that value $= 4\pi$ only when $r=0$

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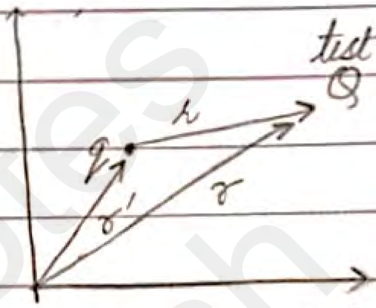
Puffin
Date 11/09/12
Page

Chapter - 2
ELECTROSTATICS

★ Coulomb's Law



$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{r}$$

$r = r - r'$

ϵ_0 : permittivity of free space
$= 8.854 \times 10^{-12}$

$\vec{F}$: Force on Q due to q.

- If several forces $q_1, q_2 \dots q_n$ at distances $r_1, r_2 \dots r_n$ from Q, total force on Q is

$$F = F_1 + F_2 + \dots = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 Q}{r_1^2}\hat{r}_1 + \dots\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \dots\right)$$

$$= QE \; ; \; \vec{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n} \frac{q_i}{r_i^2}\hat{r}_i$$

= Electric field.

★ $E = \frac{F}{Q}$ = Force p.u. charge.

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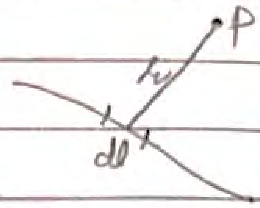
✲ CONTINUOUS CHARGE DISTRIBUTION

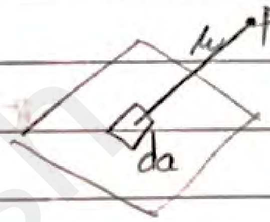
1. Line Charge $(\lambda) = \frac{\text{Charge}}{\text{Length}}$



$$E(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dl}{r^2}\hat{r}$$

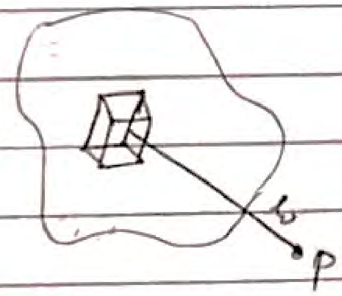
2. Surface charge $(\sigma) = \frac{\text{Charge}}{\text{Area}}$

$$E(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\, da}{r^2}\hat{r}$$

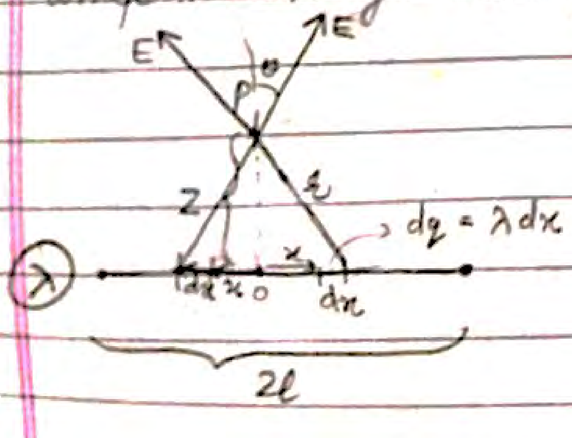


3. Volume charge $(\rho) = \frac{\text{Charge}}{\text{Volume}}$

$$E(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\, d\tau}{r^2}\hat{r}$$



ex. 2.1 Find electric field $(\vec{E})$ at a distance 'z' above mid pt. of a st. segment of length '2l' which has a uniform charge density 'λ'.



$$E_{Total} = 2E\cos\theta$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{(z^2+x^2)}$$

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$$\cos\theta = \frac{z}{\sqrt{x^2+z^2}}$$

So, $$E_{Total} = 2\left(\frac{1}{4\pi\epsilon_0}\right)\int_0^{l} \frac{\lambda\,dx}{(z^2+x^2)} \times \frac{z}{\sqrt{x^2+z^2}}$$

$$= 2\left(\frac{1}{4\pi\epsilon_0}\right)(\lambda z)\int_0^{l} \frac{dx}{(z^2+x^2)^{3/2}}$$

$$= \left(\frac{1}{2\pi\epsilon_0}\right)(\lambda z)\int_0^{\tan^{-1}(l/z)} \frac{z\sec^2\theta\,d\theta}{z^3(\sec^3\theta)} \qquad x = z\tan\theta \Rightarrow dx = z\sec^2\theta\,d\theta$$

$$= \frac{\lambda}{2\pi\epsilon_0 z^2}\int_0^{\theta} \cos\theta\,d\theta$$

$$= \frac{\lambda}{2\pi\epsilon_0 z^2}\left[\sin\theta\right]_0^{\theta=\sin^{-1}\left(\frac{x}{\sqrt{z^2+x^2}}\right)}$$

$$= \frac{2\lambda}{4\pi\epsilon_0 z^2}\left[\frac{x}{\sqrt{z^2+x^2}}\right]_0^{l}$$

$$\Rightarrow E_{total} = \frac{2\lambda l}{4\pi\epsilon_0\sqrt{z^2+l^2}}$$

$$\Rightarrow \boxed{E_{Total} = \frac{1}{4\pi\epsilon_0}\left(\frac{2\lambda l}{\sqrt{z^2+l^2}}\right)}$$

$\hookrightarrow z >> l.$

$$E_{Total} = \frac{q}{4\pi\epsilon_0 z^2}$$

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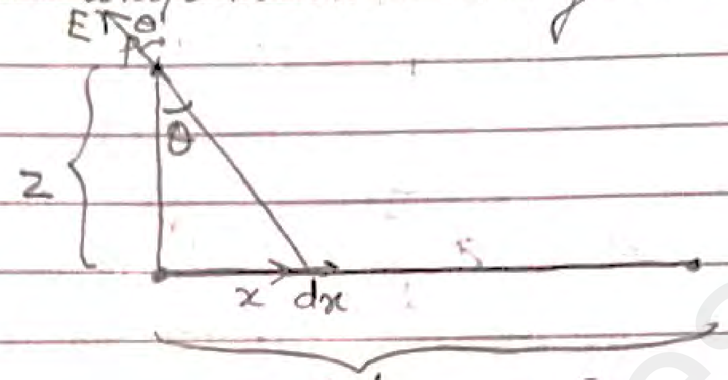
ex 2.3 Find $\vec{E}$ at a distance z above one end of a st. line segment of length 'l' which carries a uniform line charge 'λ'.



$$\text{Total } E = \int E\cos\theta + \int_{}^{l} E\sin\theta$$

$$= \int_0^l \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda\, dx}{(z^2+x^2)} \times \frac{z}{\sqrt{z^2+x^2}}\ \hat{j}$$

$$+ \int_0^l \frac{1}{4\pi\epsilon_0} \frac{\lambda\, dx}{(z^2+x^2)} \times \frac{x}{\sqrt{z^2+x^2}}\ \hat{i}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[\int_0^l \frac{z\,dx}{(z^2+x^2)^{3/2}} + \frac{1}{2}\int_0^l \frac{2x\,dx}{(z^2+x^2)^{3/2}}\right]$$

$x = z\tan\theta$ $\qquad$ $z^2+x^2 = t^2$

$$= \frac{\lambda z}{4\pi\epsilon_0}\left[\int_0^{\theta} \frac{\cancel{z}\sec^2\theta\, d\theta}{(z^3)\sec^3\theta} + \frac{1}{\cancel{z}}\int_0^{\sqrt{z^2+l^2}} \frac{\cancel{2}t\,dt}{t^3}\right]$$

$$= \frac{\lambda z}{4\pi\epsilon_0}\left[\frac{1}{z^2}\int_0^{\theta}\cos\theta\, d\theta + \left[\frac{-1}{t}\right]_0^{\sqrt{z^2+l^2}}\right]$$

$$= \frac{\lambda z}{4\pi\epsilon_0}\left[\frac{1}{z^2}\left[\frac{x}{\sqrt{x^2+z^2}}\right]_0^l + \left[-\frac{1}{\sqrt{(z^2+x^2)}}\right]_0^l\right]$$

$$= \frac{\lambda \cancel{z}}{4\pi\epsilon_0}\left[\frac{l}{z^{\cancel{2}}\sqrt{l^2+z^2}}\right] + \left[-\frac{1}{\sqrt{(z^2+l^2)}} + \frac{1}{z}\right]$$

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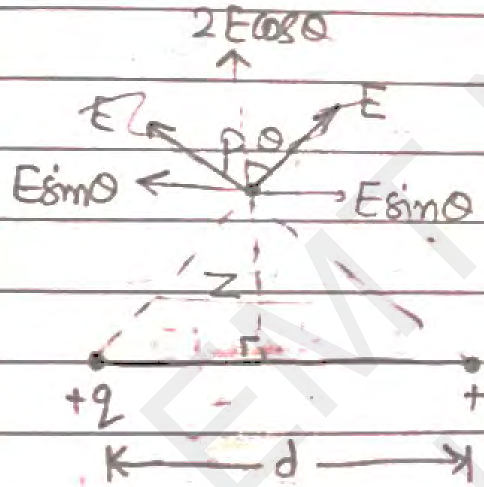
$$\Rightarrow \text{Total } E = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{l}{z\sqrt{l^2+z^2}}\right]\hat{j} + \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{(z^2+l^2)}}\right](-\hat{i})$$

$$\Rightarrow \text{Total} = \left[\frac{\lambda l}{4\pi\epsilon_0 z\sqrt{z^2+l^2}}\right]\hat{j} + \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+l^2}}\right](-\hat{i})$$

Q(a) Find $\vec{E}$ at a distance z above the mid pt. b/w 2 equal charges, q, distance d apart. Check for $z \gg d$.

(b) Repeat part (a) with charges +q & −q.

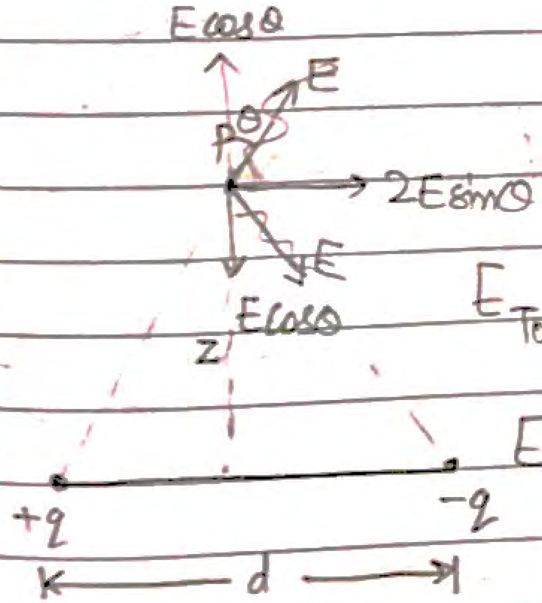
(a) 

$$E_{Total} = \left[\frac{q}{4\pi\epsilon_0\left(z^2 + d^2/4\right)}\right][2\cos\theta]$$

$$= \frac{q}{4\pi\epsilon_0(z^2+d^2/4)}\left(2\times\frac{z}{\sqrt{z^2+d^2/4}}\right)$$

$$\Rightarrow E_{Total} = \frac{2qz}{4\pi\epsilon_0\left(z^2+d^2/4\right)^{3/2}}$$

$\hookrightarrow z \gg d$: $E = \dfrac{2q}{4\pi\epsilon_0 z^2}$ (pt charge)

(b) 

$$E_{Total} = (2)\left(\frac{q}{4\pi\epsilon_0\left(z^2+\frac{d^2}{4}\right)}\right)\left(\frac{d/2}{\sqrt{z^2+d^2/4}}\right)$$

$$E_{Total} = \frac{qd}{4\pi\epsilon_0\left(z^2+d^2/4\right)^{3/2}}$$

$\hookrightarrow z \gg d$; $E = \dfrac{qd}{4\pi\epsilon_0 z^3}$

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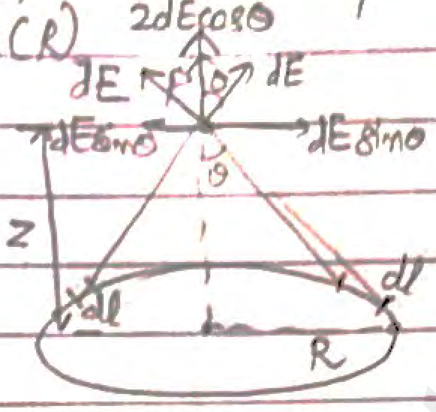
Puffin
Date
Page

2.5

Q Circle with line charge density (λ), calculate $\vec{E}$ from centre of circle 'z' distance above.

Radius (R)



$$E_{Total} = \int dE\cos\theta$$

$$= \int \frac{\lambda\, dl}{4\pi\epsilon_0 (z^2+R^2)} \times \frac{z}{\sqrt{z^2+R^2}}$$

$$= \frac{\lambda z}{4\pi\epsilon_0 (z^2+R^2)^{3/2}} \int_0^{2\pi R} dl$$

$$= \frac{\lambda z}{4\pi\epsilon_0 (z^2+R^2)^{3/2}} (2\pi R)$$

$$\Rightarrow E_{Total} = \frac{\lambda z R}{2\epsilon_0 (z^2+R^2)^{3/2}}$$

$$= \frac{(2\pi R\lambda) z}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

Total charge Q

$$= \frac{Qz}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

$\hookrightarrow z >> R$

$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0 z^2} \quad \text{(for pt. charge)}$$

26 We have a disk of surface charge density (σ). Rest from Q. 2.5

PTO

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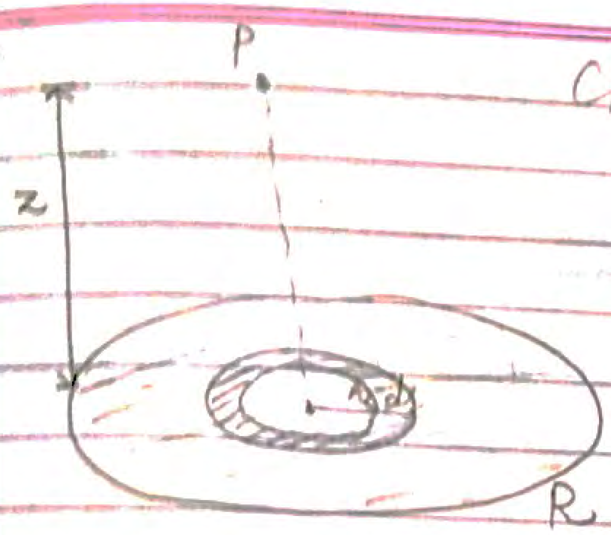

Charge on small ring,

$$q = 2\pi r\,dr\,\sigma$$

$$E_{Total} = \int E_{\text{due to ring}}$$

$$= \int \frac{Q}{4\pi\epsilon_0 (z^2+r^2)^{3/2}}$$

$$Q \to 2\pi r\,dr\,\sigma$$

$$= \int_0^R \frac{2\pi r\,dr\,\sigma}{4\pi\epsilon_0 (z^2+r^2)^{3/2}}$$

$$= \frac{\sigma}{2\epsilon_0}\int \frac{r\,dr}{(z^2+r^2)^{3/2}}$$

$$z^2 + r^2 = t^2$$

$$\Rightarrow 2r\,dr = 2t\,dt$$

$$\Rightarrow r\,dr = t\,dt$$

$$= \frac{\sigma}{2\epsilon_0}\int_z^{\sqrt{z^2+R^2}} \frac{\cancel{t}\,dt}{t^{\cancel{3}\,2}}$$

$$= \frac{\sigma}{2\epsilon_0}\left(-\frac{1}{t}\right)_z^{\sqrt{z^2+R^2}}$$

$$= \frac{\sigma}{2\epsilon_0}\left(\frac{-1}{\sqrt{z^2+r^2}}\right)_0^R$$

$$\Rightarrow E_{Total} = \frac{\sigma}{2\epsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right)$$

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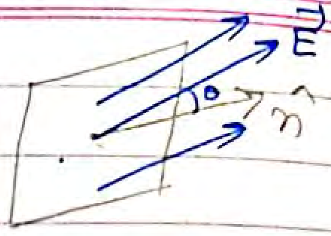
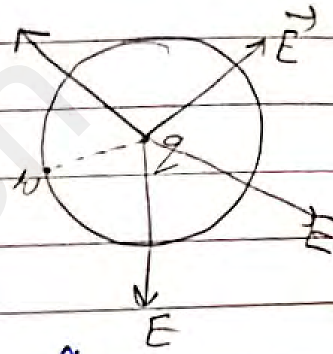
☆ Divergence of $\vec{E}$



$$\oint \vec{E}\cdot dA = \text{Flux}.$$

$= $ no. of ~~feld~~ field lines passing through surface.

☆ Charge at centre of sphere:



$$\oint_s E\cdot dA = \int \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dA$$

$$= \frac{q}{4\pi\epsilon_0 r^2}\int dA.$$

$$= \frac{q}{4\pi\epsilon_0 r^2}(4\pi r^2) = \frac{q}{\epsilon_0}.$$

$$\Rightarrow \oint_s \vec{E}\cdot\vec{dA} = \frac{q}{\epsilon_0}$$; q : charge enclosed within surface $= q_{enclosed}$.

or:- $$\boxed{\oint \vec{E}\cdot da = \frac{q_{encl}}{\epsilon_0}.}$$: GAUSS LAW

• Using divergence thm., $\oint_s \vec{E}\cdot\vec{dA} = \int_V (\nabla\cdot E)\,d\tau$

$$q_{encl} = \int_V \rho\,d\tau$$

$$\int_V (\nabla\cdot E)\,d\tau = \int_V \frac{\rho}{\epsilon_0}\,d\tau \Rightarrow \boxed{\nabla\cdot E = \frac{\rho}{\epsilon_0}.}$$ ☆

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$$\boxed{\nabla \cdot E = \frac{\rho}{\epsilon_0}}$$: Gauss Law in Differential form.

☆ Electric flux : $\phi_E = E \cdot A = EA\cos\theta$

$= \text{net flux}$

$=$ difference of field lines leaving & entering surface.

(Net flux through surface) $\propto$ (net no. of lines leaving the surface)

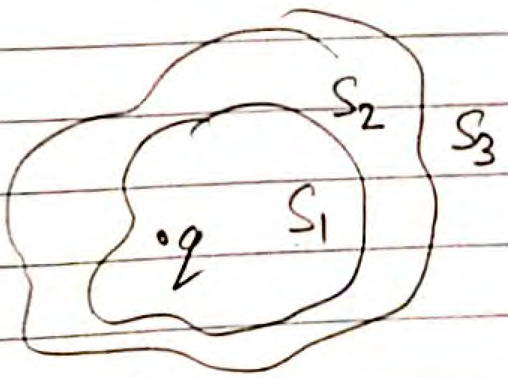
Net no. of lines = No. of lines leaving surface − no. of lines entering the surface.

examples

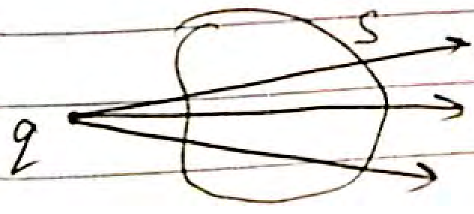
(1) 

entering = leaving

net $\phi_E = 0$.

(2) 

$\phi_{S_1} = \phi_{S_2} = \phi_{S_3}$

(3) 

net $\phi_s = 0$; net lines coming in = coming out

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☆ Application of Gauss Law:

It's useful only when charge distribⁿ is SYMMETRic in nature

- (a) Spherical Symm: Make Gaussian Surface (G.S) a concentric sphere.
- (b) Cylindrical Symm: G.S. : coaxial cylinder
- (c) Plane symm:— Use a Gaussian "pillbox" (like a cube/cuboid)

- If $\vec{E}$ & $\vec{dA}$ are in same dirⁿ, $\vec{E}\cdot dA = E(dA)$.
  (So, E can be taken out of integral)

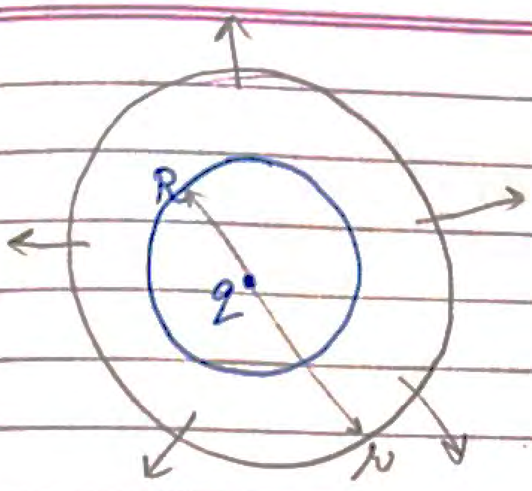
ex. Find field outside a uniformly charged solid sphere (R, q). At a pt $r\ (>R)$

S1. Draw a Gaussian surface at r.
($\vec{E}$ should be same at any pt on G.S)

S2) Apply Gauss law on G.S.

S3) Take E out of integral & evaluate (LHS)

S4) Find $q_{encl.}$ (RHS)



Gauss law :-

$$\oint E \cdot dA = \frac{q_{encl}}{\epsilon_0}.$$

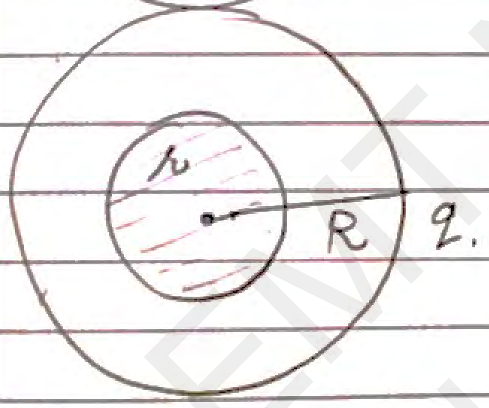
$$E \oint dA = E(4\pi r^2)$$

$$q_{encl} = q.$$

$$\Rightarrow E(4\pi r^2) = \frac{q}{\epsilon_0}.$$

$$\Rightarrow E = \left(\frac{q}{4\pi\epsilon_0 r^2}\right)\hat{r}$$

Q. $r < R$



$$\oint E \cdot dA = \frac{q_{encl}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{1}{\epsilon_0}\left(\frac{q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right)$$

$$= \frac{q r^3}{\epsilon_0 R^3}$$

$$\Rightarrow \vec{E} = \frac{q}{4\pi\epsilon_0}\left(\frac{r}{R^3}\right)\hat{r}.$$

Q. Hollow Sphere

$$\vec{E}(r > R) = \frac{q}{4\pi\epsilon_0 r^2}\hat{r} \quad \left(E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}\right)$$

$$\vec{E}(r < R) = \vec{0}$$ (charge enclosed = 0 as charge is only on surface)

$$\left(E \cdot 4\pi r^2 = 0/\epsilon_0\right)$$

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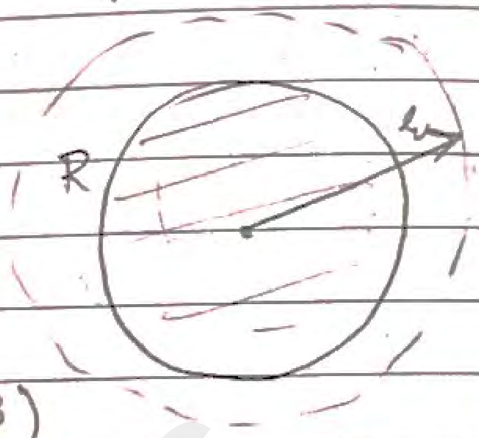
Date
Page

Q. Solid sphere (ρ given)

$\vec{E}$? $(r>R)$

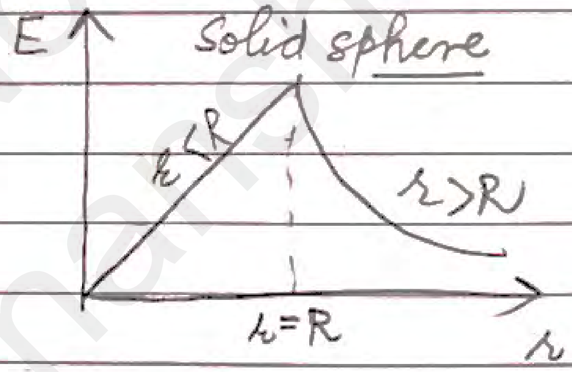


$$\oint \vec{E}\, d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow \vec{E}(4\pi r^2) = \frac{\rho\left(\frac{4}{3}\pi R^3\right)}{\epsilon_0}$$

$\because$ total charge still lying on surface.

$$\Rightarrow \vec{E} = \frac{\rho}{3\epsilon_0}\left(\frac{R^3}{r^2}\right)\hat{r}$$



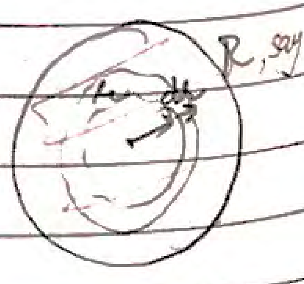
Q. $\vec{E}$? Solid sphere

Given :- $\rho = kr$, some k &

r : distance from origin.

$r<R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$$



$$\Rightarrow \vec{E}\cdot(4\pi r^2) = \frac{\rho(4\pi r^2\, dr)}{\epsilon_0}$$

$$\Rightarrow \vec{E}\cdot(4\pi r^2) = \int \frac{kr\,(4\pi r^2\, dr)}{\epsilon_0}$$

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$$\Rightarrow \vec{E}(4\pi r^2) = \frac{k(4\pi)\rho}{\epsilon_0}\int_0^r r^3\,dr$$

$$\Rightarrow \vec{E}(4\cancel{\pi} r^2) = \frac{\rho k\cancel{\pi}}{\epsilon_0}\cancel{4}\times\frac{r^4}{\cancel{4}}$$

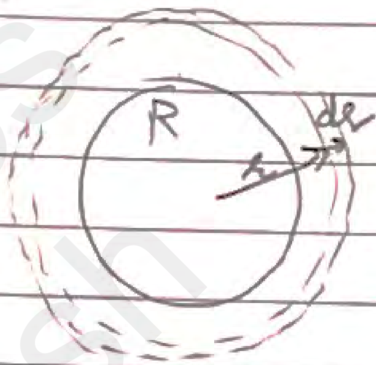
$$\Rightarrow \vec{E} = \frac{k}{4\epsilon_0}\left(\rho r^2\right)\hat{r}$$

$r > R$

$$q_{encl} = \int_0^R \rho.(4\pi r^2\,dr)$$

$$= \rho 4k\pi\int_0^R r^3\,dr$$

$$= \rho k\pi R^4.$$

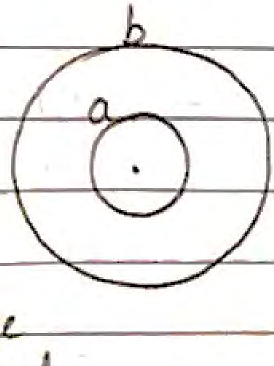


$$\Rightarrow \vec{E}(4\cancel{\pi} r^2) = \frac{\cancel{\pi}\rho k R^4}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \rho\frac{k}{4\epsilon_0}\left(\frac{R^4}{r^2}\right)\hat{r}$$

Q. Hollow spherical shell ; $\rho = \frac{k}{r^2}$ in region $a \le r \le b$

Find $\vec{E}$ (i) $r < a$
(ii) $a < r < b$
(iii) $r > b$



(i) $r < a$

$$\vec{E}(4\pi r^2) = \frac{0}{\epsilon_0}$$ → no charge enclosed

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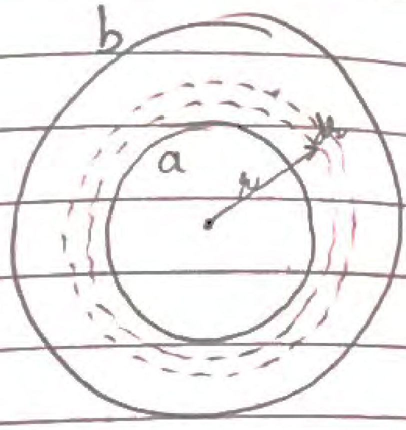
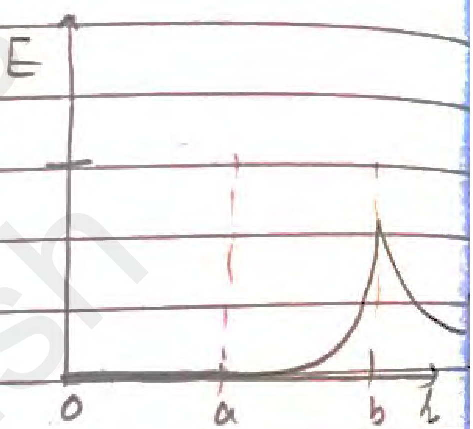
(ii) $a < r < b$



$$\vec{E}(4\pi r^2) = \int \frac{\rho(4\pi r^2 dr)}{\epsilon_0}$$

$$= \int \frac{\frac{k}{r^2}(4\pi r^2 dr)}{\epsilon_0}$$

$$= \int_a^r \frac{4k\pi dr}{\epsilon_0}$$



$$\Rightarrow \vec{E}(4\pi r^2) = \frac{4k\pi}{\epsilon_0}(r-a)$$

$$\Rightarrow \vec{E} = \frac{k}{\epsilon_0}\left(\frac{r-a}{r^2}\right)\hat{r}.$$

(iii) $r > b$

$$\vec{E}(4\pi r^2) = \int \frac{\rho(4\pi r^2)dr}{\epsilon_0}$$

$$= \int_a^b \frac{k}{r^2}\frac{(4\pi r^2)dr}{\epsilon_0}$$

$$= \frac{4k\pi}{\epsilon_0}\int_a^b dr$$

$$\Rightarrow \vec{E}(4\pi r^2) = \frac{4k\pi}{\epsilon_0}\int_a^b dr$$

$$\Rightarrow \vec{E} = \frac{k}{\epsilon_0}\left(\frac{b-a}{r^2}\right)\hat{r}$$

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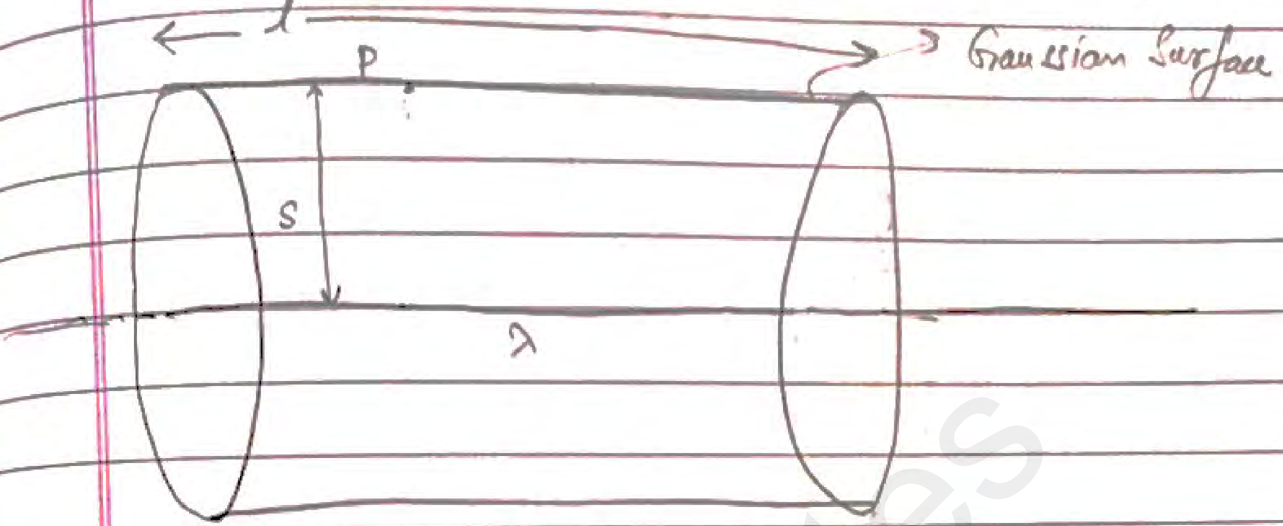
Date ______
Page ______

Q $\vec{E} = ?$ at a distance s from an infinitely long st. wire, which carries uniform line charge λ.



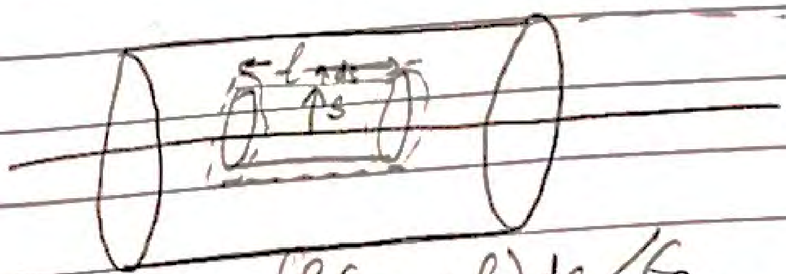
$$\oint \vec{E}\cdot\vec{dA} = \frac{q_{encl}}{\epsilon_0}$$

$$E \cdot 2\pi s l = \frac{\lambda l}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{\lambda}{2\pi s \epsilon_0}\,\hat{s}$$

Q A long cylinder carries charge density (ρ) proportional to distance from axis. $\vec{E}_{inside}$ cylinder

Given :- $\rho = ks$; k = const.



$$E(2\pi s l) = \int \rho(2\pi s l)\,ds/\epsilon_0$$

$$= \int_0^s ks(2\pi s l)\,ds/\epsilon_0$$

$$= \frac{2\pi k l}{\epsilon_0}\int_0^s s^2\,ds = \frac{2\pi k l s^3}{3\epsilon_0}$$

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$$\Rightarrow E(\cancel{2\pi s l}) = \frac{\cancel{2\pi} s^{\cancel{2}} \cancel{l} k}{3\epsilon_0}$$

$$\Rightarrow E = \frac{s^2 k}{3\epsilon_0}\hat{s}$$

Q. Long Coaxial Cable

Volume charge density $\rightarrow \rho$ on inner cylinder of radius a & surface charge density on the outer cylindrical shell (radius b).

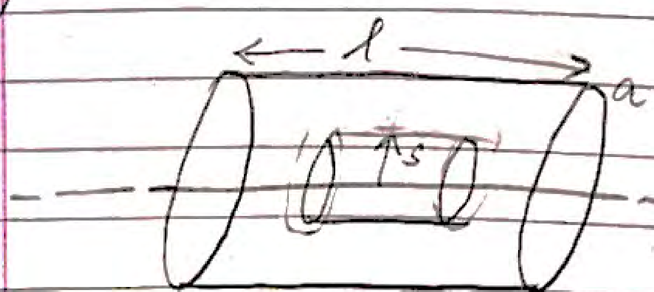
Surface charge is −ve & s.t −ve & +ve just balance

Find :- (i) $\vec{E}$ $(s<a)$

(ii) $(a<s<b)$

(iii) $(s>b)$

(i)



$$E(2\pi s l) = \frac{\rho(\pi s^2 l)}{\epsilon_0}$$

$$\Rightarrow \underset{(s<a)}{E} = \frac{\rho s}{2\epsilon_0}\hat{s}$$

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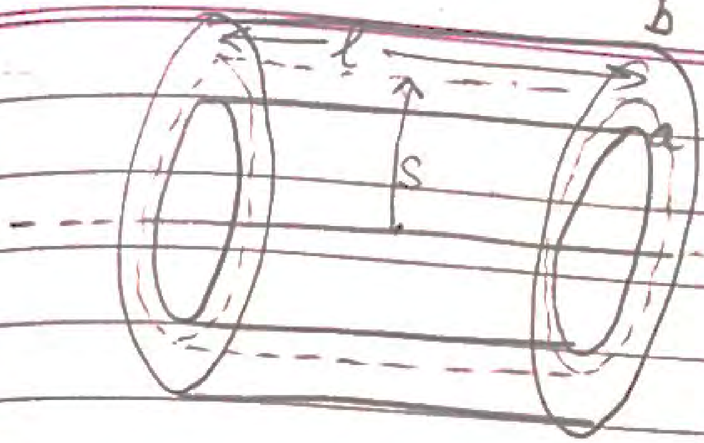
(ii)

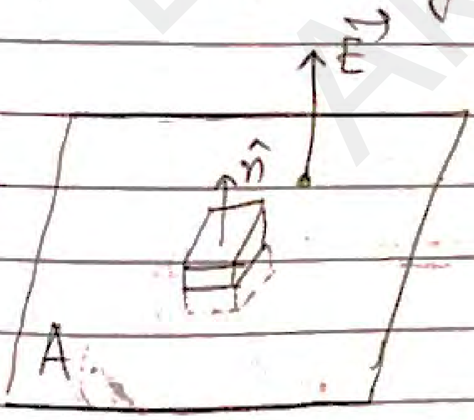


$$E(2\pi s l) = \frac{\rho(\pi a^2 l)}{\epsilon_0}$$

$$\Rightarrow \vec{E}_{a<s<b} = \frac{\rho a^2}{2\epsilon_0 s}\hat{s}$$

(iii) $\vec{E}(s>b) = \vec{0}$ (net charge inside gaussian surface $= 0$)

Q. Infinite plane
Uniform surface charge 'σ'. $\vec{E} = ?$



above & below

$$\int E \cdot dA = E(2A)$$

$$q_{encl} = \sigma A$$

$$\Rightarrow E(2\cancel{A}) = \frac{\sigma \cancel{A}}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}$$

(Independent of distance from sheet)

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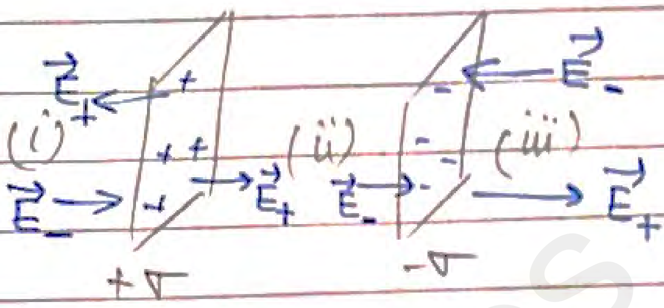
Q. 2 infinite ∥ planes carrying equal but opposite charge σ. Find $\vec{E}$ in each of the 3 regions



~~Region 1~~ $\vec{E} = \dfrac{\sigma}{2\epsilon_0}$ for thin sheet.

$$\vec{E}_{\text{region}①} = (E_+) + (E_-) = 0.$$

$$\vec{E}_{\text{region}②} = (E_+) + (E_-) = 2(E_+) = \frac{\sigma}{\epsilon_0}\ \hat{i}$$

$$\vec{E}_{\text{region}③} = 0.$$

✱ CURL OF $\vec{E}$

$$E = k\frac{q}{r^2}\hat{r} \ ; \ k = \frac{1}{4\pi\epsilon_0}.$$

$$\int_a^b \vec{E}\cdot d\vec{l} = k\int_a^b \frac{q}{r^2}\,dr = -\frac{kq}{r}\Big]_{r_a}^{r_b}.$$

$$= -k\left[\frac{q}{r_b} - \frac{q}{r_a}\right]$$

$$\oint_a^b \vec{E}\cdot d\vec{l} \Rightarrow r_a = r_b.$$

$\Rightarrow$ $\oint \vec{E}\cdot d\vec{l}$

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Stoke's thm:- $\Rightarrow \oint \vec{E} \cdot dl = \int_S (\nabla \times E) \cdot da$

$$\Rightarrow \boxed{\nabla \times E = 0}$$

ELECTRIC POTENTIAL (V_r)

$$V_{(r)} = -\int_{O}^{r} E \cdot dl .$$

O → reference pt.
anything

PD b/w 2 pts. a & b :-

$$V_{(b)} = -\int_0^b E \cdot dl \quad ; \quad V_{(a)} = -\int_0^a \vec{E} \cdot dl$$

$$V_{(b)} - V_{(a)} = -\int_0^b E \cdot dl + \int_0^a E \cdot dl$$

$$= -\int_0^b E \cdot dl - \int_a^0 E \cdot dl .$$

$$\Rightarrow V_{(b)} - V_{(a)} = -\int_a^b E \cdot dl$$

Also, $V_{(b)} - V_{(a)} = \int_a^b (\nabla V) \cdot dl .$

So, $\int_a^b (\nabla V)\, dl = -\int_a^b E \cdot dl .$

$$\Rightarrow \boxed{E = -\nabla V}$$

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Q. Find potential inside & outside a spherical shell of radius R which carries a uniform surface charge. Set reference pt at ∞.

$$E\,(r>R) = \frac{kq}{r^2}\,\hat{r}.$$

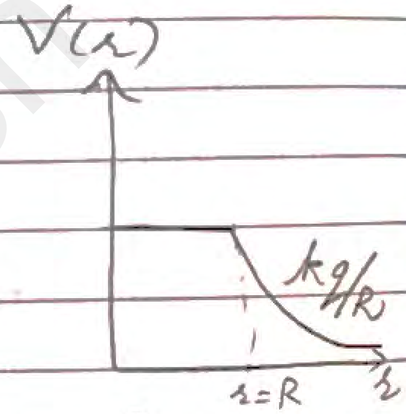
$$V(r) = -\int_{O}^{r} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{R} \frac{kq}{r^2}\,dr$$

$$V = +\frac{kq}{R}$$

Q. Spherical shell (radius R).

Reference $\longrightarrow \infty$

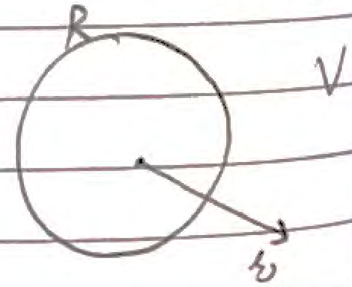
Find :- $V_{(r)}$ inside & outside



$$E(r>R) = \frac{kq}{r^2}\,\hat{r}.$$

$$V(r) = -\int_{O}^{r} E\cdot dl = -\int_{\infty}^{r} \frac{kq}{r'^2}\,dr'$$

$$\Rightarrow V_{(r)} = \frac{kq}{r}$$



$$E(r<R) = 0$$

$$\underbrace{V(r<R) = -k\int_{\infty}^{R} \frac{q}{(r')^2}\,dr' - \int_{R}^{r} (0)\,dr'}_{} = \frac{kq}{R}$$

2 integrals $\because\ \exists$ 2 kinds of $\vec{E}$.

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Q. $V_{(r)} = ?$ inside & outside Solid sphere (R, q).

$(r > R)$

$$E \cdot dA = \frac{q}{\epsilon_0}$$

$$\Rightarrow E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$\Rightarrow \vec{E_1} = \frac{kq}{r^2}\hat{r}$$

$$V_{(r)} = -\int_{\infty}^{r} E_1 \cdot dl$$

$$= -\int_{\infty}^{r} \frac{kq}{r^2} \cdot dr$$

$$= -kq\left[-\frac{1}{r}\right]_{\infty}^{r}$$

$$= kq\left[\frac{1}{r}\right] = \frac{kq}{r}$$

$r < R$

$4\pi r^2$

$$E \cdot dA = \frac{\frac{q}{\left(\frac{4}{3}\pi R^3\right)}\left(\frac{4}{3}\pi r^3\right)}{\epsilon_0}$$

$$\Rightarrow E = \frac{q r^3}{\epsilon_0 (4\pi r^2)}$$

$$\Rightarrow \vec{E_2} = \frac{q r}{4\pi\epsilon_0 R^3}\hat{r}$$

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$$V_{(r)} = -\int_{\infty}^{R} E_1\, dl - \int_{R}^{r} E_2\, dl$$

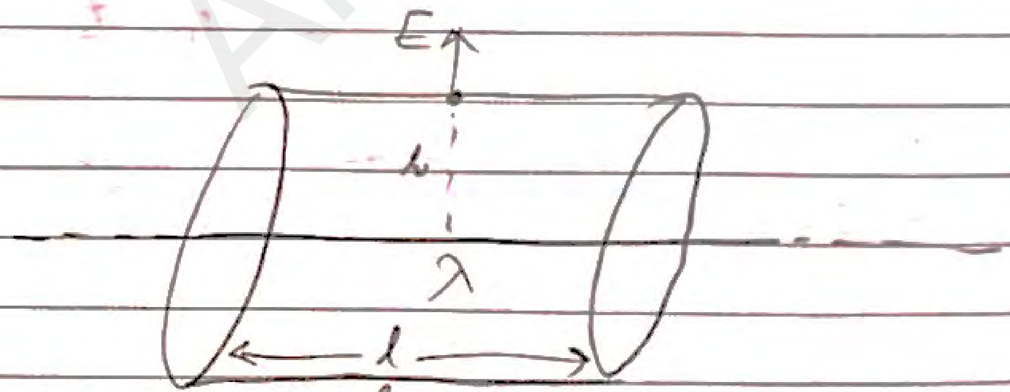
$$= -\int_{\infty}^{R} \frac{kq}{r^2}\, dr - \int_{R}^{r} \frac{kqr}{R^3}\, dr$$

$$= -kq\left[\frac{-1}{r}\right]_{\infty}^{R} - \frac{kq}{R^3}\left[\frac{r^2}{2}\right]_{R}^{r}$$

$$= \frac{kq}{R} - \frac{kq}{R^3}\left[\frac{r^2 - R^2}{2}\right]$$

$$\Rightarrow V_{(r)} = \frac{3}{2}\frac{kq}{R} - \frac{kq r^2}{2R^3}$$

Q. $V_{(r)} = ?$
At distance r from an infinitely long st. wire (λ)



$$E \cdot dA = \frac{\lambda . l}{\epsilon_0}$$

$$\Rightarrow E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{\lambda}{2\pi \epsilon_0 r} = \frac{2k\lambda}{r}\hat{r}$$

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$$\int_0^a x^2 dx + \int_a^b x^2 dx = \int_0^b x^2 dx$$

Puffin
Date
Page

$$V_{(r)} = -\int E \cdot dl$$

$$= -\int_A^r E \cdot dr$$

A → Let pt. A is a reference pt. S.t $V_{(A)} = 0$.

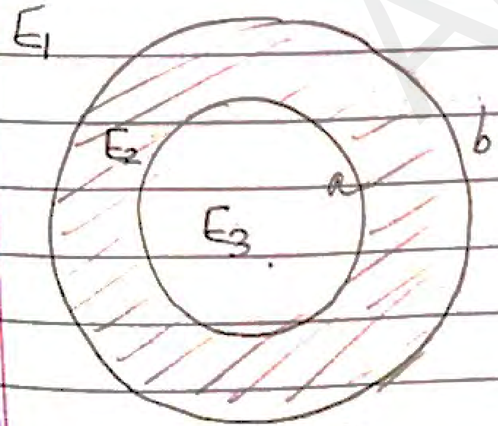
$$= -\int_A^r \frac{2k\lambda}{r} \cdot dr$$

$$= -2k\lambda \ln\left(\frac{r}{A}\right)$$

$$\Rightarrow V_{(r)} = 2k\lambda \ln\left(\frac{A}{r}\right)$$

★ Here, we didn't choose ∞ as reference pt. b'coz we have an infinite charge distribution. So, potential at ∞ is not 0, but infinite.

Q



Find :- Potential at centre.

$$V_0 = -\int_\infty^b E_1 dr - \int_b^a E_2 dr$$

$$- \int_a^0 E_3 dr$$

$$\rho = \frac{k}{r^2}$$

$$= -\int_\infty^b \frac{k(b-a)}{\epsilon_0 r^2} dr - \int_b^a \frac{k(r-a)}{\epsilon_0 r^2} dr - 0$$

$$= \frac{2ak}{\epsilon_0}\left[\frac{-1}{r}\right]_\infty^a$$

$$\int_\infty^b \frac{kb}{\epsilon_0 r^2} \ldots - \frac{k}{\epsilon_0}\int_b^a \ldots dr$$

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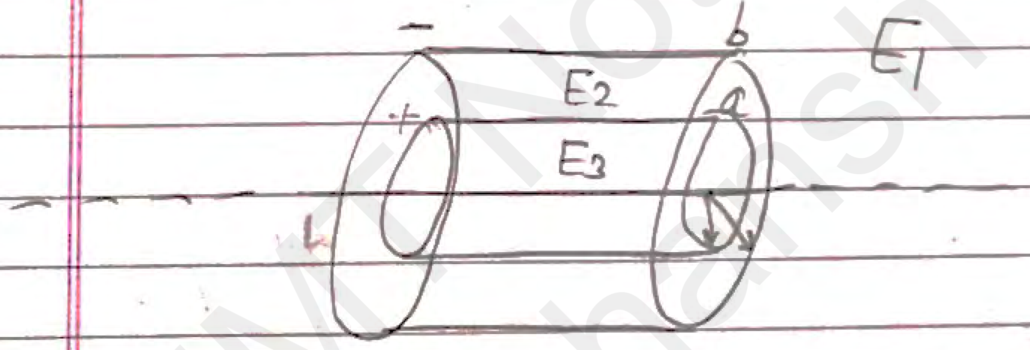
$$= \frac{2\alpha k}{\epsilon_0}\left(-\frac{1}{\alpha}\right) - \frac{kb}{\epsilon_0}\left(-\frac{1}{r}\right)_{\infty}^{b} - \frac{k}{\epsilon_0}\left[\ln r\right]_b^a$$

$$= -\frac{2k}{\epsilon_0} + \frac{k}{\epsilon_0} - \frac{k}{\epsilon_0}\ln\left(\frac{a}{b}\right)$$

$$\Rightarrow V = \frac{k}{\epsilon_0}\left(1 - \ln\left(\frac{a}{b}\right) - 1\right)$$

Check

Q. Find Potential b/w a pt. on the axis & a pt. on outer cylinder



$$V(b) - V(0) = -\int_0^b E \cdot dr$$

$$= -\int_0^a E_3\,dr - \int_a^b E_2\,dr$$

$$= -\int_0^a \frac{\rho r}{2\epsilon_0}\,dr - \int_a^b \frac{\rho a^2}{2r\epsilon_0}\,dr$$

$$= -\frac{\rho a^2}{4\epsilon_0} - \frac{\rho a^2}{2\epsilon_0}\ln\left(\frac{b}{a}\right)$$

$$= \frac{\rho a^2}{2\epsilon_0}\left[\ln\left(\frac{a}{b}\right) - \frac{1}{2}\right]$$

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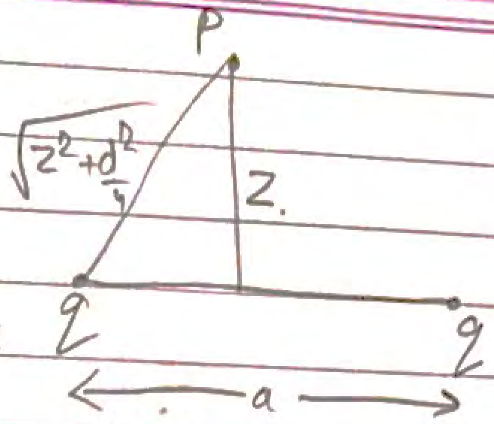
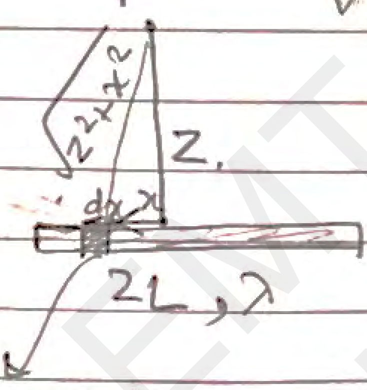
sec + tan

Q.2.25) (a) $V(P) = ?$



$$V = \left[\frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{z^2+\frac{d^2}{4}}}\right] \times 2$$ → due to 2 charges.

(b) $V(P) = ?$



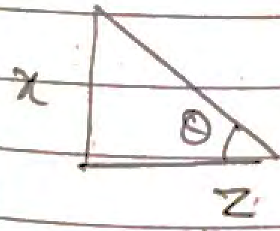
$dq = \lambda dx$

$$V = \left[\int_0^L \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{z^2+x^2}}\right] \times 2$$

$$= \left[\int_0^L \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{\sqrt{z^2+x^2}}\right] \times 2$$

$$= 2 \times \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{\sqrt{z^2+x^2}}$$

$x = z\tan\theta \Rightarrow dx = z\sec^2\theta\, d\theta$



$$= \frac{\lambda}{2\pi\epsilon_0}\int_0^L \frac{\cancel{z}\sec^2\theta\, d\theta}{\cancel{z}\sec\theta}$$

$\theta = 0$ to

$\theta = \tan^{-1}\left(\frac{x}{z}\right)$

or $\theta = \sec^{-1}\left(\frac{\sqrt{x^2+z^2}}{z}\right)$

$$= \frac{\lambda}{2\pi\epsilon_0}\int_0^L \sec\theta\, d\theta$$

$$= \frac{\lambda}{2\pi\epsilon_0}\int_0^L \frac{(\sec\theta)(\sec\theta+\tan\theta)\,d\theta}{\sec\theta+\tan\theta}$$

$$= \frac{\lambda}{2\pi\epsilon_0}\left[\ln(\sec\theta+\tan\theta)\right]$$

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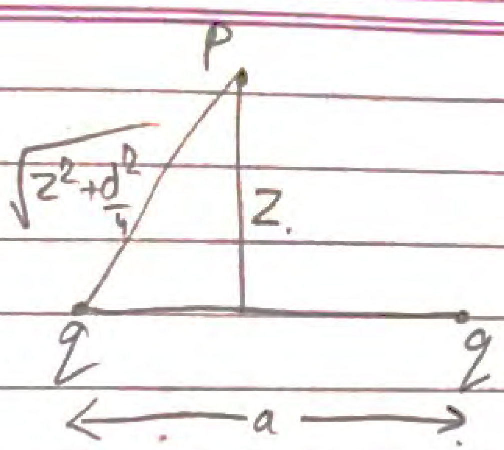
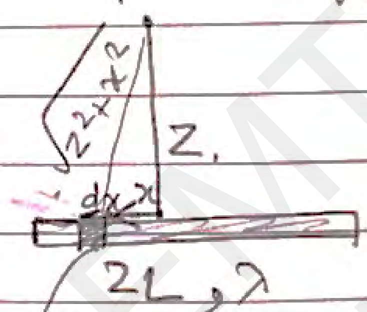
$\dfrac{\sec(\sec + \tan)}{\sec + \tan}$

Q.2.25) (a) $V(P) = ?$



$$V = \left[\frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{z^2+\frac{d^2}{4}}}\right] \times 2$$ → due to 2 charges.

(b) $V(P) = ?$

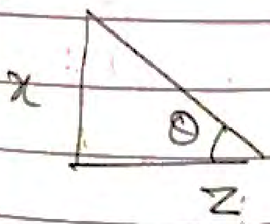


$dq = \lambda dx$

$$V = \left[\int_0^L \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{z^2+x^2}}\right] \times 2$$

$$= \left[\int_0^L \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{\sqrt{z^2+x^2}}\right] \times 2$$

$$= 2 \times \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{\sqrt{z^2+x^2}}$$



$x = z\tan\theta \Rightarrow dx = z\sec^2\theta \, d\theta$

$$= \frac{\lambda}{2\pi\epsilon_0}\int_0^L \frac{\cancel{z}\sec^2\theta \, d\theta}{\cancel{z}\sec\theta}$$

$\theta = 0$ to

$\theta = \tan^{-1}\left(\frac{x}{z}\right)$

or $\theta = \sec^{-1}\left(\frac{\sqrt{x^2+z^2}}{z}\right)$

$$= \frac{\lambda}{2\pi\epsilon_0}\int_0^L \sec\theta \, d\theta$$

$$= \frac{\lambda}{2\pi\epsilon_0}\int_0^L \frac{(\sec\theta)(\sec\theta+\tan\theta)d\theta}{\sec\theta+\tan\theta}$$

$$= \frac{\lambda}{2\pi\epsilon_0}\left[\ln(\sec\theta+\tan\theta)\right]$$

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Puffin
Date
Page

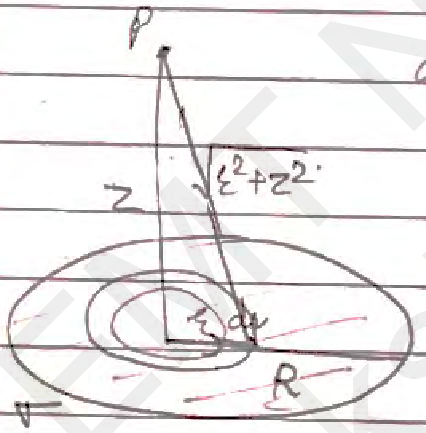
$$= \frac{\lambda}{2\pi\epsilon_0}\left[\ln(\sec\theta + \tan\theta)\right]$$

$$= \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{\sqrt{x^2+z^2}}{z} + \frac{x}{z}\right)\Bigg|_0^L$$

$$= \frac{\lambda}{2\pi\epsilon_0}\ln\left[\frac{\sqrt{L^2+z^2}}{z} + \frac{L}{z}\right]$$

$$= \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{\sqrt{L^2+z^2}+L}{z}\right)$$ Ans

(c)



$$dq = \sigma(2\pi r)dr$$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{r^2+z^2}}$$

$$V = \frac{\sigma(2\pi)}{4\pi\epsilon_0}\int_0^R \frac{r\,dr}{\sqrt{r^2+z^2}}$$

$$r^2+z^2 = t^2$$

$$\Rightarrow \cancel{2}r\,dr = \cancel{2}t\,dt$$

$$\Rightarrow V = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{\cancel{t}\,dt}{\cancel{t}}$$

$$= \frac{\sigma}{2\epsilon_0}\left[t\right]_0^R$$

$$= \frac{\sigma}{2\epsilon_0}\left[\sqrt{r^2+z^2}\right]_0^R$$

$$\Rightarrow V = \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2} - z\right)$$

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Date
Page

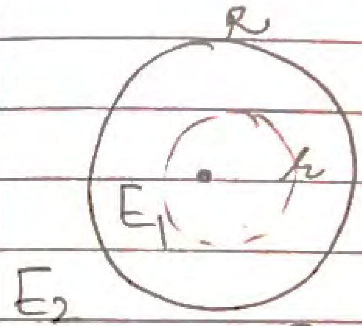
ENERGY question

Q. Solid sphere charge density $\rho(r) = kr$; k : constt.

Find Energy configur^n

$$W = \frac{\epsilon_0}{2}\int E^2 d\tau$$

(1) $\boxed{r<R}$



$$E(4\pi r^2) = \int_0^r \frac{\rho(4\pi r^2 dr)}{\epsilon_0}$$

$$= E(4\pi r^2) = \frac{k}{\epsilon_0}(4\pi)\int_0^r r^3 dr$$

$$\Rightarrow E_1 = \frac{k r^2}{4\epsilon_0}.$$

(2) $\boxed{r>R}$

$$E(4\pi r^2) = \int_0^R \rho\frac{(4\pi r^2)dr}{\epsilon_0}$$

$$\Rightarrow E(4\pi r^2) = \frac{k(4\pi)}{\epsilon_0}\frac{R^4}{4}$$

$$\Rightarrow E_2 = \frac{kR^4}{4\epsilon_0 r^2}$$

$$W = \frac{\epsilon_0}{2}\int_0^R E_1^2 d\tau + \frac{\epsilon_0}{2}\int_R^\infty E_2^2 d\tau$$

$$= \frac{\epsilon_0}{2}\int_0^R \frac{k^2 r^4}{16\epsilon_0^2}(4\pi r^2)dr + \frac{\epsilon_0}{2}\int_R^\infty \frac{k^2R^8(4\pi r^2 dr)}{16\epsilon_0^2 r^4}$$

$$= \frac{\epsilon_0 k^2\pi}{\epsilon_0^2 \times 8}\left(\frac{R^7}{7}\right) + \frac{\epsilon_0}{2\times 4}\frac{k^2R^8\pi}{\epsilon_0^2}\left[-\frac{1}{R}\right]_R^\infty$$

$$= \frac{k^2\pi}{8\epsilon_0}\left[\frac{R^7}{7} + R^8\left(\frac{1}{R}\right)\right]$$

$$= \frac{k^2\pi}{8\epsilon_0}\left(\frac{R^7}{7} + \frac{7R^7}{7}\right) = \frac{k^2\pi}{8\epsilon_0}\times\frac{8R^7}{7} = \frac{k^2\pi R^7}{7\epsilon_0}$$

Ans

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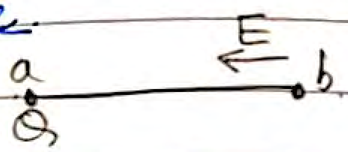
Page

☆ Work done to move a charge

Force by electric field = QE.



Work done against electric force, $W = \int_a^b F \cdot dl = -Q\int_a^b E \cdot dl = Q[V(b) - V(a)]$

$$\Rightarrow \quad W = Q[V_{(b)} - V_{(a)}].$$

$$\Rightarrow \quad V_{(b)} - V_{(a)} = \frac{W}{Q}.$$

If reference pt. is at ∞ i.e $a \to \infty$ & $b \to r$

$$\Rightarrow \quad W = Q[V_{(r)} - V_{(\infty)}] \quad V_{(\infty)} \to 0$$

$$\Rightarrow \quad W = Q(V_{(r)}).$$

or $V_{(r)} = \frac{W}{Q}$ or $W = Q\,V_{(r)}$ — W → WD to bring a charge from ∞ to a pt. r.

Note Also, potential is potential p.u. charge.

☆ ENERGY of a Continuous Charge distribution

For a volume charge density, ρ

$$W = \frac{1}{2}\int \rho V d\tau \qquad \left(\nabla \cdot E = \frac{\rho}{\epsilon_0} \to \text{gauss law in differential form}\right)$$

$$= \frac{\epsilon_0}{2}\int (\nabla E) \cdot V d\tau$$

$$= \frac{\epsilon_0}{2}\left[-\int E(\nabla V) d\tau + \oint_S V E \cdot dA\right]$$

(exercise eqn 1.59 → taken directly)

$$= \frac{\epsilon_0}{2}\left(\int_{\text{all space}} E^2 d\tau + \oint_S V E \cdot dA\right)$$

$(\because E = -\nabla V)$

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Date ___
Page ___

☆ Energy of a Point Charge distribution

↳ i.e, how much work would it take to assemble an entire collection of point charges, like $q_1, q_2, q_3, q_4, ---$

Consider the first charge to be brought from ∞
So, WD = 0 ($\because$ no opposing force on it)

$$\Rightarrow W_1 = 0 \left(= q_1(0)\right)$$

$$W_2 = q_2 (V_1)$$

→ Potential due to first charge brought

$$= \frac{q_2 q_1}{4\pi\epsilon_0 r_{12}}$$

$$W_3 = q_3(V_1) + q_3(V_2)$$

$$= \frac{q_3}{4\pi\epsilon_0}\left[\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right]$$

$$W_4 = q_4 [V_1 + V_2 + V_3]$$

$$= \frac{q_4}{4\pi\epsilon_0}\left[\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}}\right]$$

||ly, all n charges.

In general

Total work, $W = W_1 + W_2 + W_3 + W_4 + --- + W_n$

$$\Rightarrow W = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\sum_{j=1}^{n}(q_j)\left(\frac{q_i}{r_{ij}}\right)$$

$*\ j > i$

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j, i : don't count them twice.

- If we count each pair twice (eg, W_{12} & W_{21}). Then, divide by 2

$$\Rightarrow W = \frac{1}{8\pi\epsilon_0} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_j)\left(\frac{q_i}{r_{ij}}\right)$$

- or, $i \neq j$

$$W = \frac{1}{2} \sum_{i=1}^{n} q_i \left[\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{q_j}{4\pi\epsilon_0 r_{ij}} \right]$$

or $$\boxed{W = \frac{1}{2} \sum_{i=1}^{n} q_i V(r)}$$ *

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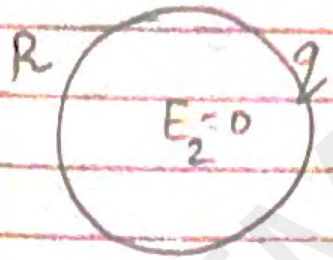
Now, as $\oint V\vec{E}\cdot d\vec{A} \to 0$ i.e., voltage tends to zero as $V \to 0$ as $r \to \infty$.

So, $$\boxed{W = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2\, d\tau}$ *

$\to$ energy density $= \frac{\epsilon_0}{2}E^2$

$\left(\frac{W}{d\tau} = \text{Energy p.u. volume}\right)$

Q) Uniform sphere shell of charge q, radius R. $W = ?$



$E_1 = \frac{kq}{r^2}$

$$W = \frac{\epsilon_0}{2}\int E^2\, d\tau$$

$$= \frac{\epsilon_0}{2}\int_R^\infty \left[\frac{kq}{r^2}\right]^2 d\tau + \int_0^R E_2^2\, d\tau$$

$$= \frac{\epsilon_0}{2}\int_R^\infty \left(\frac{kq}{r^2}\right)^2 (4\pi r^2 dr)$$

$$= \frac{\epsilon_0}{2}k^2q^2(4\pi)\int_R^\infty \frac{dr}{r^2}$$

$$= \frac{\epsilon_0}{2}k^2q^2(4\pi)\left[\frac{-1}{r}\right]_R^\infty$$

$$= \frac{\epsilon_0}{2}k^2q^2(4\pi)\left[\frac{1}{R}\right]$$

$$= \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q^2}{2R} = \frac{kq^2}{2R}$$

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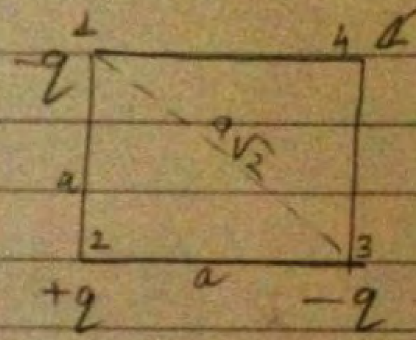
Q. (a) 3 charges are situated at corners of a square (side a). W = ? for bringing another charge +q from ∞ to 4th corner.



$+q = q_4$

(a) $W_4 = q_4 V$

$= q_4 (V_1 + V_2 + V_3)$

$= q_4 \left(-\frac{kq}{a} + \frac{kq}{a\sqrt{2}} - \frac{kq}{a} \right)$

$= q_4 \left(-\frac{2kq}{a} + \frac{kq}{a\sqrt{2}} \right)$

$\Rightarrow W_4 = \frac{q^2 k}{a} \left(-2 + \frac{1}{\sqrt{2}} \right)$

$; k = \frac{1}{4\pi\epsilon_0}$.

b) How much work does it take to assemble all 4 charges one by one

(b) $W_3 = V_1 + V_2 = (-q)\left(-\frac{kq}{a\sqrt{2}} + \frac{kq}{a} \right)$

$W_2 = V_1 = (+q)\left(-\frac{kq}{a} \right)$

$W_1 = 0$

Total work done $= W_1 + W_2 + W_3 + W_4$

$= 0 - \frac{kq^2}{a} - \frac{kq^2}{a} + \frac{kq^2}{a\sqrt{2}}$

$- \frac{2kq^2}{a} + \frac{kq^2}{a\sqrt{2}}$

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$$\Rightarrow W_{Total} = -4\frac{kq^2}{a} + \sqrt{2}\frac{kq^2}{a}$$

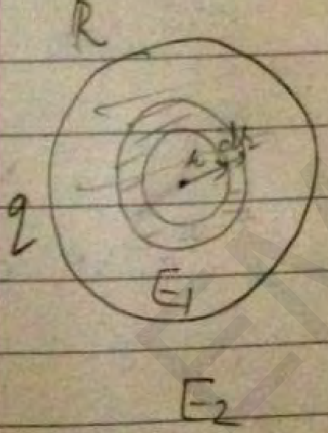
$$\Rightarrow W_{Total} = \frac{kq^2}{a}(\sqrt{2}-4) \quad ; k = \frac{1}{4\pi\epsilon_0}$$

Ans

Q. Energy stored = ?

For solid sphere ; q, R.

Use $W = \frac{\epsilon_0}{2}\int E^2 \, d\tau$



$$d\tau = 4\pi r^2 \, dr$$

$$W = \frac{\epsilon_0}{2}\left[\int E_1^2 \, d\tau + \int E_2^2 \, d\tau\right]$$

$$= \frac{\epsilon_0}{2}\left[\int_0^R \left(\frac{kqr}{R^3}\right)^2 (4\pi r^2 dr) + \int_R^\infty \left(\frac{kq}{r^2}\right)^2 (4\pi r^2 dr)\right]$$

derived before $\begin{cases} E_1 = \frac{kqr}{R^3} \\ E_2 = \frac{kq}{r^2} \end{cases}$

$$= \frac{\epsilon_0}{2}\left[\int_0^R \frac{k^2q^2r^2}{R^6}(4\pi r^2 dr) + \int_R^\infty \frac{k^2q^2}{r^4}(4\pi r^2 dr)\right]$$

$$= \frac{\epsilon_0}{2}\left[k^2q^2(4\pi)\right]\left[\int_0^R \frac{r^4}{R^6} dr + \int_R^\infty \frac{1}{r^2} dr\right]$$

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$$= \frac{\epsilon_0}{2} k^2 q^2 4\pi \left[\frac{1}{R^6}\left(\frac{R^5}{5}\right) + \left[-\frac{1}{R}\right]_R^\infty \right]$$

$$= 2\pi k^2 q^2 \epsilon_0 \left[\frac{1}{5R} + \frac{5}{5R} \right]$$

$$= \frac{2\pi\epsilon_0 q^2}{(4\pi\epsilon_0)^2} \left[\frac{6}{5R} \right]$$

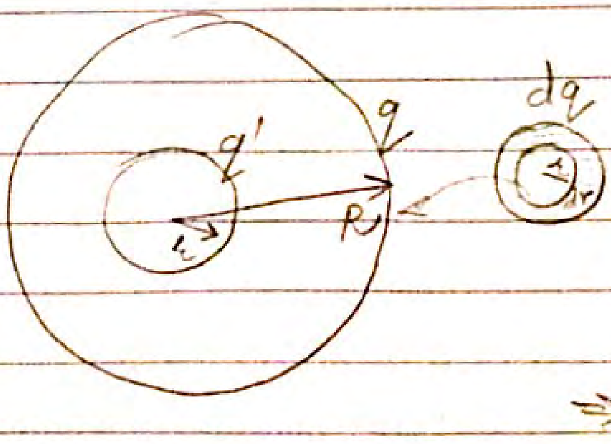
$$W_{total} = \frac{3kq^2}{5R}$$

Good!

Q) Uniformly charged solid sphere, E = ?
Assemble sphere layer by layer. Sphere has total charge q & radius is R.

Thought process:- $W\Delta = q\,dV$. So, find potentials & then integrate.

charge of solid sphere at any pt. of time. $q' = \frac{q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3 = \frac{q r^3}{R^3}$



$$W = qV$$

$$\Rightarrow dW = dq\, V_{(q')}$$

$$\Rightarrow W = \int_0^R dW$$

$$dq = \frac{q}{\frac{4}{3}\pi R^3} \times 4\pi r^2 dr$$

$$\Rightarrow dq = \frac{3 q r^2 dr}{R^3}$$

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$V(q')_{\text{at } r} = \frac{kq'}{r}$ (Potential at surface of a solid sphere q', r)

$$W = \int dW = \int dq \cdot V(q')$$

$$= \int \frac{3q r^2 dr}{R^3} \cdot \frac{1}{4\pi\epsilon_0} \frac{q'}{r}$$

$$= \int \frac{3q r^2 dr}{R^3 \times 4\pi\epsilon_0} \times \frac{q\, r^3}{R^3 \cdot r}$$

$$= \frac{3q^2}{4\pi\epsilon_0 R^6} \int_0^R r^4 dr$$

$$= \frac{3}{5(4\pi\epsilon_0)} \frac{q^2}{\times R^6} \times R^5$$

$$\Rightarrow W = \frac{3kq^2}{5R}$$

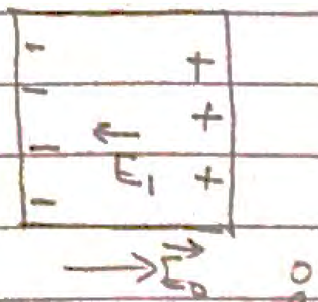
→ Same answer as done by Electrical field method (Previous ques.)

✱ CONDUCTORS : contains free e^-s

Ⓐ E_0 : External field

E_1 : Induced electric field

net $\vec{E}$ inside $= \vec{E_0} - \vec{E_1} = 0$



Ⓑ If $\vec{E}_{net} = 0 \Rightarrow \rho$ is zero inside conductor $\left(\because \nabla \cdot E = \frac{\rho}{\epsilon_0}\right)$

Ⓒ ⇒ any net charge resides on the surface

Ⓓ $V(a) - V(b) = -\int_a^b E \cdot dl = 0 \Rightarrow V(a) = V(b)$.

⇒ Conductor

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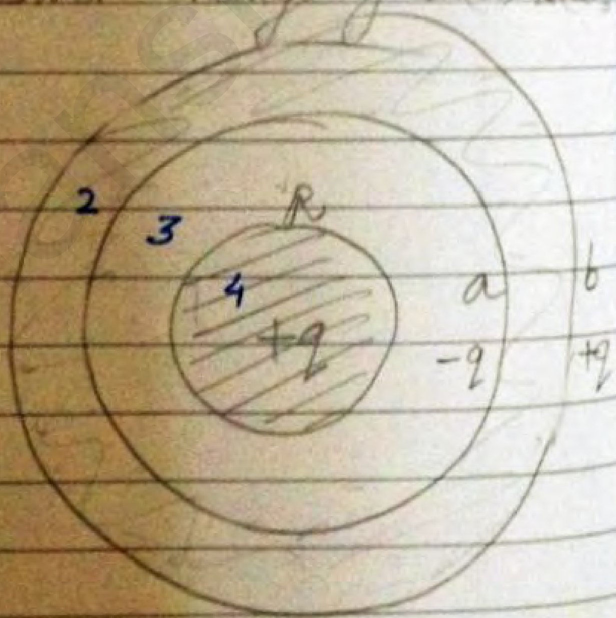
- Inside a conductor, electric field $(\vec{E}) = 0$ & hence, ~~to~~ its equipotential inside at any pt.

Q. A metal sphere of radius R carrying charge q is surrounded by a thick concentric shell (inner radius a, outer radius b). Shell carries no net charge.
(a) Find σ at R, a & b.
(b) V at center (∞ : Reference pt.)
(c) If outer surface is touched to grounding wire, how would answers change for (a) & (b)?



(a) $\sigma_R = \dfrac{q}{4\pi R^2}$

$\sigma_a = \dfrac{-q}{4\pi a^2}$

$\sigma_b = \dfrac{q}{4\pi b^2}$

(b) $V(0) = -\displaystyle\int_{\infty}^{b} \frac{kq}{r^2}\,dr - \int_{b}^{a} (0)\,dr - \int_{a}^{R} \frac{kq}{r^2}\,dr - \int_{R}^{0} (0)\,dr$

$= kq\left[\left(\dfrac{1}{b}\right) + \dfrac{1}{R} - \dfrac{1}{a}\right]$

(c) $\sigma_b = 0$, rest same.

$V(0) = 0 - 0 - \displaystyle\int_{a}^{R} \frac{kq}{r^2}\,dr - 0 = kq\left(\frac{1}{R} - \frac{1}{a}\right)$

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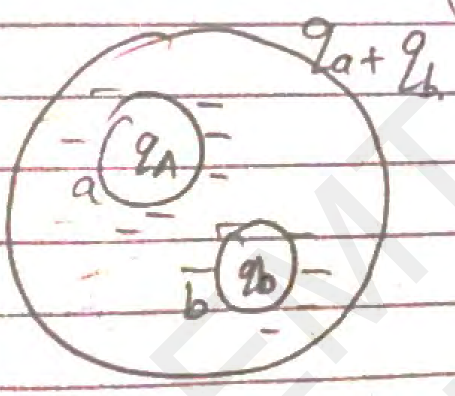
Q 2 spherical cavities of radius a & b are hollowed out from the interior of a (neutral conducting sphere, radius R). At center of cavities 2 charge placed, q_A & q_B.

(a) σ_A, σ_b, σ_R ?
(b) Field outside conductor
(c) Field within each cavity
(d) force on q_a & q_b (due to each other)
(e) What change when q_c is brought near conductor



(a) $\sigma_a = \dfrac{-q_a}{4\pi r_a^2}$

$\sigma_b = \dfrac{-q_b}{4\pi r_b^2}$

$\sigma_R = \dfrac{q_a+q_b}{4\pi R^2}$

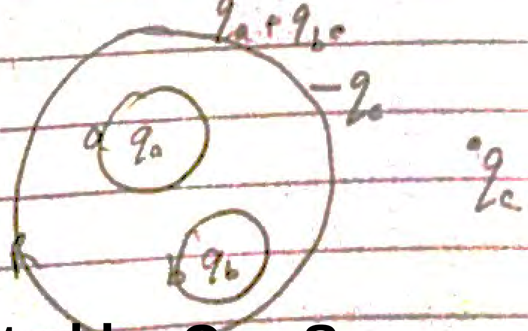
(b) $\vec{E}(r>R) = \dfrac{q_a+q_b}{4\pi\epsilon_0 r^2}\hat{r}$

(c) $\vec{E}(r<R_a) = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_a}{r^2}\hat{r}$.

$E(r<r_b) = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_b}{r^2}\hat{r}$

(d) $F=0$

(e) If q_c comes, only surface charge will change. So, σ_R change. So, $\vec{E}(r>R)$ changes.



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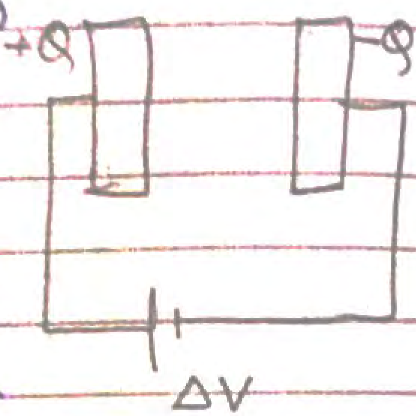
CAPACITANCE

$Q \propto \Delta V$

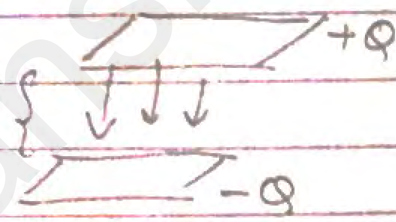
$\Rightarrow \boxed{Q = C\Delta V.}$

depends only on geometry $\leftarrow$ $C = Q/\Delta V$



- Inside capacitor, energy stored in the form of electric field.

Q. Given :- ∥ plate capacitor → 2 metal surfaces of area A, distance 'd' apart
Find :- capacitance



$\vec{E}$ b/w plates $= \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$

$= \frac{Q}{A\epsilon_0} \quad (\sigma = \frac{Q}{A})$

$\Delta V = E \cdot d = \frac{Qd}{\epsilon A} \Rightarrow \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d} = C$

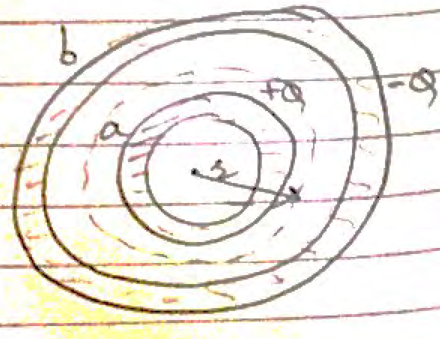
$\Rightarrow \boxed{C = \frac{A\epsilon_0}{d}}$

Q. Find capacitance:
2 concentric metallic shells, a & b radii.

$\Delta V = -\int_b^a E \cdot dl$

$= -\int_b^a \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r^2}\right) dr$

$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$



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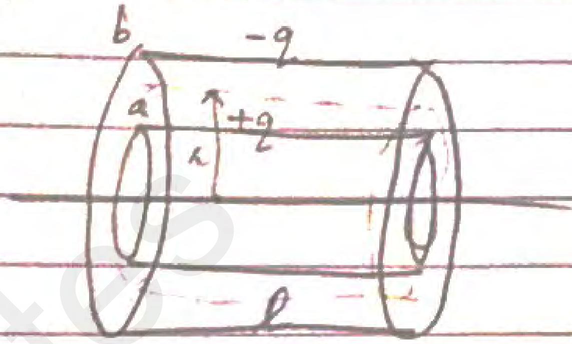
$\Rightarrow C = \frac{Q}{V} = 4\pi\epsilon_0 \left(\frac{ab}{b-a}\right)$ Ans

Q. Cylindrical capacitor: a & b radii. Capacitance = ?



$$E \cdot dA = \frac{q}{\epsilon_0}$$

$$\Rightarrow E(2\pi r l) = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{2\pi r l \epsilon_0}$$

$$\Delta V = -\int_b^a E \cdot dl = -\int_b^a \frac{q}{(2\pi r l \epsilon_0)}\, dr$$

$$= -\frac{q}{2\pi l \epsilon_0} \ln\left(\frac{a}{b}\right)$$

$$\Delta V = \frac{q}{2\pi\epsilon_0 l} \ln\left(\frac{b}{a}\right)$$

$$\Rightarrow \frac{q}{\Delta V} = \boxed{\frac{2\pi\epsilon_0 l}{\ln(b/a)} = C}$$

Capacitance per length $= \frac{C}{l} = \frac{2\pi\epsilon_0}{\ln\left(\frac{b}{a}\right)}$

Q. Capacitance of earth = ? Consider earth as an isolated conducting sphere of radius $R = 6370$ Km

Ans - One missing plate is of conducting sphere of

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infinite radius.

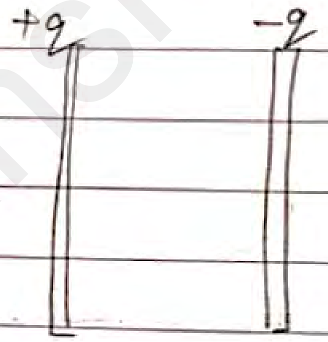
So, $b \to \infty$, $a \to R$

$$\boxed{C = 4\pi\epsilon_0 R} \to \text{isolated sphere.}$$

$$C = 4\pi \times 8.85 \times 10^{-12} \times 6.37 \times 10^{6}\ m$$
$$= 710\ \mu F$$

$\because$ for spherical capacitor,
$$C = 4\pi\epsilon_0 \frac{ab}{b-a}$$

☆ Energy stored in a capacitor.



Work done in order to move charge dq from +ve plate to negative plate

$$= dW = dq \cdot V$$
$$= dq\left(\frac{q}{C}\right)$$

$$V = \frac{q}{C}$$

Total work to go from $q = 0$ to $q = Q$ is

$$W = \int_0^Q dW = \int_0^Q \frac{q}{C}\, dq = \frac{Q^2}{2C}$$

= Energy stored.

$$\Rightarrow W = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}Q(\Delta V)$$

↳ final potential of capacitor

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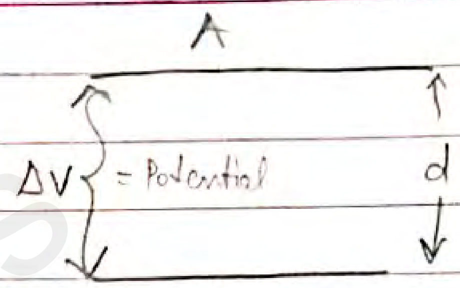
Page

In parallel plate,



$$\text{energy density} = \frac{W}{\text{Volume}}$$

$$= \frac{\frac{1}{2} C (\Delta V)^2}{A d}$$

$$= \frac{\frac{1}{2}\left(\frac{\cancel{A}\epsilon_0}{d}\right)(\Delta V)^2}{\cancel{A} d}$$

$$= \frac{1}{2} \frac{\epsilon_0 (\Delta V)^2}{d^2}$$

$$= \frac{1}{2} \epsilon_0 \left(\frac{\Delta V}{d}\right)^2$$

$$\Rightarrow \boxed{\text{Energy density} = \frac{1}{2} \epsilon_0 E^2}$$

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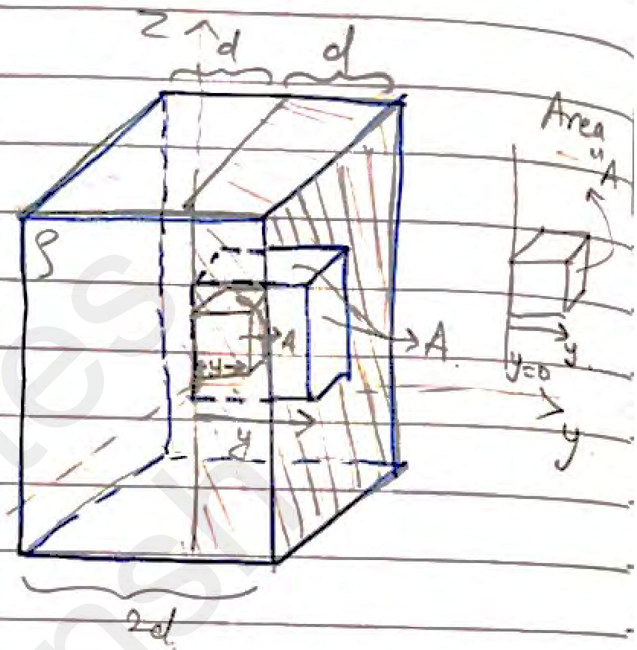
Q. An infinite plane slab of thickness 2d carries a uniform volume charge density ρ. Find E as a f^n of y, $y=0$ at centre.

$E(y<d) = ?$

$E(y>d) = ?$



By Gauss Law.

$$\oint E \cdot dA = \frac{q_{encl.}}{\epsilon_0}$$

For $y<d$

$$E \cdot \cancel{A} = \frac{\rho(\cancel{A}y)}{\epsilon_0}$$

$$\Rightarrow \bar{E} = \frac{\rho y}{\epsilon_0}\hat{y}$$

∵ For slab/sheet, Gaussian surface is Gaussian pillbox.
Volume of pillbox = $A \cdot y$
So charge inside gaussian pillbox = $\rho(Ay)$

For $y>d$

$$E(\cancel{A}) = \frac{\rho(\cancel{A}d)}{\epsilon_0}$$

$$\Rightarrow \bar{E} = \frac{\rho d}{\epsilon_0}\hat{y}$$

∵ For $y>d$, the charge included inside Gaussian surface = $\rho(Ad)$

not y ∵ as $y>d$, charge is only till d.

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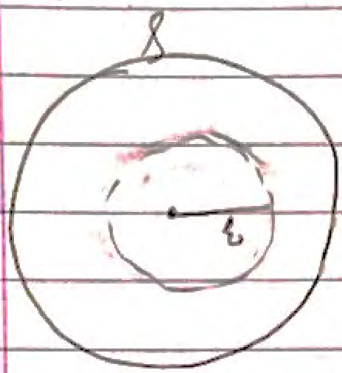
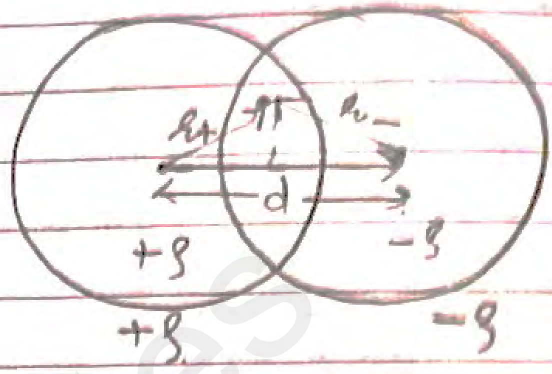
Q. 2 spheres, each of radius R and carrying uniform charge densities $+\rho$ and $-\rho$, are placed so that they partially overlap. What is $\vec{E}$ in the overlapping region?



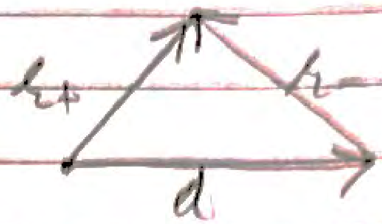


$$E(4\pi r^2) = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\epsilon_0}.$$

field at any pt. inside. $\Rightarrow \vec{E} = \frac{\rho r}{3\epsilon_0}\hat{r}$.

$$\vec{E}_+ = \frac{\rho r_+}{3\epsilon_0}$$

$$\vec{E}_- = \frac{-\rho r_-}{3\epsilon_0}.$$



$d + r_- = r_+$

$\Rightarrow (r_+) - (r_-) = d$

$$E_T = E_+ + E_-$$

$$= \frac{\rho}{3\epsilon_0}\left(r_+ - r_-\right)$$

$$\Rightarrow E_T = \frac{\rho}{3\epsilon_0}(\vec{d})$$

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Page

Q) One of these is an impossible electric field. Which one? (Use :- $\nabla \times \vec{E} = 0$)

(a) $\vec{E} = k[xy\,\hat{x} + 2yz\,\hat{y} + 3xz\,\hat{z}]$

(b) $\vec{E} = k[y^2\,\hat{x} + (2xy + z^2)\,\hat{y} + 2yz\,\hat{z}]$

, $k \to$ constant

(a) $\nabla \times \vec{E} =$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix}$$

$$= \hat{x}(0 - 2y) - \hat{y}(3z - 0) + \hat{z}(0 - x)$$

$$\Rightarrow \nabla \times \vec{E} = -2y\,\hat{x} - 3z\,\hat{y} - x\,\hat{z}$$

(b) $\nabla \times \vec{E} =$ $\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy + z^2 & 2yz \end{vmatrix}$

$$= \hat{x}(2z - 2z) - \hat{y}(0 - 0) + \hat{z}(2y - 2y)$$

$$= 0$$

$\nabla \times \vec{E}$ should come as 0.

So, (b) is correct

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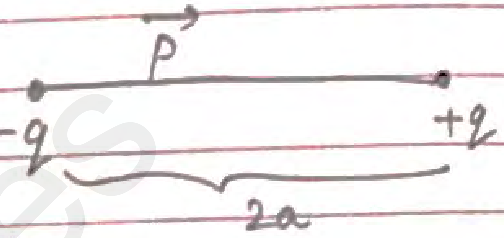
Chapter – 4.

ELECTROSTATIC FIELD IN A MATTER

* A polarized object comprises of n no. of dipoles put together.

* ELECTRIC DIPOLE

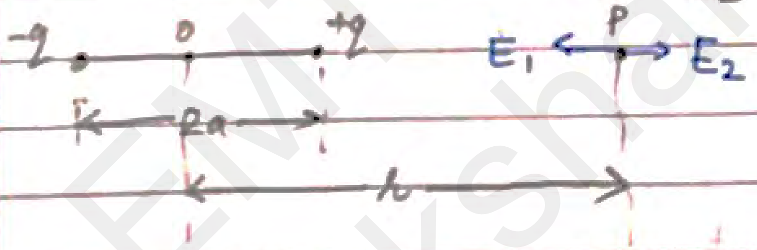


$\vec{p} = q(2\vec{a})$

→ dirⁿ of $\vec{p}$: –ve to +ve

* Field Intensity on Axial line due to electric dipole.

$\rightarrow E = E_2 - E_1$



$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}$, $E_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}$

$\Rightarrow E_2 - E_1 = \frac{q}{4\pi\epsilon_0}\left[\frac{(r+a)^2-(r-a)^2}{(r^2-a^2)^2}\right]$

$= \frac{q}{4\pi\epsilon_0}\left[\frac{4ra}{(r^2-a^2)^2}\right]$

$= \frac{2r}{4\pi\epsilon_0}\cdot\frac{q\cdot(2a)}{(r^2-a^2)^2}$ → $\vec{p}$

$\Rightarrow \vec{E}_2 - \vec{E}_1 = \frac{2\vec{p}\,r}{4\pi\epsilon_0(r^2-a^2)^2}$

→ $2a \ll r$:– $\vec{E} = \frac{2\vec{p}}{4\pi\epsilon_0 r^3}$

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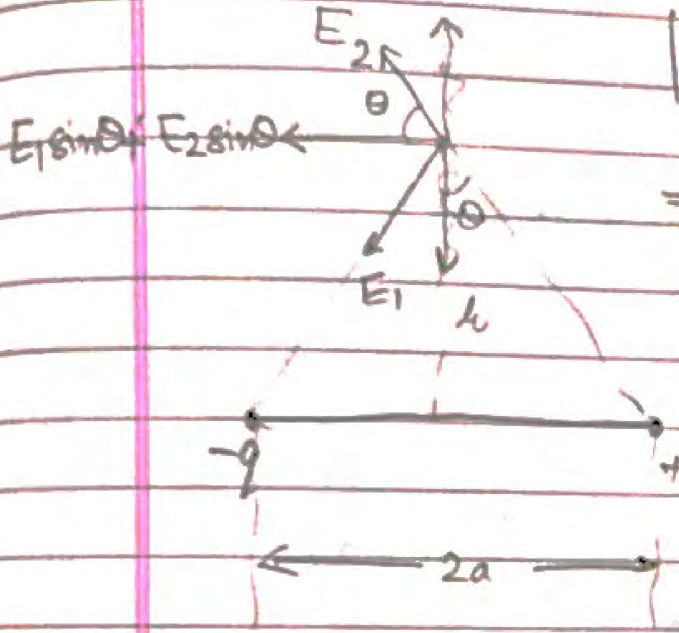
☆ Field Intensity on Equitorial line of dipole.



$$|E_1| = |E_2|$$

$$= \frac{2}{4\pi\epsilon_0}\left(\frac{q}{r^2+a^2}\right)\cdot\frac{a}{\sqrt{r^2+a^2}}$$

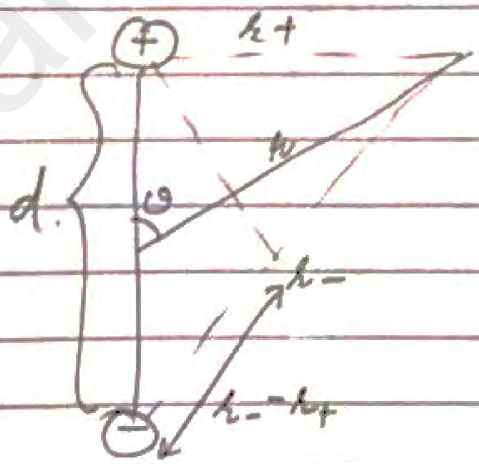
$$= \frac{\vec{p}}{4\pi\epsilon_0 (r^2+a^2)^{3/2}}.$$

$\rightarrow 2a << r$

$$\vec{E}_{Total} = \frac{\vec{p}}{4\pi\epsilon_0 r^3}$$

☆ Potential due to Electric dipole :-

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_+} + \frac{-q}{r_-}\right]$$



$(r_-) - (r_+) = d\cos\theta$

If $r >> d$

$r_- \cdot r_+ \simeq r^2$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left(\frac{r_- - r_+}{r^2}\right) = \frac{q\, d\cos\theta}{4\pi\epsilon_0 r^2}$$

$$\boxed{V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}}$$

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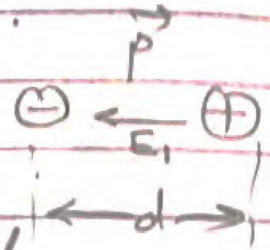
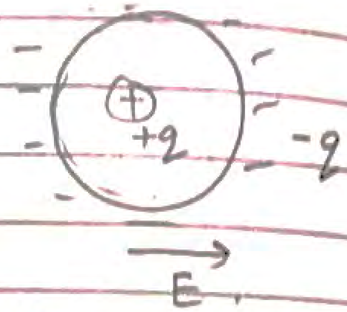
★ DIELECTRIC (Insulator) in $\vec{E}$

$\vec{p} \propto \vec{E}$

or $p = \alpha E \rightarrow (1)$

→ Atomic polarizability





$E_{e\,\text{equitorial}} = \frac{1}{4\pi\epsilon_0}\frac{qd}{a^3} = \frac{p}{4\pi\epsilon_0 a^3}$

$\Rightarrow p = \cancel{\alpha} q d = (4\pi\epsilon_0 a^3) E \rightarrow (2)$

$\Rightarrow (1)\ \&\ (2)$

$\Rightarrow \boxed{\alpha = 4\pi\epsilon_0 a^3}$

$\oint \vec{E}\, dA = \frac{q_{enc}}{\epsilon_0}$

$\Rightarrow E(4\pi d^2) = \frac{q d^3}{\epsilon_0 a^3}$

$\Rightarrow E = \frac{qd}{4\pi\epsilon_0 a^3}$

At d, $E = E_1 \Rightarrow$ Equilibrium situation

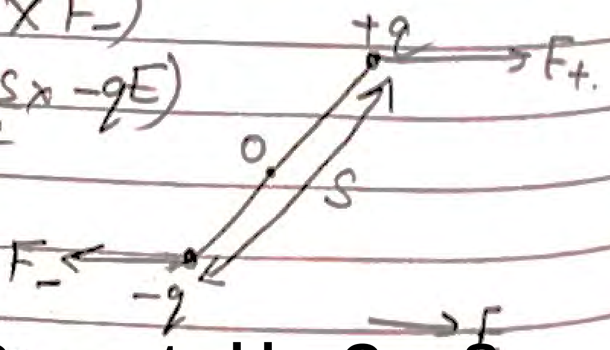
★ Polar molecules in $\vec{E}$ (uniform)

Polar ⇒ ~~bad~~ built in permanent dipole moment

Torque :-

$N_0 = (r_+ \times F_+) + (r_- \times F_-)$

$\Rightarrow N = \left(\frac{s}{2} \times qE\right) + \left(\frac{-s}{2} \times -qE\right)$



$\Rightarrow N = qs \times E$

$\Rightarrow N = p \times E$

$\Rightarrow N = pE\sin\theta$ → angle b/w E & dipole

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* If the field is NOT uniform, then, F_+ & F_- will be different. There will be a net force on the dipole, in addⁿ to torque.

* Energy of a dipole in an electric field.

$$W = U = \int \tau \cdot d\theta = \int r \times F \, d\theta = \int r F \sin\theta \, d\theta$$

$$= -rF \left[\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$\Rightarrow U = rF(\cos\theta_1 - \cos\theta_2)$$

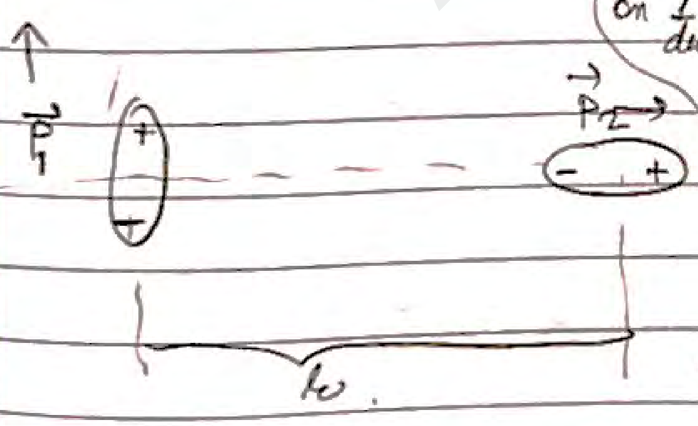
For reference pt, take $\theta_1 = \frac{\pi}{2}$, $\theta_2 = \theta$

$$\Rightarrow U = -rF\cos\theta$$

$$\Rightarrow \boxed{U = -r \cdot F}$$

Q. $\vec{p_1}$ & $\vec{p_2}$ are dipoles at a distance r apart. What is the torque on p_1 due to p_2 & p_2 due to p_1?

$$\tau = \vec{p} \times \vec{E}$$



on 1 due to 2 $\tau_{1\to2} = \vec{p_1} \times \vec{E_2}$

$= p_1 E_2 \sin\theta$ → 90°

$= p_1 E_2$ → equitorial formula

$$\Rightarrow \tau_{1\to2} = \left(\frac{p_1}{4\pi\epsilon_0 r^3}\right) E_2$$

$$\tau_{2\to1} = \vec{p_2} \times \vec{E_1} = p_2 E_1 \sin\theta \to 1$$

$$= \left(\frac{2p_2}{4\pi\epsilon_0 r^3}\right) E_1$$

axial formula

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* Remember :- Polarized object ≡ Dielectric

Puffin Date Page

* Electric displacement ($\vec{D}$)

attached with object

arises due to polarizⁿ

Gauss law in Presence of dielectric

Let $\rho = \rho_b + \rho_f$ ($\rho_b = -\nabla\cdot\rho$) b → bound charge

So, Gauss' law becomes

ρ_f : free charged (one embedded in the dielectric)

$$\nabla\cdot E = \frac{\rho}{\epsilon_0} = \frac{\rho_b + \rho_f}{\epsilon_0}$$

Polarizⁿ vector

$$\Rightarrow \epsilon_0(\nabla\cdot E) = \rho_b + \rho_f = -\nabla\cdot P + \rho_f$$

Proof done next

$$\cancel{\epsilon_0(\Delta E)} \Rightarrow \nabla\cdot(\epsilon_0 E + P) = \rho_f$$

$$\Rightarrow \boxed{\nabla\cdot D = \rho_f} \;;\; \text{let } \boxed{D = \epsilon_0 E + P} = \text{Electric displacement}$$

→ Gauss Law in presence of dielectric.

Change → $E \equiv D$, $\frac{\rho}{\epsilon_0} \equiv \rho_f$

Integral form :- $\int(\nabla\cdot D)\,d\tau = \int \rho_f \, d\tau$.

(Divergence thm. on LHS)

$$\Rightarrow \boxed{\oint D\cdot dA = Q_{free\ enclosed}} = \text{total free charge enclosed in the volume}$$

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Puffin
Date
Page

☆ Field of a Polarized object.

* Let P = Dipole moment p.u. volume = Polarizⁿ. $= \vec{P}$

$\mathcal{V} \rightarrow$ volume

Now, a polarized object ≡ we have n no. of dipoles.

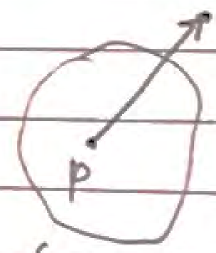
For single dipole, $V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$ (Potential)

$= \frac{1}{4\pi\epsilon_0}\frac{\hat{r}\cdot p}{r^2}$ $\quad p = P\,d\tau$

Total potential $= V = \frac{1}{4\pi\epsilon_0}\int_{vol}\frac{P\cdot\hat{r}}{r^2}\,d\tau$

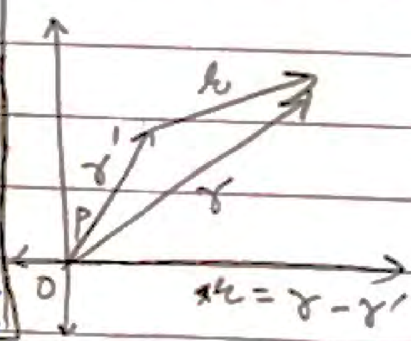
$V = \frac{1}{4\pi\epsilon_0}\int P\cdot\nabla\left(\frac{1}{r}\right)d\tau.$

$= \frac{1}{4\pi\epsilon_0}\left[\int_V \nabla\cdot\left(\frac{P}{r}\right)d\tau - \int_V \frac{1}{r}(\nabla\cdot P)\,d\tau\right]$



$(\because \vec{p} = \vec{P}\,d\tau)$

$\left(\because \nabla\left(\frac{P}{r}\right) = P\left(\nabla\cdot\left(\frac{1}{r}\right)\right) + \left(\frac{1}{r}\right)\nabla\cdot P\right)$



$r = r - r'$

$\Rightarrow V = \frac{1}{4\pi\epsilon_0}\int_S \frac{1}{r}\,P\cdot dA - \frac{1}{4\pi\epsilon_0}\int_V \frac{1}{r}(\nabla\cdot P)\,d\tau$

Behaves like σ behaves like ρ

(using divergence thm)

$\nabla\left(\frac{1}{r}\right) = \frac{d}{dr'}\left(\frac{1}{r-r'}\right)$

$= -\left(\frac{1}{r-r'}\right)^2(-1)$

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So, from previous formula, we can say that due to polarizⁿ, some charges have come on surface (in the form of σ) & others inside the sphere (in the form of ρ).

Comparing with std. formula,
1st term : Potential due to surface charge
2nd term: Potential due to volume charge.

Let $\boxed{\sigma_b = P.\hat{n}}$ & $\boxed{\rho_b = -\nabla.P}$.

$$\Rightarrow \boxed{V = \frac{1}{4\pi\epsilon_0}\int_S \frac{1}{\varepsilon}\sigma_b . dA + \frac{1}{4\pi\epsilon_0}\int_V \rho_b \frac{.d\tau}{\varepsilon}}$$

bound charge

☆ Sum of surface charges & bound charges for volume (inside) = ZERO
($\because$ object was neutral initially)

→ So, if we draw a Gaussian surface, $q_{enclosed} = 0$. But inside sphere, it will have some charge (due to ρ_b).

→ Note :- ρ_f is zero in these cases.

$\Rightarrow$ Displacement (D) = 0 $(= \epsilon_0 E + P)$

$\Rightarrow \epsilon_0 E + P = 0 \Rightarrow E = -\frac{P}{\epsilon_0}$

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Puffin
Date ______
Page ______

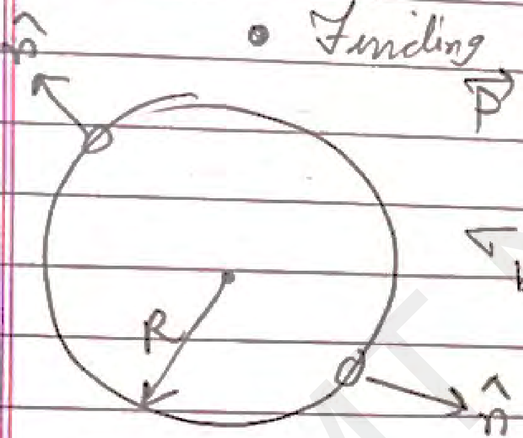
Q. A sphere of radius R carries a polarization $P = kr$, where k is constt. & r is vector from centre.

(a) Find bound charges σ_b & ρ_b

(b) Find field inside & outside sphere.

(a) $\sigma_b = \vec{P}\,\hat{n}$, $\rho_b = -\overline{\nabla}\cdot\overline{P}$; $\hat{n}$ = Area vector.

• Finding σ_b



$$\vec{P} = kr\,\hat{r}$$

$$\sigma_b = \vec{P}\cdot\hat{n} = kr(\hat{r}\cdot\hat{n})\Big|_{r=R}$$

$$= kr(1)\cos 0\Big|_{r=R}$$

$$= kr\Big|_{r=R}$$

$$= kR.$$

$$\Rightarrow \boxed{\sigma_b = kR}$$

= bound surface charge.

• Finding ρ_b

(components only along r)

$$A = (A_r, \cancel{A_\theta}, \cancel{A_\phi})$$

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + 0 + 0$$

$$\rho_b = -\nabla\cdot P.$$

$$= -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2(kr)\right) = \frac{-k}{r^2}(3r^2) = -3k$$

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$\Rightarrow$ bound volume charge density

$$= \boxed{\rho_b = -3k}$$

Now, sum of all bound charges = 0.

Total surface charge = $\sigma_b (4\pi R^2)$

$= 4k\pi R^3 \longrightarrow ①$

Total volume charge = $\rho_b \left(\frac{4}{3}\pi R^3\right)$

$= (-3k)\left(\frac{4}{3}\pi R^3\right)$

$= -4k\pi R^3 \rightarrow ②$

So, from ① & ②,

① + ② = 0 ✓ (Sum of all bound charges)

(b) $\vec{E}_{\substack{outside \\ (r>R)}} = 0$ ($\because q_{Total} = 0$, proved above)

$$E(r<R) = \frac{\rho r}{3\epsilon_0} = \frac{(-3k)r}{3\epsilon_0} = -\frac{kr}{\epsilon_0}$$

Q. Using the previous problem with $\vec{P} = \frac{k}{r}\hat{r}$

Find: (a) Locate all bound charges & use Gauss law to calculate field.

(b) Calculate D, ... then E.

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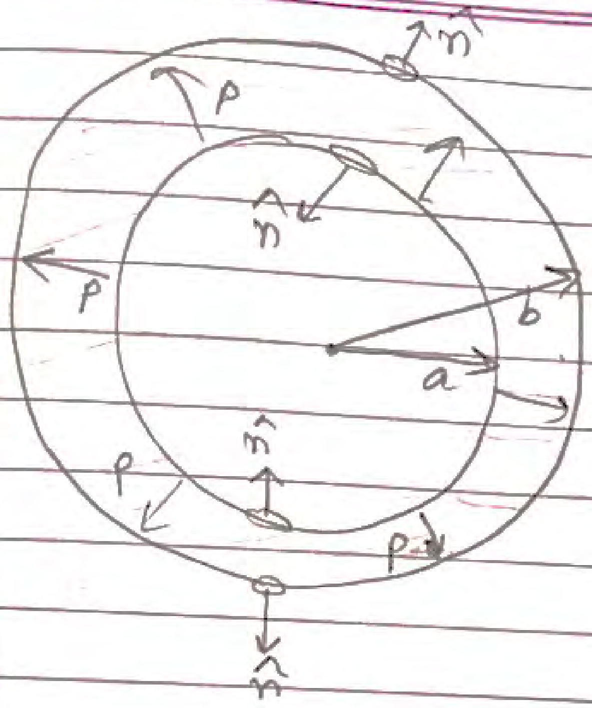


$\rho_f = 0$

$\left(\oint D \cdot da = Q_f\right)$

$\boxed{P - \text{-ve to +ve}}$

$\sigma_b \Big|_{r=a} = \bar{P} \cdot \hat{n}$

$= \left(\frac{k}{r}\right) \hat{r} \cdot \hat{n}$ See fig

$= \frac{k}{r}(1)\cos\pi \Big|_{r=a}$

$= -\frac{k}{a}$ ($\because$ $\angle$ b/w $\hat{r}$ & $\hat{n}$ at $r = a$ is π, $\cos\pi = -1$)

$\sigma_b \Big|_{r=b} = \left(\frac{k}{r}\right) \hat{r} \cdot \hat{n} \Big|_{r=b}$

$= \frac{k}{b}(1)\cos 0$

($\because \theta = 0$)

$\rho_b = -\bar{\nabla} \cdot \bar{P}$

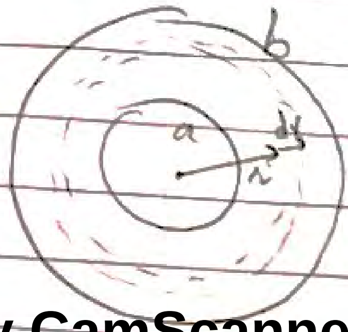
$= -\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \cdot \frac{k}{r}\right)$

$\Rightarrow \boxed{\rho_b = -\frac{k}{r^2}}$

(a) $E(r<a) = 0$

$E(r>b) = 0$ (Total charge = 0)

$E(a<r<b) = ?$



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Puffin
Date
Page

M1 By Gauss Law

$$\oint E\cdot da = \frac{q}{\epsilon_0}$$

$$\Rightarrow E(4\pi r^2) = \frac{1}{\epsilon_0}\left[\int_a^r \rho_b\,(4\pi r^2\,dr) + \sigma_b\Big|_{r=a}(4\pi a^2)\right]$$

$$= -\frac{4\pi k}{\epsilon_0}\int_a^r dr + \left(-\frac{k}{a}\right)\frac{(4\pi a^2)}{\epsilon_0}$$

$$= -\frac{4\pi k}{\epsilon_0}(r-a) - \frac{(4\pi k)a}{\epsilon_0}$$

$$\Rightarrow E(4\pi r^2) = -\frac{4\pi k r}{\epsilon_0}$$

$$\Rightarrow \boxed{\vec{E} = -\frac{k}{\epsilon_0 r}\,\hat{r}}$$

M2 $P = \frac{k}{r}$

Gauss Law

free space $\rightarrow \oint E\cdot dA = \frac{q_{enc}}{\epsilon_0}$

$\sigma_b = \vec{P}\cdot\hat{n}$ In dielectric D $\rightarrow \oint D\cdot dA = Q_f$ → free charge

$\rho_b = -\nabla\cdot\bar{P}$.

- $(r<a)$

  $D=0$
- $a<r<b$

  $D=0$ ($\because Q_f = 0$)

  ($\because \nexists$ no free charge : Given).
- $r>b$:

  $D=0$.

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$$\bar{D} = \epsilon_0 \bar{E} + \bar{P}.$$

Since, $D = 0 \ (\forall \ r)$

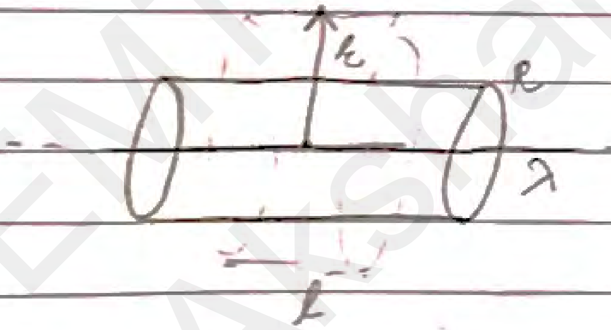
$$\Rightarrow \bar{E} = -\frac{\bar{P}}{\epsilon_0}$$

$r < a$ $\quad \bar{E} = 0 \quad$ ($\because P = 0$: no material to be polarised)

$a < r < b$: $\bar{E} = -\frac{k}{\epsilon_0 r}\hat{r}$

$r > b$: $\bar{E} = 0 \ (\bar{P} = 0)$

Q. A long straight wire, carrying uniform line charge λ, is surrounded by rubber insulⁿ out to a radius R. Find the electric displacement, $D(r > R)$. What is $E(r > R)$?



$$\oint D \cdot dA = Q_f$$

$$\Rightarrow D(2\pi r l) = \lambda l$$

$$\Rightarrow \bar{D} = \frac{\lambda}{2\pi r}\hat{r}$$

$\bar{D} = E\epsilon_0 + P$. $\rightarrow 0$ ($\because$ only free charge, nothing polarized)

$$\Rightarrow \bar{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r} \quad \text{Ans}$$

$\epsilon = \epsilon_r \epsilon_0$

Note: $E(r < R) \Rightarrow \bar{E} = \frac{\lambda}{2\pi\epsilon r}\hat{r}$ ← Permittivity of rubber

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Q. A sphere of linear dielectric material has embedded in it a uniform FREE charge density ρ. Find :- Potential at centre of sphere (relative to infinity), if radius R, dielectric constt. ϵ_r.

As from question, $\exists$ no $\bar{P}$

Find $\bar{E}$ first

$$E_T = E_1 + E_2$$

$$V_0 = -\int_{\infty}^{R} E_2 \cdot dr - \int_{R}^{0} E_1 \cdot dr$$

$$\oint D \cdot dA = Q_f$$

→ $r < R \Rightarrow D \cdot (4\pi r^2) = \rho\left(\frac{4}{3}\pi r^3\right)$

$$\Rightarrow D = \frac{\rho r}{3}$$

Now, $\bar{D} = \epsilon E$

$$\bar{D} = (\epsilon_0 \epsilon_r) E$$

$$\Rightarrow E_1 = \frac{\rho r}{3\epsilon_0 \epsilon_r} \longrightarrow (1)$$

→ $r > R \Rightarrow D(4\pi r^2) = \rho\left(\frac{4}{3}\pi R^3\right)$

$$\Rightarrow \bar{D} = \frac{\rho R^3}{3r^2}$$

$$D = \epsilon E$$

$$\Rightarrow \bar{D} = \epsilon_0 \cancel{\epsilon_r} \bar{E}$$

$$\Rightarrow \bar{E}_2 = \frac{\rho R^3}{3\epsilon_0 \cancel{\epsilon_r} r^2} \longrightarrow (2)$$

In free space, $\epsilon_r = 1$ ← i.e. $\epsilon = \epsilon_0$

Finding Potential :

$$V_0 = -\int_{\infty}^{R} \frac{\rho R^3}{3\epsilon_0} \frac{dr}{r^2} - \int_{R}^{0} \frac{\rho r}{3\epsilon_0 \epsilon_r} dr$$

$$= -\frac{\rho}{3\epsilon_0}\left[\int_{\infty}^{R} \frac{R^3}{r^2} dr + \int_{R}^{0} \frac{r}{\epsilon_r} dr\right]$$

$$= -\frac{\rho}{3\epsilon_0}\left\{\left[\frac{-R^3}{R}\right] + \left[\frac{-R^2}{2\epsilon_r}\right]\right\}$$

$$= \frac{\rho R^2}{3\epsilon_0} + \frac{\rho R^2}{2\epsilon_0 \epsilon_r}$$

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* Free charge always lies inside dielectric medium.

Puffin
Date
Page

* Note :-

Polarizability, $\bar{P} \propto$ Electric field $\bar{E}$.

$\Rightarrow$ * $\boxed{\bar{P} = \epsilon_0 \chi_e \bar{E}}$

χ_e → electrical susceptibility : how susceptible it is to get polarized, how strong the electrons are bound, all physical properties

Now, $D = \epsilon_0 E + P$
$= \epsilon_0 \bar{E} + \epsilon_0 \chi_e \bar{E}$
$= \epsilon_0 \bar{E}(1 + \chi_e)$
$\Rightarrow D = \epsilon E$

→ $\epsilon = \epsilon_0(1 + \chi_e)$ = Permittivity of medium.

→ $\epsilon = \epsilon_0 \epsilon_r$

→ $\epsilon_r = 1 + \chi_e$ = Relative permittivity

Q) The space b/w the plates of a ‖ plate capacitor is filled with 2 slabs of linear dielectric material. Each slab has thickness 'a', so that total distance b/w plates is 2a. Slab 1 has dielectric constt. 2 & Slab 2 has $\epsilon = 1.5$. Free charge density on upper plate σ & lower plate $-\sigma$.
Find : (a) $\bar{D}$ in each slab
(b) $\bar{E}$ " "
(c) $\bar{P}$ " "
(d) Potential diff. b/w plates
(e) Find loc' [illegible] bound charge

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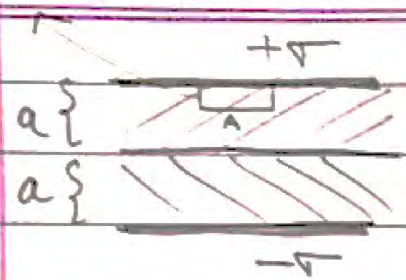
Gaussian pillbox



Gauss law in dielectric

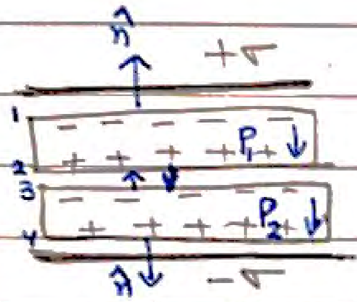
$$\oint \vec{D}\cdot d\vec{A} = Q_{free}$$

(a) $\Rightarrow DA = \sigma A$

$\Rightarrow D_1 = \sigma$

& ||ly, $D_2 = \sigma$

Total 4 surfaces



1 : Plate/slab 1

2 : Slab 2

(b) Now, $D = \epsilon E = \epsilon_0 \epsilon_r E$

$\Rightarrow E = \dfrac{D}{\epsilon}$

$\Rightarrow E_1 = \dfrac{\sigma}{2\epsilon_0}$

$|E_2| = \left|\dfrac{-\sigma}{1.5\epsilon_0}\right| = \dfrac{\sigma}{1.5\epsilon_0}$

(c) $P = D - \epsilon_0 E$

$\Rightarrow P_1 = \epsilon_0 E(\epsilon_r - 1)$

$= \epsilon_0 E_1 (\epsilon_r - 1)$

$= \cancel{\epsilon_0}\left(\dfrac{\sigma}{2\cancel{\epsilon_0}}\right)(2-1)$

$P_1 = +\dfrac{\sigma}{2}$

& $P_2 = \epsilon_0 E(\epsilon_r - 1)$

$= \cancel{\epsilon_0}\left(\dfrac{\sigma}{1.5\cancel{\epsilon_0}}\right)(1.5-1)$

$\Rightarrow P_2 = \dfrac{\sigma}{3}.$

(d) $V = E_1 A + E_2 A$.

$= (E_1 + E_2)a = \left(\dfrac{\sigma}{2\epsilon_0} + \dfrac{2\sigma}{3\epsilon_0}\right)a$

$\Rightarrow V = \dfrac{7}{6}\dfrac{\sigma a}{\epsilon_0}$

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See $\hat{n}$ of dielectric & not slab (dir^n)

(e) $\sigma_b = \vec{P} \cdot \hat{n} = P n \cos\theta$

$\sigma_1 = P_1 \cdot (1) \cos 180^\circ$

$\sigma_1 = -\frac{\sigma}{2}$

$\sigma_2 = P_1 \cdot (1) \cos 0$

$= +\frac{\sigma}{2}$

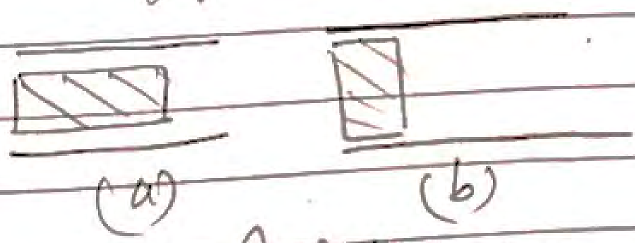
$\sigma_3 = P_2 (1) \cos 180^\circ$

$= -\frac{\sigma}{3}$

$\sigma_4 = P_2 (1) \cos 0^\circ$

$\Rightarrow \sigma_4 = +\frac{\sigma}{3}$.

Q) Suppose you have enough linear dielectric material of dielectric constt ϵ_r to half fill a ∥ plate capacitor. By what fraction is the capacitance increased when you distribute the material as in fig (a) & fig (b).



$C = \frac{A\epsilon_0}{d} \xrightarrow[\text{dielectric}]{\text{In presence of}} \frac{A\epsilon}{d} = \frac{A\epsilon_0 \epsilon_r}{d}$

★ Note: Notation :- $\epsilon_r = K$ → dielectric constt

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(a)

$C_1 : \epsilon_r$

$C_2 : \epsilon_0$

$\equiv$ like, 2 capacitors in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C = \frac{C_1 C_2}{C_1 + C_2}$$

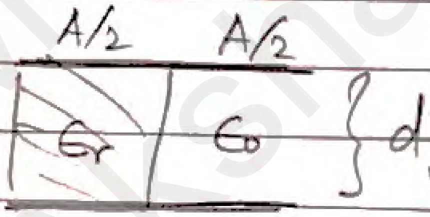
$$\to C_1 = \frac{A\epsilon_0 \epsilon_r}{d/2}$$

$$\to C_2 = \frac{A\epsilon_0}{d/2}$$

$$\Rightarrow C = \frac{A\epsilon_0\epsilon_r \left(\frac{2}{d}\right)}{\epsilon_r + 1}$$

$$C_A = \frac{2A\epsilon_0\epsilon_r}{d(\epsilon_r + 1)} \longrightarrow (1)$$

(b)



: Connected in parallel

$$C = C_1 + C_2$$

$$\to C_1 = \frac{\left(\frac{A}{2}\right)\epsilon_0\epsilon_r}{d}$$

$$\to C_2 = \frac{\left(\frac{A}{2}\right)\epsilon_0}{d}$$

$$\Rightarrow C_B = \left(\frac{A}{2}\frac{\epsilon_0}{d}\right)(\epsilon_r + 1) \longrightarrow (2)$$

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Puffin
Date
Page

Now, $C_0 = \frac{A \epsilon_0}{d}$

So, from (1)

$$\frac{C_A}{C_0} = \frac{2\epsilon_r}{\epsilon_r + 1}$$

from (2)

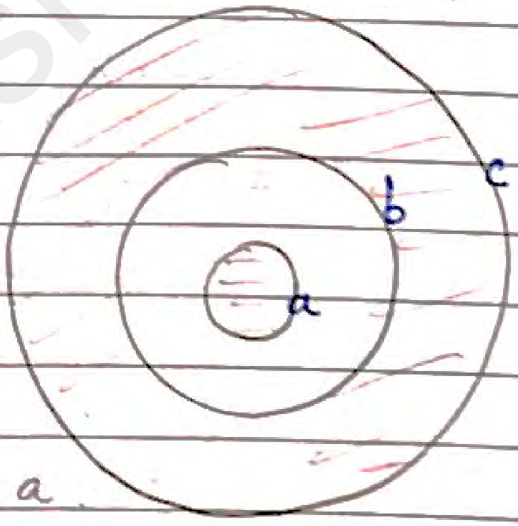
$$\frac{C_B}{C_0} = \frac{\epsilon_r + 1}{2}$$

Q. A certain coaxial cable consists of copper wire, radius 'a', surrounded by a concentric copper tube of inner radius b. The space between, is partially filled (from b to c) with material of dielectric constt. ϵ_r. Find Capacitance p.u length of the cable.

Basically, try finding

V + regions in terms of Q & then

$Q = CV$.



Put some charge +Q on a

$$V = \frac{Q}{C} \qquad \frac{1}{C}$$

Now, $(V_{ac}) = (\ ) Q$

Find this

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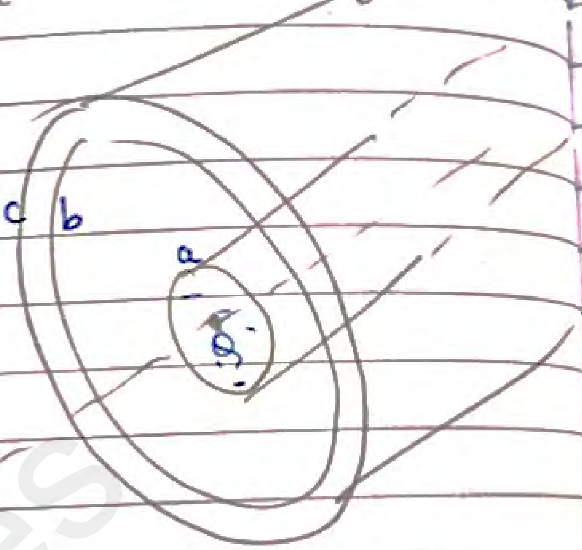
$$V_{ac} = \int_a^c E \cdot dr = \int_a^b E_1\, dr + \int_b^c E_2\, dr$$

$E_1 = E(a<r<b)$

$E_2 = E(b<r<c)$



$$\oint D \cdot da = Q_f$$

$$\Rightarrow D \cdot (2\pi r l) = Q$$

$$\Rightarrow D = \frac{Q}{2\pi r l}$$

$$E_1 = \frac{D}{\epsilon_0} = \frac{Q}{2\pi r l \epsilon_0}$$

$$E_2 = \frac{D}{\epsilon_0 \epsilon_r} = \frac{Q}{2\pi r l (\epsilon_0 \epsilon_r)}$$

$$\Rightarrow V_{ac} = \frac{Q}{2\pi l \epsilon_0}\left[\int_a^b \frac{dr}{r} + \int_b^c \frac{dr}{r\epsilon_r}\right]$$

$$= \frac{Q}{2\pi l \epsilon_0}\left[\ln\frac{b}{a} + \frac{1}{\epsilon_r}\ln\left(\frac{c}{b}\right)\right]$$

$$= \underbrace{\left\{\left[\ln\frac{b}{a} + \frac{1}{\epsilon_r}\ln\left(\frac{c}{b}\right)\right] \times \frac{1}{2\pi l \epsilon_0}\right\}}_{\frac{1}{C}\ (\text{not } C)} \times Q$$

$$\Rightarrow C = \frac{2\pi l \epsilon_0}{\ln\left(\frac{b}{a}\right) + \frac{1}{\epsilon_r}\ln\left(\frac{c}{b}\right)}$$

Ans

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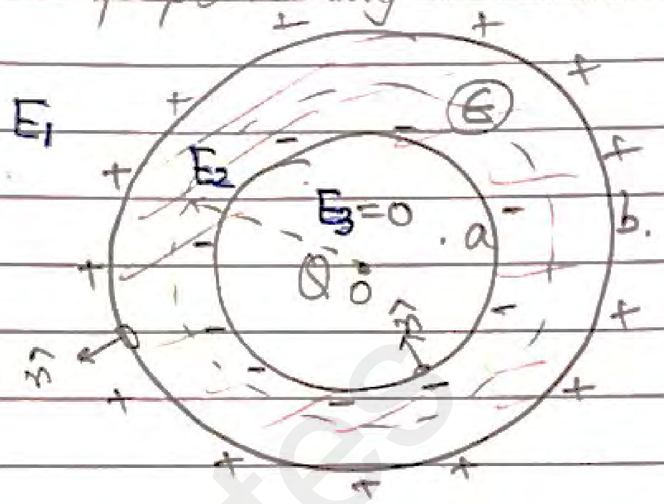
Date
Page

Q.4.5 A metal sphere of radius a, carries a charge Q. It is surrounded out to radius b by linear dielectric material of permittivity ϵ. Find potential at centre.



$$V_{(0)} = -\int_{\infty}^{b} E_1 \cdot dr - \int_{b}^{a} E_2 \cdot dr - \int_{a}^{0} E_3 \cdot dr$$

$\int_a^0 E_3 \cdot dr \rightarrow 0$ ($\because$ inside metal, $E = 0$)

By Gauss Law,

$$\oint D \cdot dA = Q_{free}$$

$$V(0) = -\int_{\infty}^{b} E_1 . dr - \int_{b}^{a} E_2 . dr$$

$$Q_{free} = Q$$

$$\Rightarrow \oint D \cdot dA = Q$$

For $a < r < b$ $\quad D \cdot 4\pi r^2 = Q$

$$\Rightarrow D = \frac{Q}{4\pi r^2}$$

Now, $D = \epsilon E$

$$\Rightarrow E = \frac{D}{\epsilon} = \frac{Q}{4\pi\epsilon r^2} = E_2$$

$r > b$ $\quad D = \epsilon_0 E + P$, $P \rightarrow 0$.

$$\Rightarrow E = \frac{D}{\epsilon_0} = \frac{Q}{\ldots} = E_1$$

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So, $V_{(0)} = -\int_{\infty}^{b} \frac{Q}{4\pi\epsilon_0 r^2}\, dr - \int_{b}^{a} \frac{Q}{4\pi\epsilon r^2}\, dr$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{b}\right] + \frac{Q}{4\pi\epsilon}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$= \frac{Q}{4\pi}\left[\frac{1}{\epsilon a} + \frac{1}{\epsilon_0 b} - \frac{1}{\epsilon b}\right]$$

Ans

Now finding σ_b & ρ_b

↳ Find P

$$D = \epsilon_0 E + P$$

$$\Rightarrow P = D - \epsilon_0 E$$

$$= D\Big|_{at\ a<r<b} - \epsilon_0 E$$

$$= \frac{Q}{4\pi r^2} - \epsilon_0\left[\frac{Q}{4\pi r^2 \epsilon}\right]$$

$$\Rightarrow P = \frac{Q}{4\pi r^2}\left(1 - \frac{\epsilon_0}{\epsilon}\right)$$

$$\Rightarrow P = \frac{Q}{4\pi r^2}\left(\frac{\epsilon - \epsilon_0}{\epsilon}\right) = \frac{A}{r^2}, \text{ say.}$$

Now,

$$\sigma_b = \bar{P}\cdot\hat{n} \quad \& \quad \rho_b = -\nabla\cdot\bar{P}$$

$$\rho_b = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot P_{\text{radial component}}\right) + 0 + 0$$

θ components ϕ components

$$\Rightarrow \rho_b = \frac{-1}{r^2}\frac{\partial}{\partial r}\left(\cancel{r^2} \times \frac{Q}{4\pi \cancel{r^2}}\left(\frac{\epsilon - \epsilon_0}{\epsilon}\right)\right) = 0$$

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* Note :- Bound charges will exist only inside the dielectric. Whereas, surface charges will exist on inside and outside surfaces of dielectric section (from $a<r<b$, in this problem).

Now

$$\sigma_b\Big|_{r=a} = \bar{P}\cdot\hat{n} = -\frac{A}{a^2}$$

$$\sigma_b\Big|_{r=b} = \bar{P}\cdot\hat{n} = \frac{A}{b^2}.$$

Ans

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8 ENERGY IN DIELECTRIC

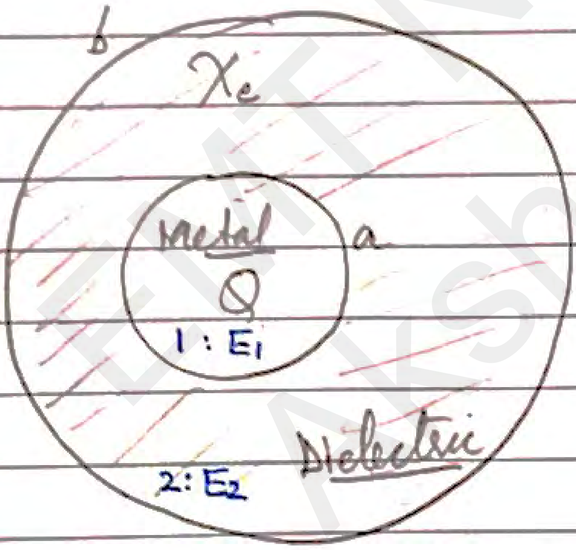
$$W = \frac{\epsilon_0}{2}\int E^2 d\tau \quad : \text{free space}$$

Dielectric $W = \dfrac{\epsilon}{2}\displaystyle\int E^2 d\tau$

$$= \frac{1}{2}\int (\epsilon E)\, E d\tau .$$

$$\Rightarrow \boxed{\text{Dielectric } \underbrace{W}_{W_D} = \frac{1}{2}\int D\cdot E\, d\tau}$$

Q. 4.26 Find $W_D = ?$



3 : E_3

$$W_D = \frac{1}{2}\int D\cdot E d\tau$$

$$= \frac{1}{2}\int_0^a D\, E_1 d\tau \rightarrow 0$$

$$+ \frac{1}{2}\int_a^b D\, E_2 d\tau$$

$$+ \frac{1}{2}\int_b^\infty D\, E_3 d\tau$$

$$D = \frac{Q_f}{4\pi r^2}$$

$$E_2 = \frac{D}{\epsilon} = \frac{Q}{4\pi r^2 \epsilon} = \frac{Q}{4\pi r^2 \epsilon_0 (1+\chi_e)}$$

$\epsilon = \epsilon_0(1+\chi_e)$

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$$E_3 = \frac{D}{\epsilon_0} = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\Rightarrow W_D = \frac{1}{2}\int_a^b \frac{Q^2}{(4\pi r^2)^2 \epsilon_0 (1+\chi_e)} d\tau$$

$$+ \frac{1}{2}\int_b^\infty \frac{Q^2}{(4\pi r^2)^2 \epsilon_0} d\tau$$

$$= \frac{1}{2}\frac{Q^2}{(4\pi)^2 \epsilon_0}\left[\int_a^b \frac{\overset{4\pi r^2 dr}{d\tau}}{r^4(1+\chi_e)} + \int_b^\infty \frac{\overset{4\pi r^2 dr}{d\tau}}{r^4}\right]$$

$$= \frac{Q^2}{8\pi\epsilon_0}\left[\int_a^b \frac{dr}{r^2(1+\chi_e)} + \int_b^\infty \frac{dr}{r^2}\right]$$

$$\Rightarrow W_D = \frac{Q^2}{8\pi\epsilon_0}\left[\frac{1}{(1+\chi_e)}\left[\frac{1}{a} - \frac{1}{b}\right] + \frac{1}{b}\right]$$

Ans.

☆ Gauss Law : SUMUP.

- Free space
- $\oint E \cdot dA = \frac{q_{encl}}{\epsilon_0}$.
- $W = \frac{\epsilon_0}{2}\int E^2 d\tau$

- Dielectric.
- $\oint D \cdot dA = q_{free}$

$D = \epsilon_0 E + P$
$= \epsilon E$
$= \epsilon_0(1+\chi_e)E$
$= \epsilon_0 \epsilon_r E$.

- $W = \frac{1}{2}\int D \cdot E \, d\tau$.

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★ If P (Polarizability) is given in any problem,
eg. $P = \frac{k}{r^2}\hat{r}$, say. Then, $q_{free} = 0$.

$\Rightarrow D = 0$.

$\Rightarrow E = -\frac{P}{\epsilon_0}$.

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Chapter - 5
Magnetostatics

§ LORENTZ FORCE :

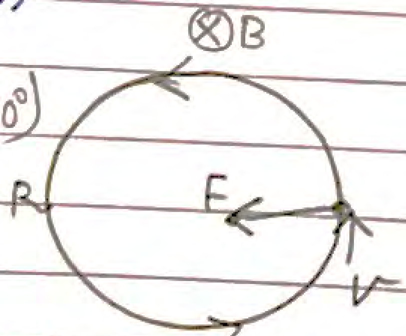
Any charge particle Q moving with velocity 'v' in presence of B will experience a force,

$$F_{mag} = Q(v \times B) \Rightarrow \text{Lorentz force law.}$$

In presence of both $\vec{E}$ and $\vec{B}$.

$$F = Q(E + (v \times B))$$

§ Cyclotron motion :- (1) $\boxed{V \perp B}$ $(\theta = 90^\circ)$



$$QvB = \frac{mv^2}{R}$$

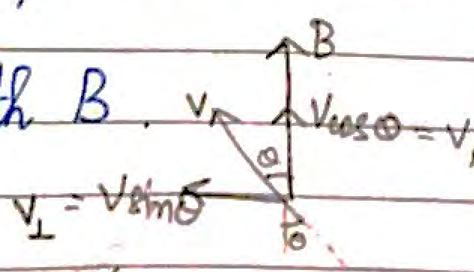
$$\Rightarrow QB = \frac{mv}{R} = \frac{P}{R} \Rightarrow \boxed{p = QBR}$$

= momentum of particle.

B is into the page
⇒ Circular motion

only ⊥ component responsible for radial path.

(2) V makes some angle (θ) with B.



$$\frac{m(V_\perp)^2}{R} = Q V_\perp B.$$

$$\Rightarrow R = \frac{mV_\perp}{QB} = \frac{mV\sin\theta}{QB} \Rightarrow$$

Pitch = $V_{\parallel} \times T$

T : Time Period

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- $V_{\parallel}$: component of vel. $\parallel$ to magnetic field (B)
- $V_{\perp}$: component of vel. $\perp$ to $\vec{B}$.

- When particle enters at an angle in a region (angle with $\vec{B}$), a helical path is seen

✶ CURRENTS : $I = \frac{q}{t} = \frac{\text{Charge}}{\text{time}}$.

① $\lambda = \frac{\text{charge}}{\text{length}}$ (1D)

$\lambda(v\Delta t)$ — Δt

Total charge $= \lambda(v\Delta t)$

distance = length $= v\Delta t$.

$$\Rightarrow I = \frac{\lambda(v\Delta t)}{\Delta t} = \underline{\underline{\lambda v}}.$$

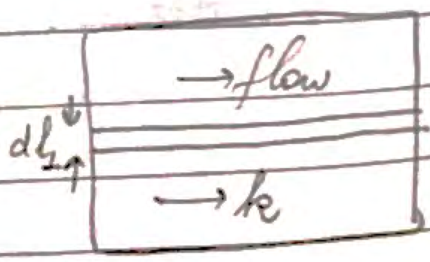
② <u>2D</u>

Let k : surface current density

$k \Rightarrow$ current p.u. width $\perp$ to flow.

$$k = \frac{dI}{dl_{\perp}} = \sigma v \text{ (}v \to \text{vel.)}$$

(σ : surface charge density)



③ <u>3D</u> J : ρv : volume ~~charge~~ current density.

= current p.u. area $\perp$ to flow.

(ρ : volume charge density) $J = \frac{dI}{dA_{\perp}}$

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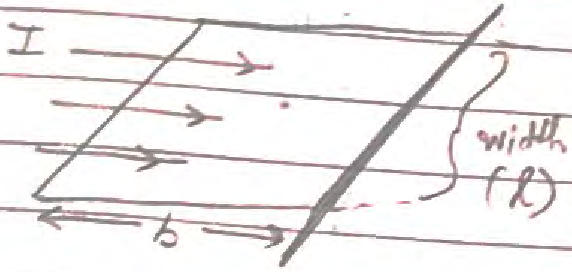
§ CONCEPT

★ 2D



$l \perp I$

So, current passing through the width is surface current density.

So, $K = \frac{I}{l} = \frac{dI}{dl}$

$= \frac{(\sigma A/t)}{l}$

$= \frac{\sigma(\cancel{l}b)}{\cancel{l}t} = \sigma\left(\frac{b}{t}\right)$ vel.

$\Rightarrow \boxed{K = \sigma v}$ ★

- Its different; if we have q instead of I, then,

$$\sigma = \frac{q}{A}$$

★ 3D

$A \perp I$

So, current passing through area is volume current density

$J = \frac{I}{A} = \frac{dI}{dA}$

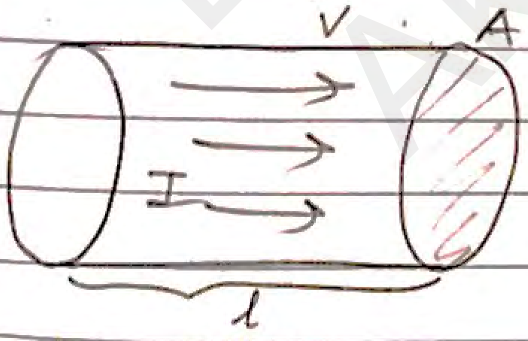
$= \frac{(\rho \cdot Vol)/t}{A}$

$= \frac{\rho \cdot A \cdot l}{t \cdot A} = \rho\left(\frac{l}{t}\right)$ vel.

$\Rightarrow \boxed{J = \rho \cdot v}$ ★

- Its different from ρ, volume charge density.

$$\rho = \frac{q}{V}$$



★ Note :- l & A are the $\perp$ lengths and areas.

★ If K or J are f^{ns} of l or r respectively, then, integrate and solve (Taking small dl or dA).

eg :- If $J = kr$ $\Rightarrow I = JA$

$= \int_0^R kr(2\pi r\,dr)$

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Q. Current I flows down a wire of radius a.
(a) If its uniformly distributed a surface, $K = ?$
(Note: here, current flowing on surface)
(b) If its distributed s.t $J \propto \frac{1}{r}$ → distance from axis

$J = ?$

(a) $K = \sigma v = \frac{I}{l} = \frac{I}{2\pi a}$

(b) $J \propto \frac{1}{r}$; $J = \frac{k}{r}$ → (1)

$I = J.A$

$\Rightarrow \int dI = \int J \cdot dA$

$\Rightarrow \int dI = \int_0^a \left(\frac{k}{\cancel{r}}\right)(2\pi \cancel{r}\, dr)$

$\Rightarrow I = 2\pi k(a)$

$\Rightarrow k = \frac{I}{2\pi a}$

$\Rightarrow \vec{J} = \frac{I}{(2\pi a)}\left(\frac{1}{r}\right)\hat{r}$ (from (1))

→ k

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☆ Note :-

$k = \frac{I}{\partial l}$: Here, I is $\perp$ to l.
So, although I is a scalar qty., still, here it will be taken as a vector

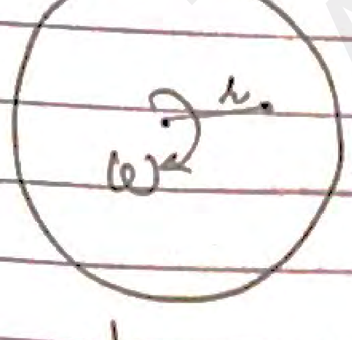
That gives :- k : vector

• Also, as $\exists$ Area vector. so, J : vector.

Q (a) A phonograph record carries a uniform density of σ. If it rotates at angular vel. ω, what is K at a distance r from centre.

(b) A uniformly charged solid sphere of radii R & total charge Q is centered at the origin & spinning at angular vel ω about z axis. Find $J(r, \theta, \phi)$ within the sphere?

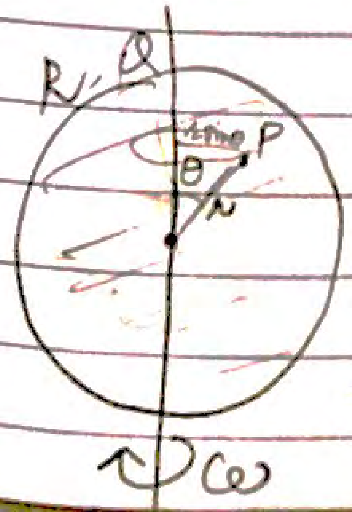
(a)



$K = \sigma \underbrace{v}_{r\omega}$

$= \sigma r \omega$

(b)



$J = \underbrace{\rho}_{\frac{Q}{\frac{4}{3}\pi R^3}} \underbrace{v}_{(r\sin\theta)\omega}$ → vel will be tangential to the ring formed. $\vec{v} = \vec{\omega} \times \vec{r}$

$\Rightarrow \vec{J} = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)(r\sin\theta\,\omega)\,(\hat{\phi})$

☆☆ $= \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)(\vec{\omega} \times \vec{r})\,(\hat{\phi})$

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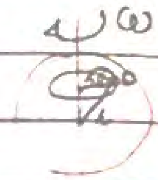
Concept $J = \left(\dfrac{Q}{\frac{4}{3}\pi R^3}\right)\left(\bar{\omega} \times \bar{r}\sin\theta\right)\hat{\phi}$.



along R — varies. (its outwards)

$\bar{\omega} \times \bar{r}\sin\theta = |\bar{\omega}||\bar{r}\sin\theta| \sin 90^\circ$

$= \underline{\omega r \sin\theta}$ — angle between ω & $r\sin\theta$.

Dir^n will be $\perp$ to both ω & $r\sin\theta$

i.e, along $\hat{\phi}$

Concept

☆ Finding dir^n of $\bar{B}$ when current is flowing through the wire

1) For straight wire

By Right hand thumb rule

Take dir^n of current at thumb.
Curling of fingers give dir^n of $\bar{B}$.

2) For circular wire.
Take dir^n of current by curling fingers
Dir^n in which thumb points = dir^n of $\bar{B}$.

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Date
Page

★ Magnetic force on a current carrying wire:

1) $F = \int (V \times B) dq = \int (V \times B) \lambda dl = \int (I \times B) dl.$

or $F = \int I (dl \times B) = I \int (dl \times B)$

(I & dl have same dir^n)

2) $F = \int (V \times B) \sigma dA = \int (K \times B) dA$.

3) $F = \int (V \times B) \rho \, d\tau = \int (J \times B) dA$.

★ Continuity eq^n (Just for info, not in syllabus).

$J = \frac{dI}{da_{\perp}} \Rightarrow I = \int_s J \cdot da$

$\oint_S J \cdot da = \int_V (\nabla \cdot J) d\tau \Rightarrow I = -\frac{dq}{dt} = -\frac{d}{dt} \int_V \rho \cdot d\tau$

$\Rightarrow \boxed{\nabla \cdot J = -\frac{d\rho}{dt}}$ ($\because$ Charge is concerned)

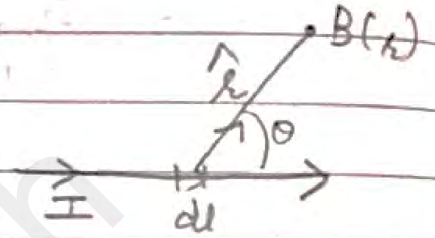
BIOT-SAVART LAW

$\mu_0 = 4\pi \times 10^{-7}\ N/A^2$.

$$\bar{B} = \frac{\mu_0}{4\pi}\int \frac{I \times \hat{r}}{r^2}\, dl = \frac{\mu_0 I}{4\pi}\int \frac{dl \times \hat{r}}{r^2}.$$

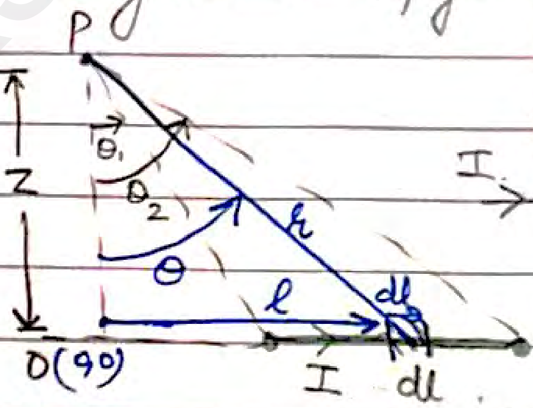
Note :- I and dl are in same dir^n.

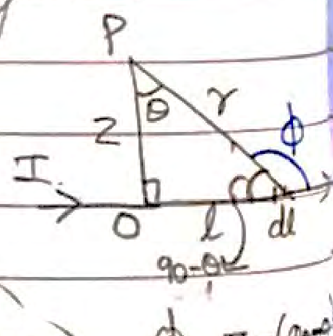
$\Rightarrow dl(\bar{I} \times \hat{r}) = I(d\bar{l} \times \hat{r})$



ex 5 Find the magnetic field at a distance Z above a long straight wire carrying a steady current I as given in figure.

Considering a small segment dl at a length l from the pt just below P.





$$dl \times \hat{r} = dl \sin\phi \quad (\phi \to 90+\theta)$$
$$= dl \cos\theta.$$

$$l = z\tan\theta$$

$$dl = z\sec^2\theta\, d\theta = \frac{z}{\cos^2\theta}\, d\theta.$$

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$$\cos\theta = \frac{z}{r} \Rightarrow \frac{1}{r^2} = \frac{\cos^2\theta}{z^2}$$

Now

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\theta_2} \underbrace{\left(\frac{\cos^2\theta}{z^2}\right)}_{1/r^2} \underbrace{\left(\frac{z}{\cos^2\theta}\right)\cos\theta\, d\theta}_{dl \times \hat{r}}$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi z}\int_{\theta_1}^{\theta_2} = \frac{\mu_0 I}{4\pi z}(\sin\theta_2 - \sin\theta_1)$$

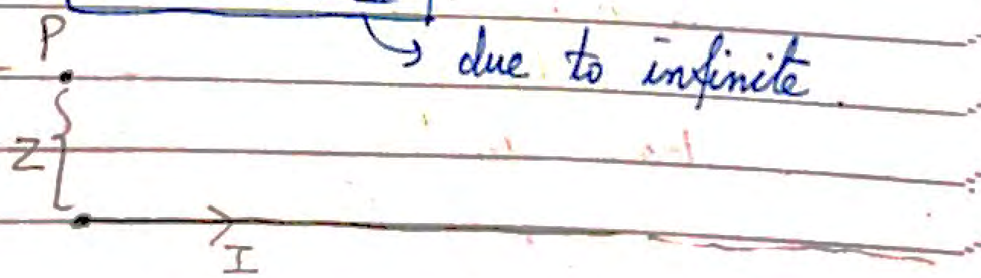
$$\Rightarrow \boxed{B = \frac{\mu_0 I}{4\pi z}(\sin\theta_2 - \sin\theta_1)}$$

→ If wire → ∞ (infinite wire)
$\theta_2 \to \frac{\pi}{2}$ & $\theta_1 \to -\frac{\pi}{2}$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi z}(1-(-1))$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi z}}$$ → due to infinite.

★ Note:-



For semi infinite $\theta_1 = 0,\ \theta_2 = \frac{\pi}{2}$.

$$\Rightarrow B = \left(\frac{\mu_0 I}{4\pi z}\right) = \left(\frac{\mu_0 I}{2\pi z}\right)\left(\frac{1}{2}\right) = \frac{1}{2}\,(\text{Due to infinite})$$

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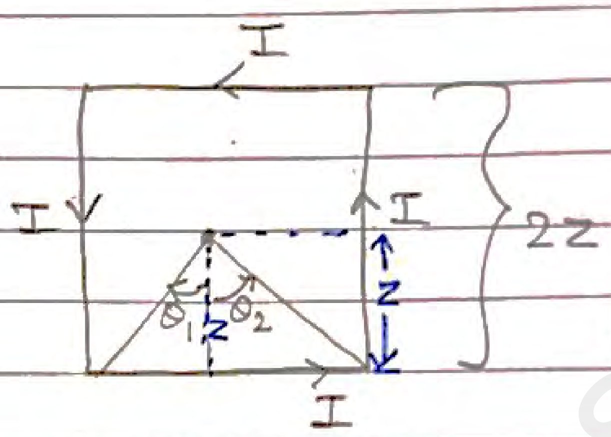
Q. Given a square



$\theta_1 = -\frac{\pi}{4}$

$\theta_2 = \frac{\pi}{4}$.

$$B = \frac{\mu_0 I}{4\pi z}(\sin\theta_2 - \sin\theta_1)$$

$$= \frac{\mu_0 I}{4\pi z}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

due to 1 wire $B = \frac{\mu_0 I(\sqrt{2})}{4\pi z}$.

$$\text{Total } B = 4 \times B = \frac{4 \times \mu_0 I(\sqrt{2})}{4\pi z}$$

$$\Rightarrow B_{Total} = \frac{\sqrt{2}\mu_0 I}{\pi z}.$$ ⊙ outwards

Q. Find B at a distance z above the centre of a circular loop of radius R which carries a steady current

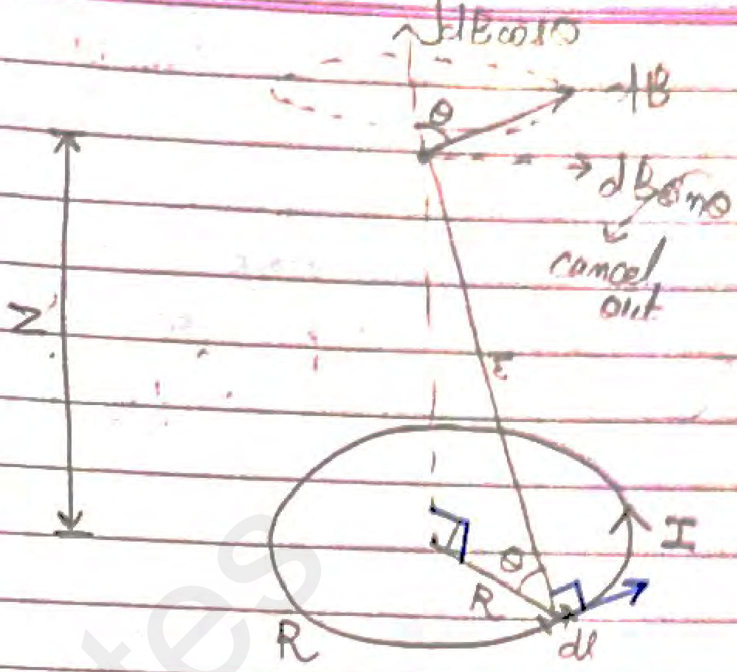
Idea :- Apply Biot Savart law :- Take $\underline{dl}$.

① $\boxed{dl \text{ \& } \hat{r} \text{ are } \perp}$ *

② Horizontal components cancel out.

$d\vec{l} \times \hat{r} = dl \sin 90^\circ$

($\angle$ b/w $d\vec{l}$ & $\hat{r}$ is 90°)



Only $\cos\theta$ components survive

So,

$$\int dB\cos\theta = \left[\frac{\mu_0 I}{4\pi}\int\frac{dl}{r^2}\right]\cos\theta$$

$$= \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{r^2}\right)\int dl$$

$$= \frac{\mu_0 I}{2}\,\frac{R^2}{(z^2+R^2)^{3/2}}$$

$\left(\because \int dl = 2\pi R\right.$

$\cos\theta = \frac{R}{(z^2+R^2)^{3/2}}$

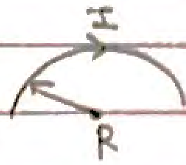
$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2}\left(\frac{R^2}{(R^2+z^2)^{3/2}}\right)}$$

→ If $z=0$ (at centre of loop)

$\Rightarrow B = \frac{\mu_0 I}{2R}$

→ For a semicircular loop

$B = \frac{1}{2}(\text{Full loop}) = \frac{\mu_0 I}{4R}$



→ For quarter

$B = \frac{1}{4}(\text{Full}) = \frac{\mu_0 I}{8R}$

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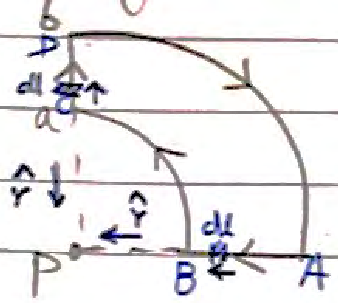
Q.5.9 Find $\vec{B}$ at pt. P for each of the steady current configuration :-

(a)



For a) $B_a = \frac{1}{4}(\text{Full}) = \frac{1}{4}\left(\frac{\mu_0 I}{2a}\right)$

For b) $B_b = \frac{1}{4}\left(\frac{\mu_0 I}{2b}\right)$

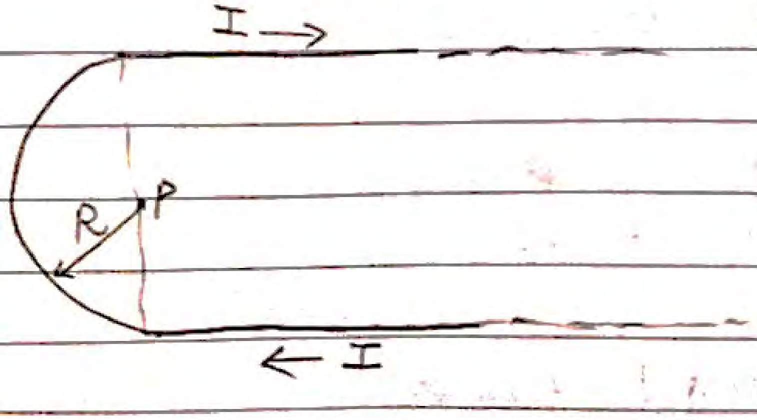
$B_b < B_a$

$B_{Total} = B_a - B_b \odot$

$= \frac{\mu_0 I}{8}\left(\frac{1}{a} - \frac{1}{b}\right) \odot$

[Note :- $\bar{B}$ at AB & CD $= 0$
$\because$ dl, r are in the same line
So, $(dl \times \hat{r})_{AB} = dl \sin 0° = 0$
$(dl \times \hat{r})_{CD} = dl \sin 180° = 0$]

(b)



$$B_{Total} = \underset{\text{(Semi circular loop)}}{\bar{B}} + \underset{\text{(Semi infinite wire)}}{\bar{B}} + \underset{\text{other (Semi infinite wire)}}{\bar{B}}$$

$$= \frac{1}{2}(\text{Full}) + \frac{1}{2}(\text{full}) + \frac{1}{2}(\text{full})$$

$$= \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right) + \frac{1}{2}\left(\frac{\mu_0 I}{2\pi z}\right) + \frac{1}{2}\left(\frac{\mu_0 I}{2\pi z}\right)$$

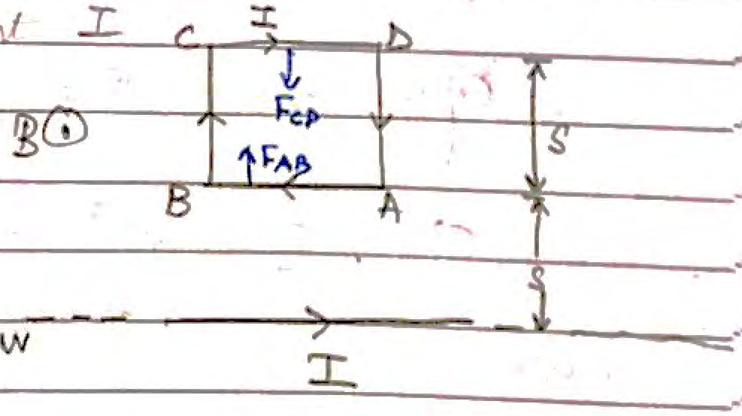
$\otimes$ $\quad$ R $\otimes$ $\quad$ R $\odot$

Same magnitude & opp. dir^n

$$= \frac{\mu_0 I}{4R}$$

$$\Rightarrow B_{Total} = \frac{\mu_0 I}{4R} \otimes$$

Q. Find the force on a sq. loop placed as shown in fig near an infinite st. wire. Both loop & wire carry a steady current I



$$B(s) = \frac{\mu_0 I}{2\pi s}$$

$$B(2s) = \frac{\mu_0 I}{2\pi(2s)}$$

$$B_{\text{due to wire}} = \frac{\mu_0 I}{2\pi z}$$

$$F_{BC} = -F_{AD}$$

$$\bar{F}_{CD} = I(l \times B) = I(s)\,B(2s) = Is\left(\frac{\mu_0 I}{4\pi s}\right) \cdot \sin 90^\circ \;(\downarrow)$$

unift. throughout length of wire.

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~~Magn~~ $F_{AB} = I(s)\, B(s) = Is \cdot \frac{\mu_0 I}{2\pi s} \sin 90^\circ \; (\uparrow)$

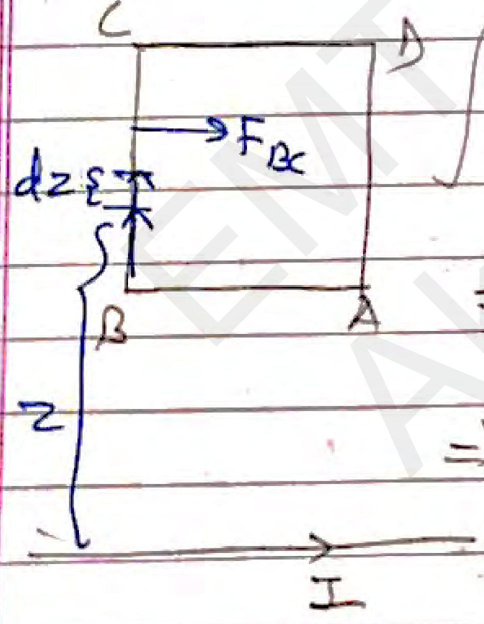
$$F_{AB} = \frac{\mu_0 I^2}{2\pi} \uparrow$$

$$F_{net} = F_{AB} - F_{CD}$$

$$= \frac{\mu_0 I^2}{\pi}\left(\frac{1}{2} - \frac{1}{4}\right) \; (\uparrow\uparrow - \downarrow)$$

$$\Rightarrow \bar{F}_{net} = \frac{\mu_0 I^2}{4\pi} \; (\uparrow)$$

* Now, suppose, we need to find F_{BC}



$$\int dF_{BC} = \int_s^{2s} I(dz) \frac{\mu_0 I}{2\pi z}$$

$$\Rightarrow = \frac{\mu_0 I^2}{2\pi} \ln\left(\frac{2s}{s}\right)$$

$$\Rightarrow \bar{F}_{BC} = \frac{\mu_0 I^2}{2\pi} \ln 2 \; (\longrightarrow)$$

$$= F_{AD} \; (\longleftarrow)$$

Good!

Q.5. 11) Find B on the axis of a tightly wound solenoid consisting on N turns per unit length wrapped around a cylindrical tube of radius R, current I. Find $(\bar{B})_P$.

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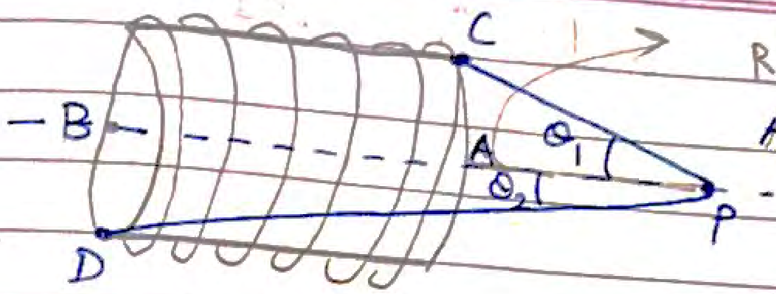
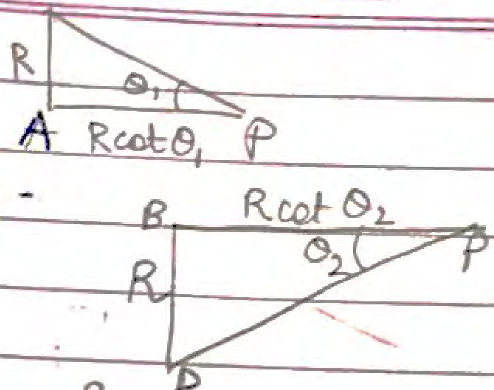
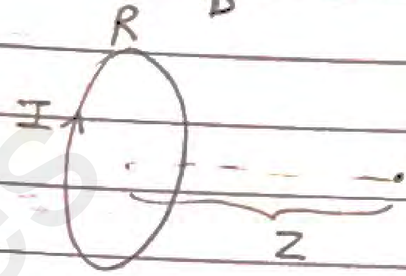
Puffin
Date
Page

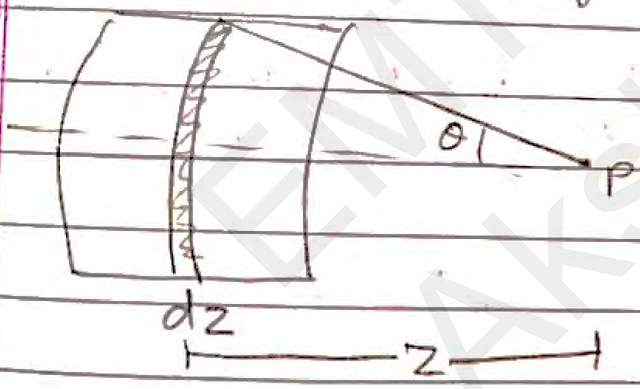




$B_{\text{circular loop at a distance z above}} = \dfrac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$



Idea: Consider a thin segment of the cylinder. It'll be a circular loop. Find B due to loop on P. Then, integrate

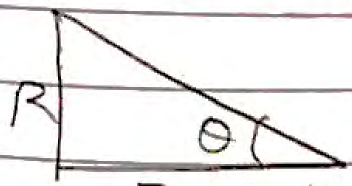


no. of turns = $N dz$

Current = $I \times N dz$

So, replace I by $I(N dz)$ in B_{loop}.

$$\Rightarrow B_{(dz)(loop)} = \frac{\mu_0 (I \times N dz) R^2}{2(R^2+z^2)^{3/2}}$$



$z = R\cot\theta$.

$dz = R(-\operatorname{cosec}^2\theta)d\theta$

So,

$$(R^2+z^2)^{3/2} = (R^2+R^2\cot^2\theta)^{3/2}$$
$$= R^3\operatorname{cosec}^3\theta$$
$$(\cot^2\theta + 1 = \operatorname{cosec}^2\theta)$$

$$= \frac{\mu_0 (IN) R^2 \cdot (-R\operatorname{cosec}^2\theta\, d\theta)}{2(R^3\operatorname{cosec}^3\theta)}$$

dz

$(R^2+z^2)^{3/2}$

EMT Notes Akshansh

Puffin
Date
Page

$$\Rightarrow B_{\substack{(loop) \\ (dz)}} = \frac{\mu_0 N I R^3 (-\operatorname{cosec}^2\theta)\, d\theta}{2R^3 \operatorname{cosec}^3\theta}$$

$$\Rightarrow B_{\substack{(loop) \\ (dz)}} = -\frac{\mu_0 N I}{2} \sin\theta \, d\theta$$

$$B_{solenoid} = \left(-\frac{\mu_0 N I}{2}\right) \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

θ_1 to θ_2

$\because$ we took element w.r.t initial pt. (C)

$$\Rightarrow \boxed{B_{solenoid} = \frac{\mu_0 N I}{2}\left[\cos\theta_2 - \cos\theta_1\right]}$$ ★

dir^n Along axis

For infinite solenoid

$\theta_1 = \pi$, $\theta_2 = 0$

$$\Rightarrow \boxed{B_{\substack{\infty \\ (solenoid)}} = \mu_0 N I}$$

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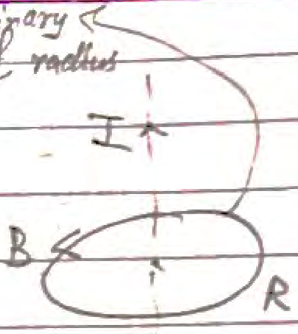
* Note: Electrostatics
  - Gauss law
  - q distributed uniformly

  Magnetostatics
  - Ampere's law
  - I distributed uniformly

Puffin
Date
Page

Divergence & Curl of B



$$\bar{B}_{\text{due to wire}} = \frac{\mu_0 I}{2\pi R}$$

$$\oint B \cdot dl = \oint \left(\frac{\mu_0 I}{2\pi R}\right) dl = \frac{\mu_0 I (2\pi R)}{2\pi R}$$

$$= \mu_0 I.$$

$$\Rightarrow \boxed{\oint B \cdot dl = \mu_0 I_{\text{enclosed}}}$$

→ I_{en} : current enclosed inside the loop.

→ AMPERE'S LAW.

Applying Stoke's Thm. on LHS,

$$\oint B \cdot dl = \int (\nabla \times B) dA.$$

$$\Rightarrow \int (\nabla \times B) dA = \mu_0 I_{en}$$

$$= \mu_0 \left[\int J . da \right]$$

→ $I_{enclosed}$

J → Volume current density

$$\Rightarrow \boxed{\int (\nabla \times B) dA = \mu_0 \int J . dA}$$

→ Ampere's law in Integral form.

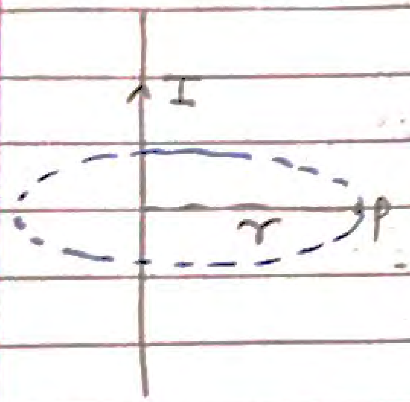
→ or Differential form.

$$\int (\nabla \times B) \cancel{dA} = \mu_0 I_{encl.}$$

Applicable for symm. current distrib^n.
(Gauss law : Just like it : for symm. charge distrib^n)

Puffin
Date
Page

eg



If we have to find magnetic field at P (B_P).

S1) Create Amperial loop (AL)
($\equiv$ Gaussian Surface)

S2) Apply Ampere's Law on AL
($\equiv$ Apply Gauss Law)

$$\oint B.dl = \mu_0 I_{encl}$$

$$\Rightarrow \oint B.dl\cos\theta = \mu_0 I_{encl}$$

$\cos 0$ $\because$ B & dl have 0° b/w them

$$\Rightarrow \boxed{B \oint dl = \mu_0 I_{encl}}$$

$\oint dl$ → Length of AL ($\equiv$ Area of GS)

I_{encl} → current enclosed by AL ($\equiv$ charge inside GS)

S3) Give dir^n to $\bar{B}$.

$$\oint B.dl = \mu_0 I_{encl}$$

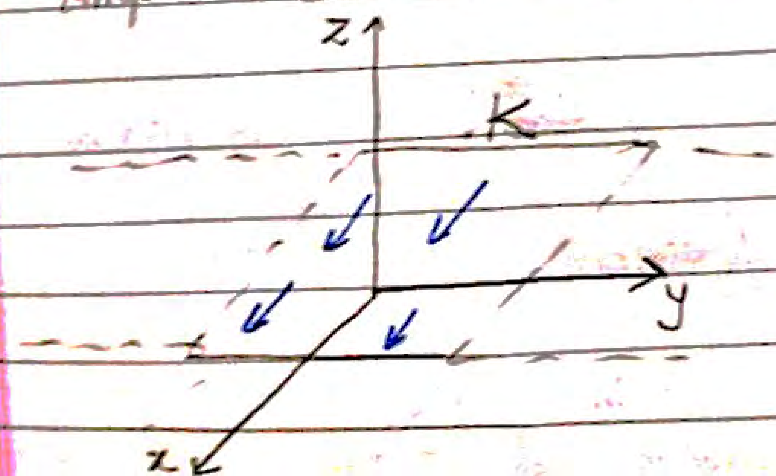
$$\Rightarrow B\int dl = \mu_0 I_{encl}$$

$$\Rightarrow B(2\pi r) = \mu_0 I$$

$$\Rightarrow \bar{B} = \frac{\mu_0 I}{2\pi r}$$

ex 5.8 Find magnetic field of infinite surface current $\bar{K} = K\hat{x}$ over x-y plane using Ampere's Law :-

K → Surface current density = current p.u. WIDTH



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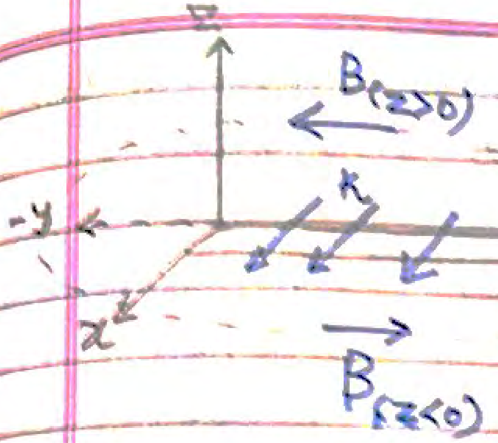
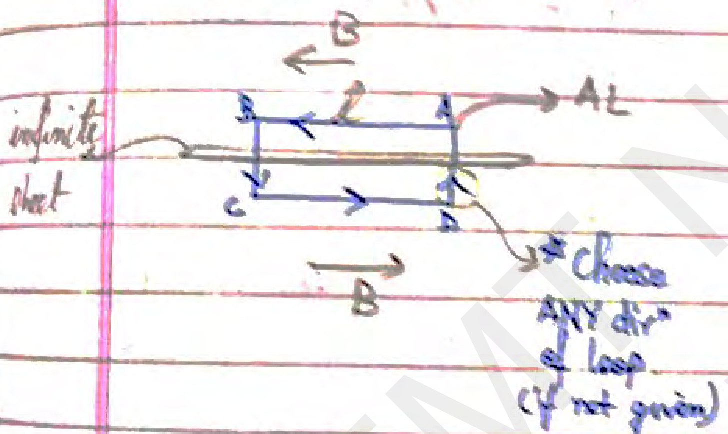


So, dir^n of $\bar{B}$

$\bar{B}_{(z<0)}$: $+\hat{y}$

$\bar{B}_{(z>0)}$: $-\hat{y}$

* Now, choose AL s.t. $\angle$ b/w B & $dl = 0$.



$$\oint B \cdot dl$$

$$= (B \times l \cos 0^\circ)_{AB} + (Bl \cos 90^\circ)_{BC} + (Bl \cos 0^\circ)_{CD} + (Bl \cos 90^\circ)_{AD}$$

$$= (Bl)_{AB} + (Bl)_{CD}$$

$$= 2Bl$$

Now

$$\oint B \cdot dl = \mu_0 I_{encl}$$

$$\Rightarrow 2Bl = \mu_0 (Kl) \qquad \left(\because K = \frac{dI}{dl_\perp}\right)$$

$$\Rightarrow B = \frac{\mu_0 K}{2}$$

$$\Rightarrow \vec{B} = \begin{cases} -\frac{\mu_0 K}{2}\hat{y}, & (z>0) \\ +\frac{\mu_0 K}{2}\hat{y}, & (z<0) \end{cases}$$

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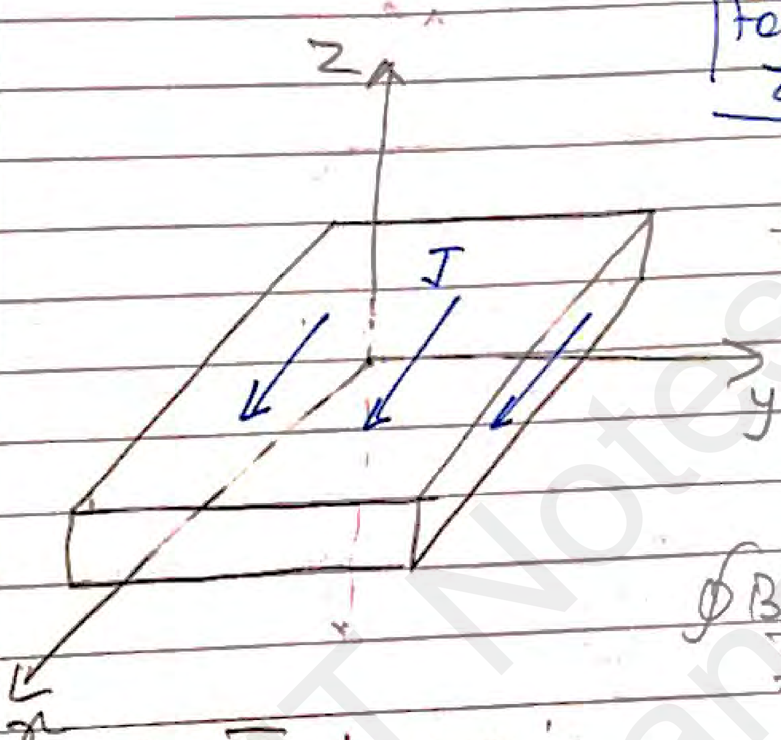
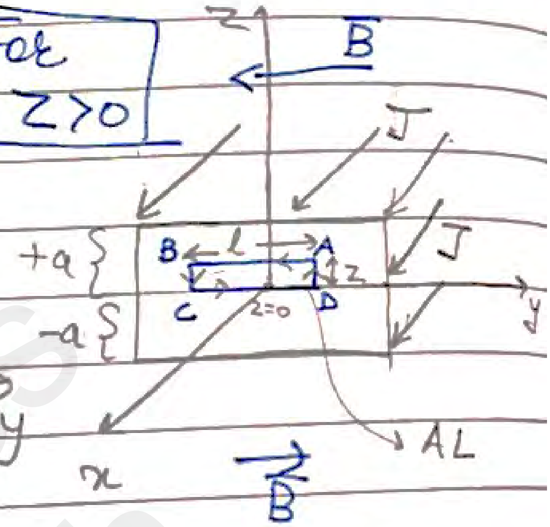
Q. 5.14 A thick slab extending from $z=-a$ to $z=+a$ carries a uniform volume current $= \bar{J} = J\hat{x}$. Find magnetic field as a f^n of z both inside & outside the slab using Ampere's law.



For $z>0$



$$\oint_{Top} \bar{B}\cdot d\bar{l} = (Bl)_{AB} + (0)_{BC} + (0)_{CD} + (0)_{AD}.$$

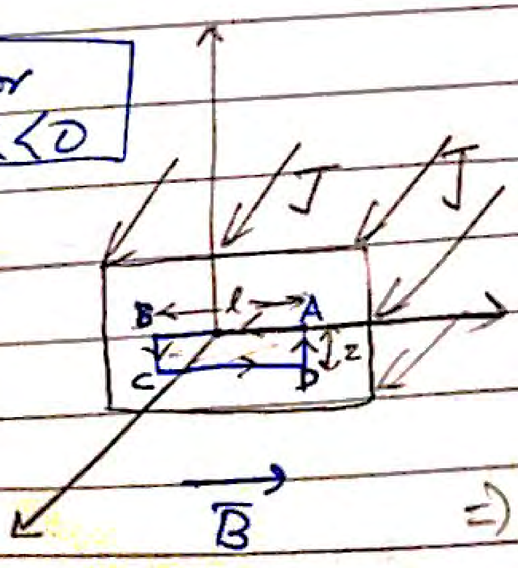
Note, $\bar{B}$ at $z=0$ is zero. So $(B_{z=0}) = 0$

($\because$ B is above & below in opp. dir^{ns}. So, in middle, its zero).

Now,

$$Bl = \mu_0 I_{enc}$$
$$= \mu_0 (J \cdot A)$$
$$\Rightarrow Bl = \mu_0 J (lz)$$
$$\Rightarrow \bar{B}_{Top} = -\mu_0 J z \,(\hat{y}) \quad (z>0)$$

For $z<0$



$$\oint_{bottom} \bar{B}\cdot d\bar{l} = (0)_{AB} + (0)_{BC} + (Bl)_{CD} + (0)_{AD}$$

$$\Rightarrow Bl = \mu_0 (J\, lz)$$

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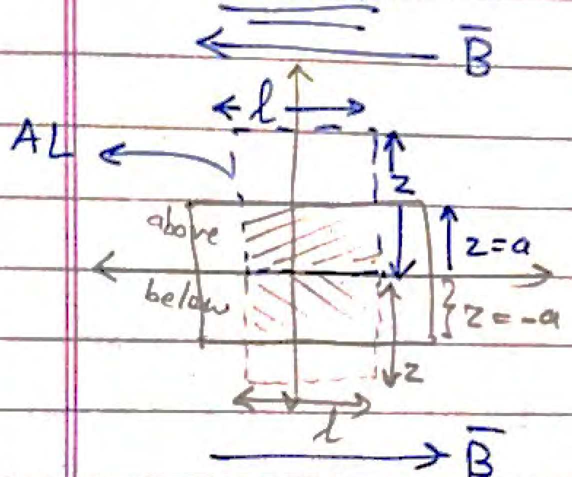
$$\Rightarrow B_{bottom} = +\mu_0 J z \, \hat{y} \; (z<0)$$

So, total inside $= \mu_0 J z (+\hat{y}) + \mu_0 J z(-\hat{y})$

For outside



Above

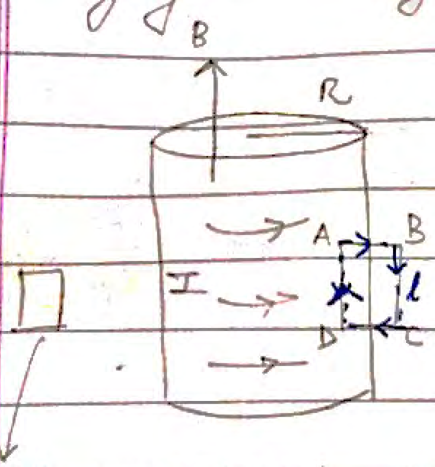
$B(l) = \mu_0 J (la)$

$$\Rightarrow \bar{B} = \mu_0 J a (-\hat{y}) \quad (z>0)$$

Total

$$\Rightarrow \bar{B} = \begin{cases} \mu_0 J a(-\hat{y}) & , z>0 \\ \mu_0 J a(+\hat{y}) & , z<0 \end{cases}$$

Q. 5.9 Find $\bar{B}$ of a very long solenoid consisting of n closely wounds/length on a cylinder of radius R & carrying a steady current I using Ampere's law.



Note :- Dir^n of I assumed.
Ampere loop is chosen so that some current passes through it.

$B \cdot dl = (0)_{AB} + (Bl)_{BC} + (0)_{CD} + (Bl)_{AD}$

AL cannot be taken here as it doesn't enclose any current

$(B.l)_{BC} = 0 \because$ B exists only inside solenoid. So, outside $B=0$

Puffin
Date
Page

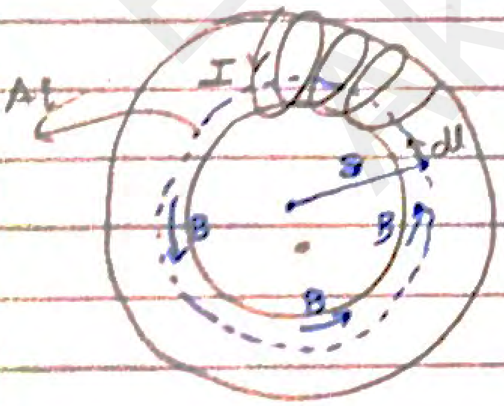
$\Rightarrow \oint B \cdot dl = (B \cdot l)_{ab}$

$= \mu_0 I_{encl}$

$\Rightarrow Bl = \mu_0 (I \times nl)$ → $\underbrace{n}_{\left(\frac{\text{no. of turns}}{\text{length}}\right)} \times \underbrace{l}_{(\text{length})}$

$\Rightarrow \vec{B} = \mu_0 n I \ \hat{k}$

Ex - 5.10 A toroid coil consists of a circular ring or "donut" along which a long wire is wrapped. It carries a current I & N is total no. of turns. Calculate B due to toroid using Ampere's law



N: Total no. of turns.
Idea for choosing Ampere's loop:
Choose it s.t current passes UNIFORMLY through it

$\oint B \cdot dl = \mu_0 I_{encl}$

$\Rightarrow B \cdot (2\pi s) = \mu_0 (\underline{NI})$

Its total current
$\because$ If you see, AL covers entire toroid

$\Rightarrow \vec{B} = \mu_0 \left(\frac{N}{2\pi s}\right) I \hat{\phi} = \mu_0 n I \hat{\phi}$

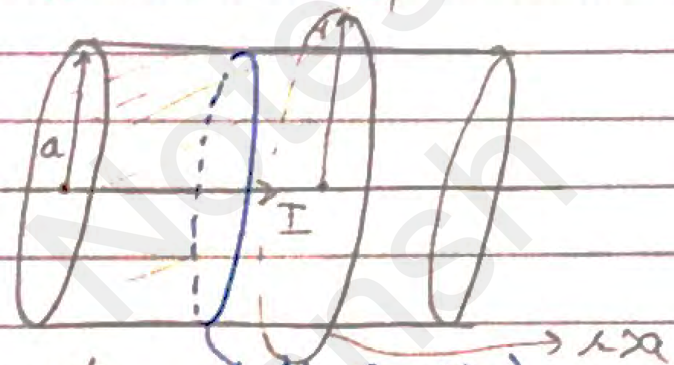
$n = \frac{N}{2\pi s}$

EMT Notes Akshansh

Puffin
Date
Page

Q.5.13 A steady steady current I flows down a long cylindrical wire of radius a. Find magnetic field both inside & outside the wire, if

(a) Current is uniformly distributed over outside surface of wire.
(b) The current is distributed in such a way that J is proportional to s, the distance from axis.



I : surface current

ΔL (a ring) $r = a$ on surface

(a) $r < a$:- $I_{encl} = 0$.
$\Rightarrow B = 0$.

$r > a$
$B \cdot (2\pi r) = \mu_0 I$
$\Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$

not asked

$r = a$:- $B(2\pi a) = \mu_0 I$
$\Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi a} \hat{\phi}$

[Note: It's fine to take on surface. no problem in that]

(b) $J \propto s \Rightarrow J = ks$; k: proportionality consth.

$I = \int J \cdot da = \int_0^a ks(2\pi s)ds \Rightarrow$
$I = \frac{2\pi k a^3}{3}$

$\oint$ $\odot^{\hat{z}}$

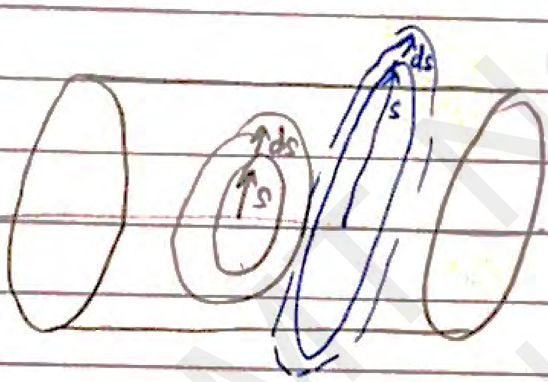
$\Rightarrow k = \frac{3I}{2\pi a^3}$ ⟶ (i)

So, $J = ks = \left(\frac{3I}{2\pi a^3}\right) \times s$

(Note, k isn't given, so, find it first)

Now,

$\underline{r < a}$ $\quad B.(2\pi s) = \mu_0 \int_0^s J \cdot da$



$= \mu_0 \int_0^s (ks)(2\pi s)ds$

$= \mu_0 \int_0^s \left(\frac{3I}{2\pi a^3}\right) s \cdot 2\pi s \, ds$

$= \mu_0 \left(\frac{\cancel{3}I}{a^3}\right)\left(\frac{s^3}{\cancel{3}}\right)$

$\Rightarrow B(2\pi s) = \frac{\mu_0 I \cancel{s^3} s^2}{a^3}$

$\Rightarrow \vec{B} = \frac{\mu_0 I s^2}{2\pi a^3} \hat{\phi}$ ⟶ whenever B is curling.

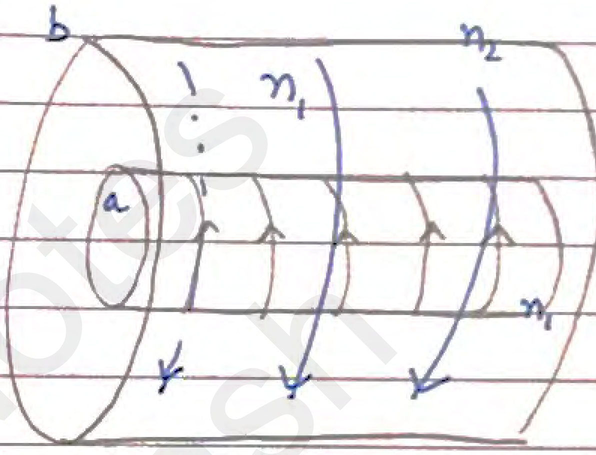
$\underline{r > a}$ $\quad B.2\pi s = \mu_0 \int_0^a J \cdot da = \mu_0 \int_0^a \left(\frac{3I}{\cancel{2\pi} a^3}\right) s \cdot \cancel{2\pi} s \, ds$

$= \frac{\cancel{3}\mu_0 I}{\cancel{a^3}} \left(\frac{\cancel{a^3}}{\cancel{3}}\right)$

$\Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi s} \hat{\phi}$ Ans

Puffin
Date
Page

Q. 5.15 Two long coaxial solenoids each carry current I but in opposite dir^n. The inner solenoid has (radius a) n_1 turns/length & outer one (radius b) has n_2.
Find
$\bar{B}$ (i) inside the inner solenoid
(ii) between them
(iii) Outside both



(a) $B = B_{\text{due to outer}} + B_{\text{inner loop}}$

$B = \underset{\longrightarrow}{\mu_0 n_2 I} + \underset{\longleftarrow}{\mu_0 n_1 I}$

$B = \mu_0 n_2 I - \mu_0 n_1 I$
(along axis)

(b) $B = B_{\text{outer}} = \mu_0 n_2 I$ (along $\underset{\longrightarrow}{\hat{x}}$)

(c) Outside $\longrightarrow B = 0$

Note :- For infinite solenoid, $\bar{B}$ outside it is zero.
So, inner loop won't contribute to $\bar{B}$ (in between them)

Q.5.8 (a) Find $\bar{B}$ at centre of sq. loop which carries steady current I. R: Distance from centre to side.
(b) Find $\bar{B}$ at centre of regular n sided polygon, carrying steady current I. R: distance from centre to side.

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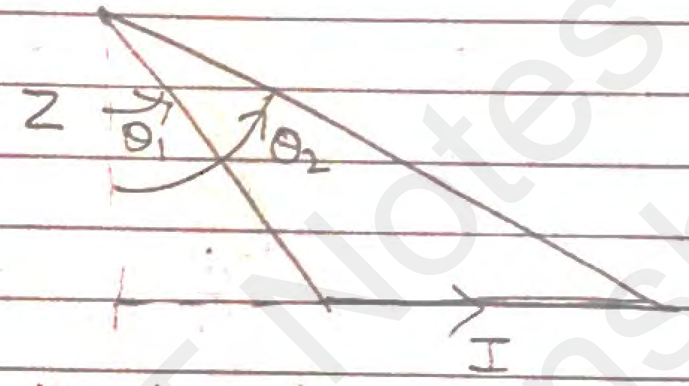
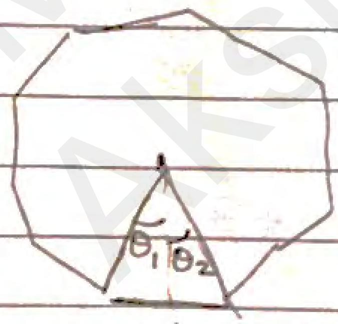
(c) Check that your formula reduces to the field at center of circular in which $n \to \infty$.

(a) Done before in Biot Savart law
(Ampere's law not applicable)

(b)



$$B = \frac{\mu_0 I}{4\pi z}(\sin\theta_2 - \sin\theta_1)$$



For n sides, total angle $\rightarrow 2\pi$

1 side $\longrightarrow \frac{2\pi}{n}$.

So, $\theta_1 + \theta_2 = \frac{2\pi}{n}$

$|\theta_1| = |\theta_2|$

So, $\theta_2 = \frac{\pi}{n}$

$\theta_1 = -\frac{\pi}{n}$

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So, $B = \frac{\mu_0 I}{4\pi R}\left(2\sin\frac{\pi}{n}\right) \times n$

for n sides

$\Rightarrow B = \frac{\mu_0 n I}{2\pi R}\left(\sin\frac{\pi}{n}\right)$

(i) For $n \to \infty$

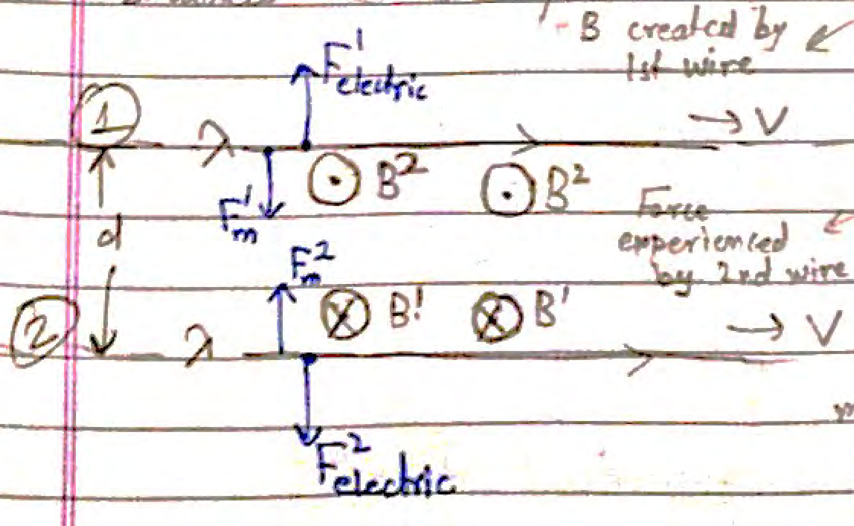
$\lim_{n\to\infty} \sin\frac{\pi}{n} \to \frac{\pi}{n}$.

$\Rightarrow B = \frac{\mu_0 \cancel{n} I}{2\cancel{\pi} R} \times \frac{\cancel{\pi}}{\cancel{n}}$.

$\Rightarrow \boxed{B = \frac{\mu_0 I}{2R}}$

↳ same as got before : $B_{\text{circular loop}}$.

Q 5.12 Suppose you have 2 infinite straight line charges λ, a distance d apart moving along a const speed v. What should be speed v so that magnetic attraction balances electric repulsion?



Magnetic part

$B_d^1 = \frac{\mu_0 I}{2\pi d} = \mu_0 \frac{(\lambda v)}{2\pi d}$

$F_{m}^2 = i\, L \times B$

$= (\lambda v) \cdot l \times B^1$

magnetic

(l : some length of wire, say)

EM Notes Akshansh

Puffin
Date
Page

$$\Rightarrow F_m^2 = \frac{\lambda v l \times \mu_0 \lambda v}{2\pi d}$$

$$\Rightarrow F_m^2 \text{ p.u length} = \frac{\lambda^2 v^2 \mu_0}{2\pi d} \left(= \frac{F_m^2}{l}\right) \longrightarrow (1)$$

experienced by 2nd due to 1st

Illy, F_m^1 experienced by 1st due to 2nd $= F_m^2$ (opp. dir^n)

Electric part

$$E^1_{\text{due to 1}} = \frac{\lambda}{2\pi\epsilon_0 d}$$

$$F_e^2 = q_2 E_1 = (\lambda l) E_1 = \frac{\lambda l \lambda}{2\pi\epsilon_0 d}$$

$$\frac{F_e^2}{l} = \frac{\lambda^2}{2\pi\epsilon_0 d} \longrightarrow (2)$$

Clearly from (1) & (2)

(1) = (2) (for repulsion = attraction, asked in ques.)

$$\Rightarrow \frac{\cancel{\lambda^2} v^2 \mu_0}{\cancel{2\pi d}} = \frac{\cancel{\lambda^2}}{\cancel{2\pi d}}\left(\frac{1}{\epsilon_0}\right)$$

$$\Rightarrow v^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$\Rightarrow v = \sqrt{\frac{1}{\mu_0\epsilon_0}} = c, \text{ speed of light.}$$

So, its possible if they move with speed of light.

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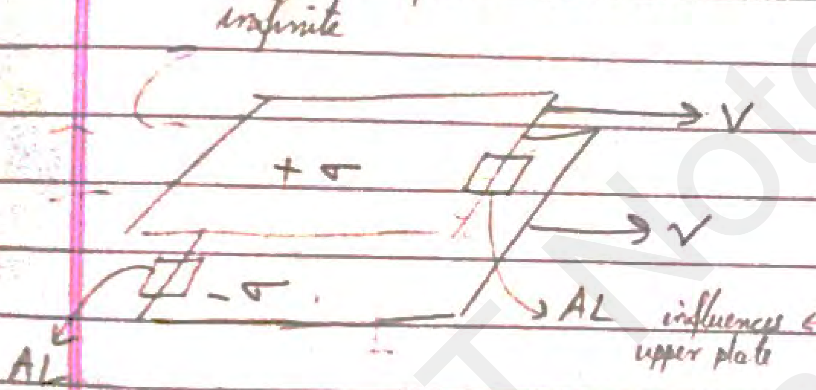
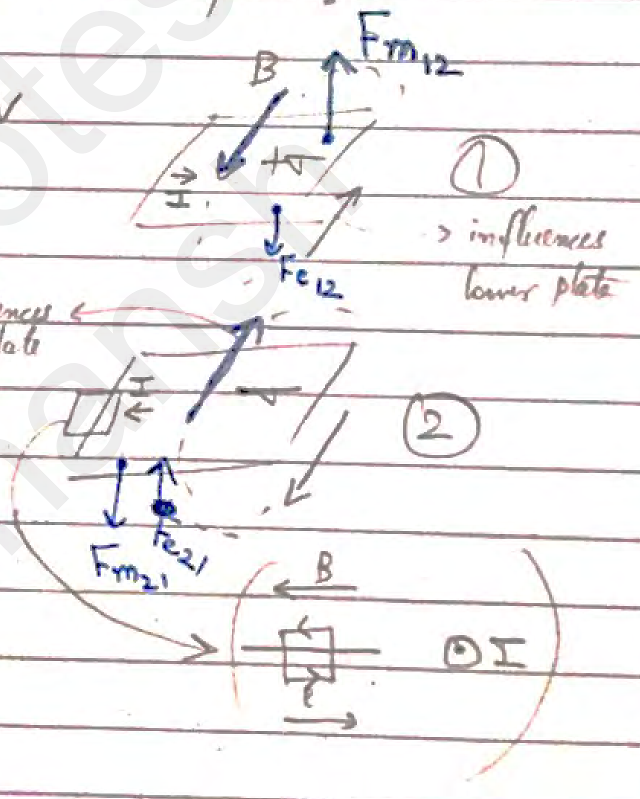
Q. 5.16 A large ∥ plate capacitor with uniform surface charge σ on upper plate & $-\sigma$ on lower is moving with a constt. speed v.

Find :- (a) $\vec{B}$ b/w plates, above & below

(b) Magnetic force p.u area on upper plate with dir^n

(c) At what speed v would the magnetic force balance electric force?





$\oint B \cdot dl = \mu_0 I_{encl}$

$\Rightarrow 2B \cdot l = \mu_0 \sigma v (l)$

$\Rightarrow B = \dfrac{\mu_0 \sigma v}{2}$

(a) Above the plates,

$\bar{B}$ will be only due to upper plate

So, $\bar{B} = \dfrac{\mu_0 \sigma V}{2}$ ⊙ or $\hat{x}$.

upper part of upper plate

Below the plates,

B will be only due to lower plate

So, $\bar{B} = \dfrac{\mu_0 \sigma V}{2}$ (⊙ or $\hat{x}$)

lower part of lower plate

Between the plates, $\bar{B} = \underbrace{B_1}_{\text{lower part of upper plate}} + \underbrace{B_2}_{\text{upper part of lower plate}} = 2\left(\dfrac{\mu_0 \sigma V}{2}\right)$ ⊗ or $-\hat{x}$

EMT Notes Akshansh

Puffin
Date
Page

$\Rightarrow \bar{B}_{b/w\ plates} = (\mu_0 \sigma v)\ (\otimes \text{ or } (-\hat{x}))$

(b) Magnetic force on upper plate $= F_{m_{12}} = i\, l \times B$

on 1 due to 2

$= (\sigma v \cdot l)\, l \times B$

$= (\sigma v\, l)\left(l \cdot \mu_0 \frac{\sigma v}{2}\right)$

$\Rightarrow F_{m_{12}} = \mu_0 \frac{\sigma^2 v^2 l^2}{2} \rightarrow$ Area term

$\Rightarrow \frac{\vec{F}_{m_{12}}}{Area} = \mu_0 \frac{\sigma^2 v^2}{2}\ \hat{z}$ (by RHTR or Flemmings left Hand rule)

$\rightarrow (1)$

(c) From previous chapter

$E_{\text{due to infinite sheet}} = \frac{\sigma}{2\epsilon_0}\hat{n} \rightarrow (2)$

On upper plate

$F_{m_{12}/Area} = F_{e_{12}/Area}$

$\Rightarrow \mu_0 \frac{\sigma^2 v^2}{2} = \frac{q\left(\frac{\sigma}{2\epsilon_0}\right)}{Area} = \frac{(\sigma \cancel{A})\left(\frac{\sigma}{2\epsilon_0}\right)}{\cancel{Area}}$

$\Rightarrow \mu_0 \frac{\cancel{\sigma^2} v^2}{\cancel{2}} = \frac{\cancel{\sigma^2}}{\cancel{2}\epsilon_0}$

$\Rightarrow v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

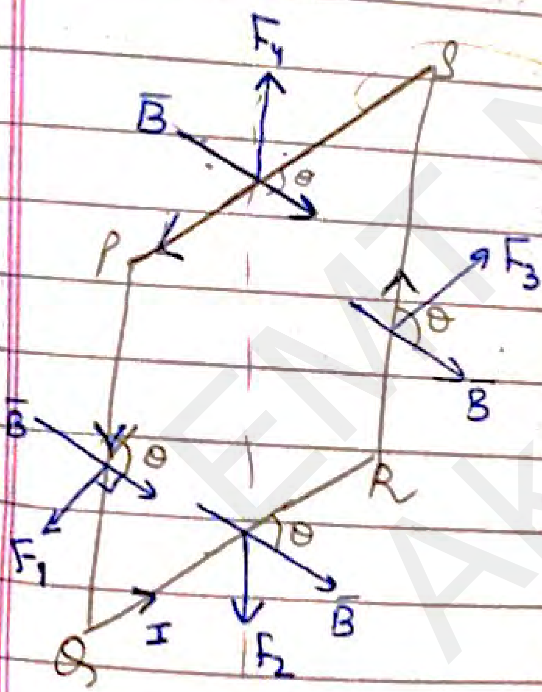
i.e, its only possible when plates move with speed of light

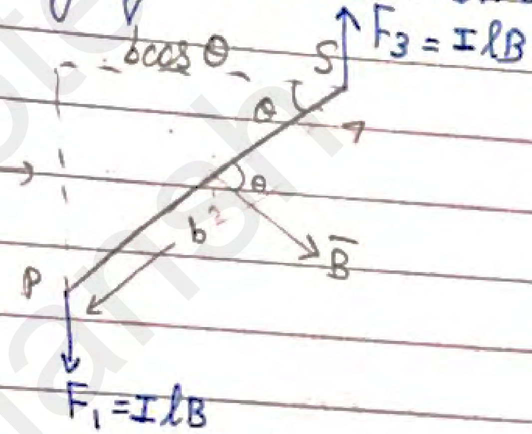
Ans

Chapter - 6

MAGNETISATION

☆ **Torque on a current carrying ~~coil~~ coil in a $\bar{B}$.**





$F_4 = I dl \times B = I(SP)B\sin(\pi-\theta)$
$= Ib(B\sin\theta)$

$F_2 = IbB\sin\theta \quad (\parallel_y)$

$F_1 = IlB\sin 90^\circ = IlB$

$F_3 = IlB \quad (\parallel_y)$

$PQ = RS = l$

$QR = SP = b$

$PQ \perp B$ & $RS \perp B$

F_2 & F_4 acting in opp. dirⁿ along same st. line So, resultant = 0.

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$F_1 = F_3 \rightarrow$ acting along diff^t line of action.

↳ so, they form a couple.

F_1 & F_3 try to rotate the coil.

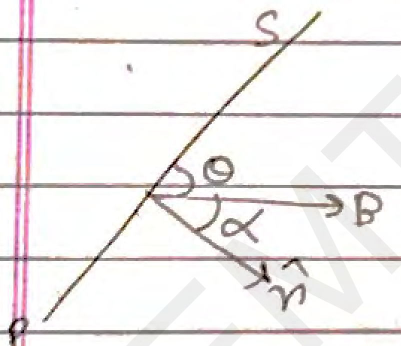
Torque, $N = r \times F = (b \cos\theta)\, I l B$

$\Rightarrow I(lb)\sin\theta = I A B \cos\theta$

$(lb = \text{Area})$

$\Rightarrow N = mB\cos\theta$

$\boxed{m = IA}$: magnetic dipole moment



From fig, $\theta + \alpha = 90^\circ$

$\Rightarrow \theta = 90^\circ - \alpha$

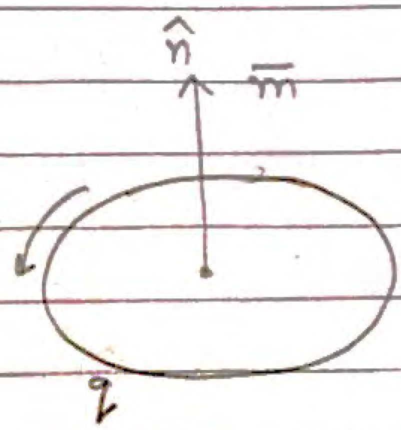
$N = mB\cos(90 - \alpha)$

$N = mB\sin\alpha$

$\Rightarrow \boxed{N = m \times B}$

Torque will try to align the dipole along the dir^n of $\overline{B}$. It is this torque that accounts for paramagnetism.

Paramagnetism takes place when there is an "unpaired e^-"



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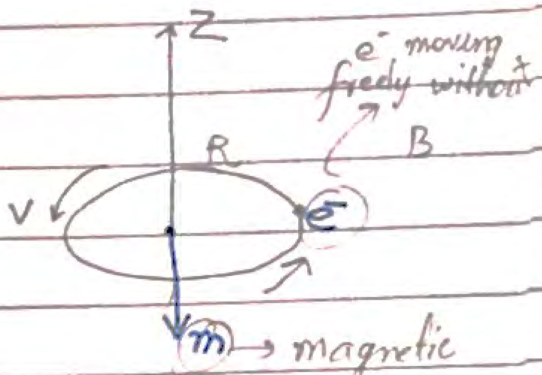
☆ Self Study: Difference & variation in properties of Para, Ferro & Dimagnetic material (eg: χ_m for them, > < or +, −, ...)

Puffin Date 19/11/12 Page

☆ Effect of Magnetic field on Atomic Orbits (Dimagnetism)



dipole moment (dirⁿ along −z → −ve particle)

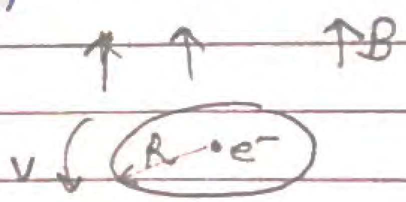
Time period $= \dfrac{2\pi R}{v}$

current $= \dfrac{e}{T} = \dfrac{ev}{2\pi R}$

Orbital dipole moment,

$$m = I \times Area$$
$$= I \times \pi R^2$$
$$= \frac{eV}{2\pi R}\pi R^2 = \frac{eVR}{2}(-\hat{z}) \longrightarrow (1)$$

When atom is placed in magnetic field B, it experiences τ $(= m \times B)$.



Force balance →

Coulomb force = Centrifugal.

$$\Rightarrow \frac{e^2}{4\pi\epsilon_0 R^2} = \frac{m_e v^2}{R} \longrightarrow (2)$$

In presence of magnetic field, add^{nal} force (Lorentz) $= (-e)(v \times B) \Rightarrow$ changes vel. (v) of e^- ($= v'$)

So,

$$\frac{e^2}{4\pi\epsilon_0 R^2} + e\bar{v}B = \frac{m_e \bar{v}^2}{R}$$

or $$\frac{m_e v^2}{R} + e\bar{v}B = \frac{m_e\bar{v}^2}{R} \quad \text{(From (2))}$$

$$\Rightarrow e\bar{v}B = \frac{m_e(\bar{v}^2 - v^2)}{R}.$$

$$\Rightarrow e\bar{v}B = \frac{m_e}{R}(\bar{v} + v)(\bar{v} - v) \simeq \frac{m_e}{R}(2\bar{v})(\Delta v)$$

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(Assuming $\bar{v}^{2} - v = \Delta v$, $\bar{v} + v = 2v$)

Then,

$$e\bar{v}B = \frac{m_e}{R}(2\bar{v})(\Delta v)$$

$$\Rightarrow \Delta \bar{v} = \frac{eR\bar{B}}{2m_e}.$$

$\Rightarrow$ when $\bar{B}$ is turned on, then e^- speeds up.

So, change in dipole moment (Δm) (from ①)

$$\Rightarrow \boxed{\Delta m = \frac{-1}{2}e(\Delta v)R\,\hat{z} = -\frac{e^2R^2\bar{B}}{4m_e}}$$

★

→ change in m is opp. to dir^n of B.

→ mechanism responsible for diamagnetism

→ It is weaker than paramagnetism & is observed mainly in atoms with ~~an~~ even no. of e^-s (where paramagnetism is usually absent.)

★ Magnetic Vector Potential : Bound Currents

($\equiv$ Dielectric material has σ_b & ρ_b)

We know :- $\nabla\cdot(\nabla\times A) = 0$

& $\nabla\cdot B = 0$

$\Rightarrow$ $B = \nabla\times A$.

→ any vector

→ magnetic field.

divergence of curl of a vector $= 0$.

$\Rightarrow$ $\boxed{\bar{\nabla} \cdot \bar{A} = 0}$ (always) (without proof)

* ↳ A : vector potential, say

By Ampere's law,

$$\nabla \times B = \mu_0 J$$ → volume current density

$$\Rightarrow \nabla \times (\nabla \times A) = \mu_0 J$$

$$\Rightarrow \nabla(\bar{\nabla} \cdot \bar{A}) - \nabla^2 A = \mu_0 J$$

$$\Rightarrow \nabla^2 A = -\mu_0 J \quad (\because \bar{\nabla} \cdot \bar{A} = 0)$$

↳ 2nd order Differential eqn

Soln (without proof)

$$\Rightarrow A = \frac{\mu_0}{4\pi} \int_{Vol} \frac{J\, d\tau}{r}$$ → Vector potential due to any kind of Vol. current

☆

||ly → $A = \frac{\mu_0}{4\pi} \int \frac{k\, d\tau}{r}$ → surface

→ $A = \frac{\mu_0}{4\pi} \int \frac{I\, dl}{r}$ → line

☆ Vector potential due to magnetic dipole moment, m, is

$$\boxed{A = \frac{\mu_0}{4\pi} \left(\frac{m \times \hat{r}}{r^2} \right) = \frac{\mu_0}{4\pi} \frac{M \times \hat{r}}{r^2} d\tau} \rightarrow (1)$$

* (without proof)

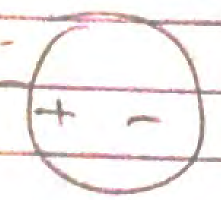
where $M = \frac{m}{V} = \frac{\text{dipole moment}}{\text{vol.}}$ = Magnetizn

(☆ $M = \frac{m}{vol}$ or $m = M d\tau$).

CONCEPT

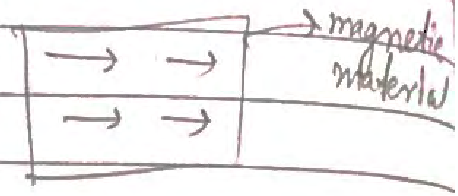
★ Electrostatics

Dielectric



Ily.

★ Magnetisation



when $\bar{E}$ applied, charges (bound) formed. So, we can say that dipole is created.
Sys. has $\bar{P}$, $\bar{p}$ → electric dipole moment
$\bar{P}$: polarisⁿ

When $\bar{B}$ given ⇒ magnetic dipole created.
Total $B = \int B_{\text{due to 1 dipole}}$
Sys. has $\bar{M}$, $\bar{m}$ → magnetic dipole moment
$\bar{M}$: magnetisⁿ

Now, A: vector potential (said like that)

& $\nabla \cdot B = 0$

& $\nabla \cdot (\nabla \times \bar{A}) = 0$

$\Rightarrow B = \nabla \times A$

★ Finding A in magnetisⁿ ≡ Find E in electrostatics

★

$$E = \int_{\text{vol}} \rho_b + \int_{\text{Surface}} \sigma_b$$

Ily, $A = \int J_b + \int K_b$

J_b → bound vol. current density

K_b → bound surface current density.

★ $\rho_b = -\nabla \cdot \bar{P}$, $P = p/_{\text{Vol}}$

$\sigma_b = P \cdot \hat{n}$;

Ily, $J_b = \nabla \times M$, $M = m/_{\text{vol}}$

$K_b = M \times \hat{n}$

(So, 1−1 correspondence)

Idea ★ Find A in any problem. So, $\nabla \times A$ will give B (req.)

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Puffin
Date
Page

From (1), we have vector potential, A due to single dipole -

So,

due to object, $A = \frac{\mu_0}{4\pi} \int_{vol.} \frac{M \times \hat{\imath}}{\imath^2} d\tau$

$\left(\nabla\left(\frac{1}{\imath}\right) = \frac{\hat{\imath}}{\imath^2} \right)$

$$\Rightarrow A = \frac{\mu_0}{4\pi} \int \left(M \times \nabla \frac{1}{\imath}\right) d\tau = \frac{\mu_0}{4\pi} \left[\int \frac{1}{\imath} (\nabla \times M) d\tau - \int \nabla \times \left(\frac{M}{\imath}\right) d\tau \right]$$

(from Rule V : $l_g - 21$)

$$= \frac{\mu_0}{4\pi} \int \frac{1}{\imath} (\nabla \times M) d\tau + \frac{\mu_0}{4\pi} \oint \frac{1}{\imath} (M \times da)$$

(from P. 1.60(b) ch-1)

$$\Rightarrow A = \frac{\mu_0}{4\pi} \int_{vol.} \frac{J_b \, d\tau}{\imath} + \frac{\mu_0}{4\pi} \oint_{surface} \frac{K_b \, da}{\imath}$$

$J_b = \nabla \times M$: bound volume current density.
$K_b = M \times \hat{n}$: bound surface current density.

(Just like it was done in electrostatics where we found ρ_b & σ_b)

☆ Note, just like in electrostatics,
$\rho_b + \sigma_b = 0$.
||ly, here,
$J_b + K_b = 0$.

EMT Notes
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★ Note :- $\nabla \times M \equiv$ curl of a vector.
So, remember eqⁿ of curl of any vector in cylindrical & polar coordinate sys.

★ Ampere's law in Magnetised Material.

Recollect, a theoretical parameter was introduced in electrostatics, D : Displacement current
||ly, here, we define (H).

Now,

$$J = J_b + J_f \quad (\equiv \rho = \rho_b + \rho_f)$$

$$\nabla \times B = \mu_0 J = \mu_0 J_b + \mu_0 J_f = \mu_0 (\nabla \times M) + \mu_0 J_f$$

$$\Rightarrow \nabla \times \left(\frac{B}{\mu_0} - M\right) = J_f.$$

$$\Rightarrow \boxed{J_f = \nabla \times H} \; ; \; H = \frac{B}{\mu_0} - M.$$

$$\left(\cancel{\equiv Q_{free} = \oint D \cdot da} \; ; \; \nabla \cdot D = \rho_f \right) \equiv$$

↳ Differential form of Ampere's law in magnetic material

Now

$$\int_S (\nabla \times H) \, da = \int J_f \cdot da \Rightarrow \boxed{\oint H \cdot dl = I_{f_{enclosed}}}$$

total free current through Ampere's loop

Ampere's law in integral form in presence of magnetic material.

$$\equiv \left(\oint D \cdot da = Q_{free} \right)$$

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✯ Magnetic Susceptibility

(how susceptible is a material w.r.t ext. magnetic field)

$$M \propto H \Rightarrow M = \chi_m H \quad \rightarrow \text{magnetic susceptibility}$$

$$(\equiv P \propto E \Rightarrow P = \chi_e E)$$

Then,

$$B = \mu_0 (H + M) = \mu_0 (H + \chi_m H) = \mu_0 (1 + \chi_m) H \quad (\equiv \epsilon_0 (1 + \chi_e) E)$$

$$\Rightarrow B = \mu H \ (\equiv \epsilon E), \quad \mu = \mu_0 (1 + \chi_m) \quad (\equiv \epsilon = \epsilon_0 (1 + \chi_e))$$

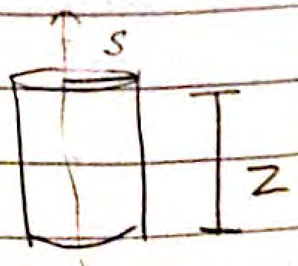
μ: permeability of material.

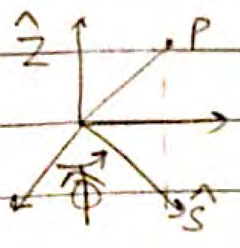
✯ In absence of material, $\chi_m = 0$.

✯ Done before: written again.
- how to find curl in cylindrical coordinate sys.

$$\nabla \times V = \left(\frac{1}{s} \frac{\partial V_z}{\partial \phi} - \frac{\partial}{\partial z} V_\phi \right) \hat{s} + \left(\frac{\partial}{\partial z} V_s - \frac{\partial}{\partial s} V_z \right) \hat{\phi}$$

$$+ \frac{1}{s} \left(\frac{\partial}{\partial s} (s V_\phi) - \frac{\partial}{\partial \phi} V_s \right) \hat{z}$$





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Puffin
Date
Page

* While finding curl of M ($\nabla \times M$) check first, what is M a f^n of, i.e M_s or M_ϕ or M_z. Then, find accordingly.

Q.6.7 An infinitely long circular cylinder carries a uniform magnetis^n M ∥ to its axis. Find $\bar{B}$, inside & outside cylinder.

M uniform ⇒ not a f^n of s, z or ϕ

So, $\nabla \times M = 0 (= J_b)$

$K_b = \bar{M} \times \hat{n} = M(\hat{z} \times \hat{n})$ along $\hat{s}$

$\Rightarrow K_b = M\hat{\phi}$ tangent along surface

So, only K_b exists !!!! solenoid

$\bar{B}_{(outside)} = 0$ (for solenoid)

$\bar{B}(inside) = \mu_0 n I$ (proved before) no. of turns

Now, $K_b = \frac{I}{l}$ or $I\left(\frac{N}{l}\right)$ or In.

$\Rightarrow \bar{B}_{(inside)} = \mu_0 K_b = \mu_0 M(\hat{z})$

found using right hand thumb rule. (don't take dir^n of K_b ($\hat{\phi}$ here))

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☆ Remember: $\hat{\phi}$ acts along tangent & is responsible for circular effect.

Puffin
Date
Page

Q 6.8 A long circular cylinder of radius R carries a magnetisation $M = ks^2 \hat{\phi}$; k: const, s: distance from axis ; ϕ: unit vector.
Find $\vec{B}$ inside & outside cylinder.

Now,

given :- $M = ks^2 \hat{\phi}$
$= f^n$ of s
$=$ along $\hat{\phi}$

$\Rightarrow M_\phi (s)$ is given.

Using formula for cylindrical coordinates, (taking only V_ϕ)

$$J_b = \nabla \times M = \left(-\frac{\partial}{\partial z} V_\phi\right)\hat{s} + \frac{1}{s}\left(\frac{\partial}{\partial s}(sV_\phi)\right)\hat{z}$$

f^n of z $= 0$; f^n of s

$$\Rightarrow \nabla \times M = \frac{1}{s}\left(\frac{\partial}{\partial s}\left(s \times ks^2\right)\right)\hat{z}$$

$$= \frac{1}{s}\left(\frac{\partial}{\partial s}(ks^3)\right)\hat{z}$$

$$= \frac{3s^2(k)}{s}\hat{z}$$

$$\Rightarrow \nabla \times M = 3ks\,\hat{z} = J_b \rightarrow (1)$$

$$K_b = M \times \hat{n} = ks^2\,\hat{\phi} \times \hat{n}$$

$$\Rightarrow K_b = ks^2(-\hat{z})$$

by RHTR (curl fingers from $\hat{\phi}$ to $\hat{n}$)

at $S = R$ $\rightarrow$ we are at surface now

$$\Rightarrow K_b = kR^2(-\hat{z}) \rightarrow (2)$$

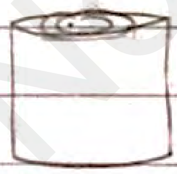
extra: Calculate total bound current ($K_b + J_b$ total)

$J_b = 3ks(\hat{z})$ $\qquad I_b = \int J_b \cdot dA + \int K_b \cdot dl_\perp$

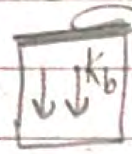
$K_b = KR^2(-\hat{z})$

$$I_b = \underbrace{\int_0^R (3ks)(2\pi s\,ds)}_{dA} + \underbrace{\int (-KR^2)(2\pi R)}_{dl_\perp}$$

dA
(∵ J is changing w.r.t s)



$dl_\perp$ is $\perp$ to flow. Now, cut the cylinder s.t



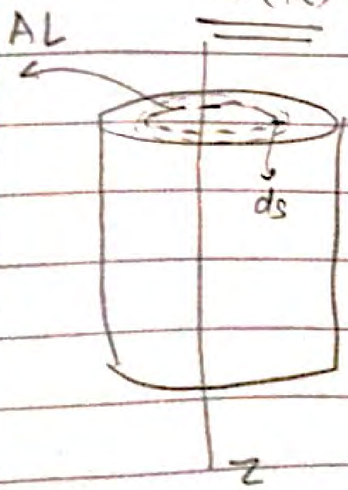
$\to dl_\perp = 2\pi R$

$$= 6\pi k\frac{R^3}{3} - 2\pi KR^3$$

$$= 0.$$

Now, $B_{(s>R)} = 0$ ($I_{encl} = 0$)

$B(s<R)$: $\oint B\cdot dl = \mu_0 I_{encl}$



$$\Rightarrow B(2\pi s) = \mu_0\int_0^s J_b \cdot da$$

$$= \mu_0\int 3ks(2\pi s)ds$$

$$\Rightarrow B(\cancel{2\pi s}) = 6\pi\mu_0 k\frac{s^3}{3} = \cancel{2\pi}\mu_0 ks^3$$

$$\Rightarrow B = \mu_0 ks^2\,\hat{\phi} \rightarrow \because J_b \text{ is along } \hat{z} \text{ (RHT)}$$

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$B = \mu_0 (Ks^2)\hat{\phi}$
$B = \mu_0 M \hat{\phi}$, $M = ks^2$, given

if it gets magnetised by any means

Q.6.12 An infinitely long cylinder of radius R carries a frozen in magnetisation, parallel to the axis
$M = ks\hat{z}$, k: constt ; s: distance from axis,
∃ no net current anywhere.
Find: (a) B → inside & outside
(b) All bound currents (J_b & K_b)

$J_b = \nabla \times M \quad (M_z(s))$

$= \left(\frac{1}{s}\frac{\partial}{\partial\phi} M_z\right)\hat{s} - \frac{\partial}{\partial s} M_z \hat{\phi}$

$= 0 - \frac{\partial}{\partial s}(ks)\hat{\phi}$

$\Rightarrow J_b = -k\hat{\phi}$

$K_b = M \times \hat{n} = ks(\hat{z} \times \hat{n})$

$\Rightarrow K_b = ks\Big|_{s=R}\hat{\phi} = kR\hat{\phi}$

$B_{(s<R)}$: $\oint B \cdot dl = \mu_0 I_{encl}$

$\Rightarrow B(2\pi s) = \mu_0 \int_0^s J_b \cdot dA$

$= \mu_0 \int_0^s (-k)(2\pi s)\,ds$

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$$\Rightarrow B(\cancel{2\pi s}) = -\cancel{2\pi} k\mu_0 \left(\frac{s^{\cancel{2}}}{2}\right)$$

$$\Rightarrow B = -k\frac{\mu_0 S}{2}\hat{z}$$

$$\Rightarrow \bar{B}_{\text{inside}} = k\frac{\mu_0 S}{2}(-\hat{z}) \rightarrow ①$$

Show, Sum of all bound currents = 0.

$$I_b = \int J_b \cdot dA + \int K_b \cdot dl_\perp$$

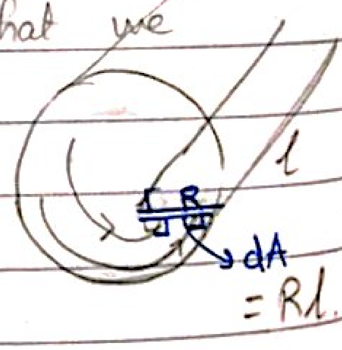
$$= \int_0^R J_b \, dA + K_b \cdot l$$

J_b is in dir^n of $(-\hat{\phi})$.

So, find dA s.t J_b should cut $\perp$ to area that we choose.

$R \cdot l$

(Note: l is infinite in this case. So, $I_{(J_b)} + I_{(K_b)} \neq 0$)

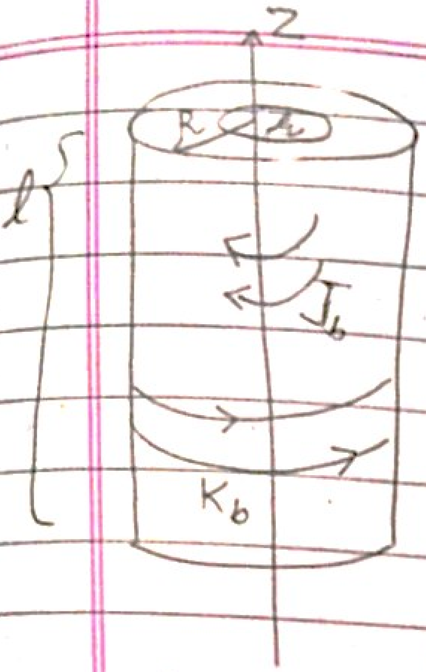


Suppose finite:

$$\Rightarrow I_b = -k(Rl) + (kR)l$$
$$= 0$$

Q 6.12 An infinitely long cylinder of radius R carries a frozen in magnetis^n parallel to the axis $M = ks$ where k: constt. & s: distance from axis. ∃ no free current anywhere. Find $\bar{B}$ inside & outside the cylinder. Locate all bound currents.

Puffin
Date
Page



$M = ks\hat{z} = M_z(s)$

Find J_b & K_b (M given)

$J_b = \nabla \times M$

$= \left(\frac{1}{s}\frac{\partial}{\partial\phi} V_z\right)\hat{s} - \left(\frac{\partial}{\partial s} V_z\right)\hat{\phi}$

$= 0 - k\hat{\phi} = -k\hat{\phi}$

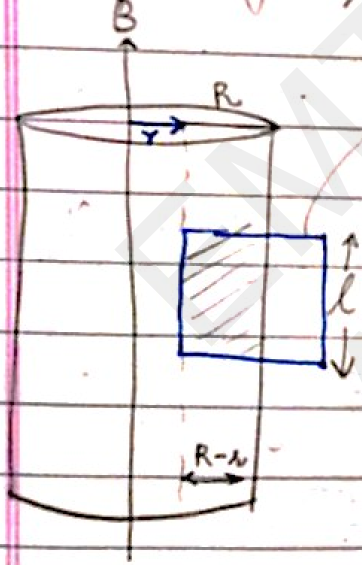
$K_b = M \times \hat{n}$

$= ks(\hat{z} \times \hat{n})$

$= ks\,\hat{\phi}\Big|_{s=R} = kR\,\hat{\phi}$

Note dir^ns of J_b & K_b

Create surface / length $\perp$ to current flow for AL



AL created at a distance r from axis.

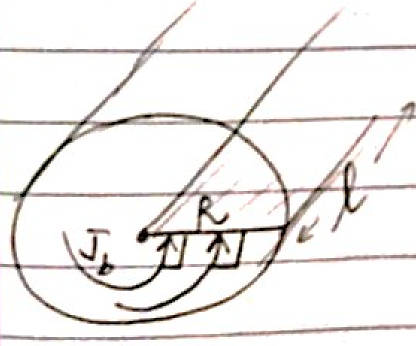
Now

$I_b = \int J_b \cdot dA + K_b \cdot dl_\perp$

$= (-k)(Rl) + (kR)\,l$

$= 0$

So, Total bound currents $= 0$



Finding B

From Ampere's law

$\oint B \cdot dl = \mu_0 I_{encl}$

($\oint H \cdot dl = I_{free}$: this formula not applicable $\because I_{free} = 0$, given)

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Akshansh

$B(r>R) = \bar{0}$ ($\because I_{enc} = 0$)

$B(r<R) =$

$$B\cdot l = \mu_0\left[\int J_b\, dA + K_b \cdot dl_\perp\right]$$

$$= \mu_0\left[(-k)(l\cdot(R-r)) + kR\cdot l\right]$$

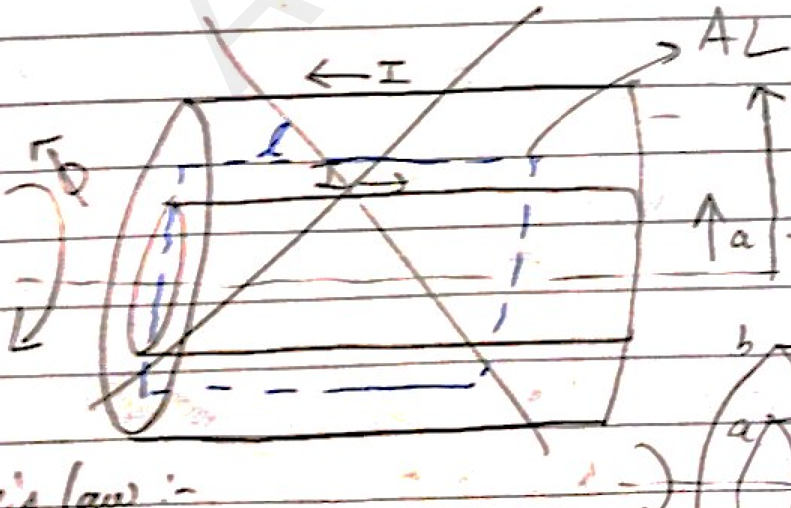
$$\Rightarrow B\cdot l = \mu_0\left[+krl\right]$$

$$\Rightarrow B = \mu_0 kr\,(-\hat{z})$$

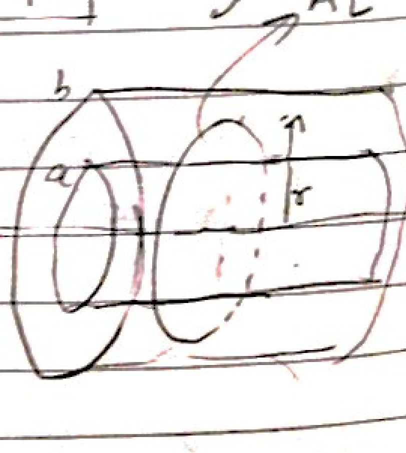
$\because J_b$ is in $-\hat{\phi}$ (RHTR)

Q.6.16 A coaxial cable consists of two very long cylindrical tubes, separated by insulating material of magnetic susceptibility χ_m. A current I flows down the inner cylinder & conductor and returns along outer one.

Find $B(a<r<b)$. Also calculate bound currents also.



Use this diagram instead



Ampere's law :-

$$\oint H\cdot dl = I_{free},\ \hat{\phi}$$

$$\Rightarrow H(2\pi r) = I.$$

EMT Notes Akshansh

Date
Page

$\Rightarrow H = \frac{I}{2\pi r}\hat{\phi}$

$B = \mu H = \mu_0(1+\chi_m)H$

$$\Rightarrow \boxed{\vec{B} = \mu_0(1+\chi_m)\left(\frac{I}{2\pi s}\right)\hat{\phi}}$$

Now, to find K_b & J_b, find M

$M = \chi_m H = \chi_m \cdot \frac{I}{2\pi s}\hat{\phi} = M_\phi(s)$

s

&

$J_b = \nabla \times M = -\frac{\partial}{\partial z}V_\phi\hat{s} + \frac{1}{s}\left(\frac{\partial}{\partial s}(sV_\phi)\right)\hat{z}$

$= 0 \quad \left(\because \frac{\partial}{\partial z}\left(\frac{I}{2\pi s}\right) = \frac{\partial}{\partial s}\left(\frac{I s}{2\pi s}\right) = 0\right)$

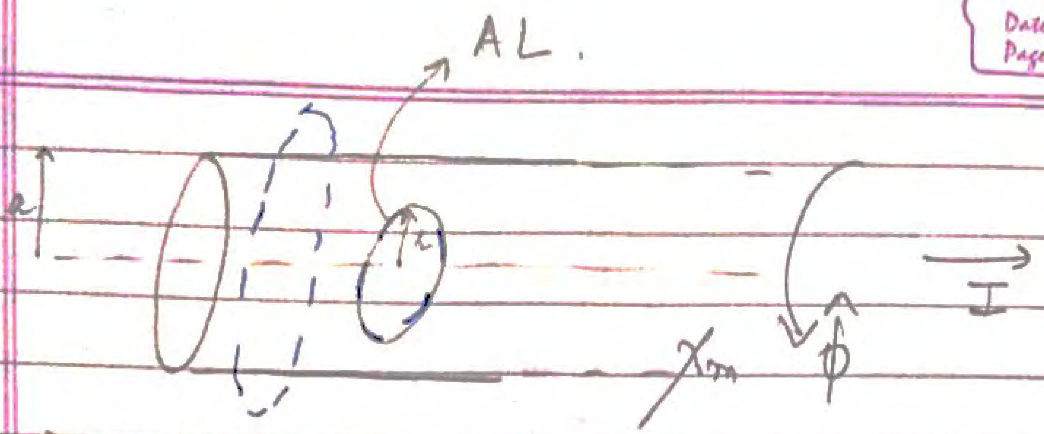
$K_b = M \cdot \hat{n} = \frac{I}{2\pi s}\Big|_{s = \text{surfaces a \& b}} \hat{z}$

$$= \begin{cases} \frac{\chi_m I}{2\pi a}(\hat{z}) & \text{at } s = a \\ \frac{\chi_m I}{2\pi b}(-\hat{z}) & \text{at } s = b \end{cases}$$

Q.6.17 A current I flows down a long st. wire of radius a. If the wire is made of linear material of susceptibility χ_m & current is distributed uniformly.

Find :- $\bar{B}$ at a distance s from axis.

- Bound currents.
- Net bound current flowing down the wire.



inside

(a) $\oint H \cdot dl = I_{free}$ current through AL.

$\Rightarrow H(2\pi r) = \frac{I}{(\pi a^2)} \times \pi r^2$

$\Rightarrow \bar{H} = \frac{I r}{2\pi a^2} \hat{\phi}$; $r < a$

outside ($r > a$)

$\oint H \cdot dl = I_{free}$

$\Rightarrow H(2\pi r) = I$

$\Rightarrow \bar{H} = \frac{I}{2\pi r} \hat{\phi}$

Now

$B = \mu \cdot H = \mu_0 (1 + \chi_m) H$

$\Rightarrow \bar{B} = \mu_0 (1 + \chi_m) \frac{I r}{2\pi a^2} \hat{\phi}$; $r < a$ (inside)

$\chi_m = 0$, $r > a$ $\quad = \mu_0 (1 + \chi_m) \frac{I}{2\pi r}$

$= \mu_0 \frac{I}{2\pi r} \hat{\phi}$; $r > a$

(b) Bound currents

$$M = \chi_m H = \begin{cases} \chi_m \left(\dfrac{I s}{2\pi a^2}\right)\hat{\phi} & \text{inside } (s<a) \\ \chi_m \dfrac{I}{2\pi s}\hat{\phi} & \text{outside } (s>a) \end{cases}$$

$$\equiv M_\phi(s)$$

$$J_b = -\frac{\partial}{\partial z} M_\phi \hat{s} + \frac{1}{s}\frac{\partial}{\partial s}(s M_\phi)\hat{z}$$

$$= 0 + \frac{1}{s}\frac{\partial}{\partial s}\left(\frac{I s^2 \chi}{2\pi a^2}\right)\hat{z}$$

$$= \frac{1}{\cancel{s}}(2\cancel{s})\frac{I\chi_m}{2\pi a^2}\hat{z}$$

$$\Rightarrow J_b = \frac{\chi_m I}{\pi a^2}\hat{z} \quad ; \quad J_b(s>a) = 0$$

$$K_b = M \cdot \hat{n}\Big|_{s=a}$$

$$= \chi_m \frac{I a}{2\pi a^2}(\hat{\phi}\times\hat{n})$$

$$K_b = \chi_m \frac{I}{2\pi a}(-\hat{z})$$

(c) $$I_b = \int J_b \cdot dA + K_b \cdot dl$$

$$= \frac{\chi_m I}{\pi a^2}\cdot \pi a^2 + \left(-\frac{\chi_m I}{2\pi a}\right)\cdot 2\pi a$$

$$= 0.$$

Hence, Proved

EMT Notes Akshansh

Date 5/12/12

Chapter - 7
Electrodynamics

done so far. { • ElectroSTATICS → charges were at rest
• MagnetoSTATICS → const. current flowing & finding B.

• ElectroDYNAMICS.
↳ Charge moving with some vel. v, find E.
or
Current is changing with time t, find B

★ Changing $\bar{B}$ creates $\bar{E}$. i.e, $B(t) \longrightarrow E(t)$
Changing $\bar{E}$ creates $\bar{B}$. or, $E(t) \longrightarrow B(t)$
induced fields
⇓
E & B are coupled together.
⇓
We get electromagnetic waves or EM wave.

★ Note :- Here, E is changing with time.
So, $\nabla \times E \neq 0$ (curl ≠ 0).
So, notation : $E_{induced}$ or E_{ind}. (instead of E).
$\bar{E}$: radially outward : +ve charge
radially inward : -ve charge.

EMT Notes Akshansh

OHM's LAW

volume current density

$$J \propto E \Rightarrow \boxed{J = \sigma E}$$

reason.

conductivity of material.

$\sigma = \frac{1}{\rho}$ → Resistivity

→ Diff^t for diff^t material.

eg: Suppose we have

[conductor]. If $\vec{E}$ applied across ends. So, charges will experience force (qE) & current generated. So, at the end, $J \left(= \frac{I}{A}\right)$ is seen.

More E, more J.

So, $J \propto E$

Self study : example 7.1, 7.2.

electromotive force. (Note. It's not a force. It's potential diff)

MOTIONAL EMF

emf = ~~potential~~ notation : $\mathcal{E} = \frac{\text{work done}}{\text{charge}}$

defin^n. $\mathcal{E} = \oint f \, dl$

→ force due to battery on each charge. It's induced electric field (not force)

(We know, $W = qV$
$= q \oint E \cdot dl$.
$\Rightarrow \frac{W}{q} = \oint E \, dl \Rightarrow \mathcal{E} = \oint f \, dl$)

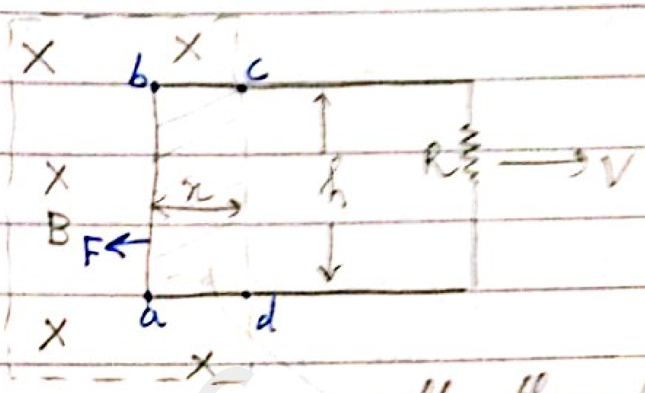
$\frac{W}{q} \to \mathcal{E}$

EMT Notes Akshansh

- $\text{Flux} = \phi = \int B \cdot da.$

$\star\ F = q(v \times B) = qE = q(f) \Rightarrow \boxed{f = v \times B}$ new notation

✩ GENERATOR.



$\phi = \int B \cdot da$

$\Rightarrow \phi = B\,h\,x_{(t)}$

$\Rightarrow \dfrac{d\phi}{dt} = Bh\dfrac{dx}{dt}$

$\Rightarrow \dfrac{d\phi}{dt} = -Bhv \rightarrow (1)$

$\because x \downarrow$ with time

flux through region

$\phi = B \cdot da$

$= B \cdot x_{(t)} \cdot h$

$$\mathcal{E} = \oint f \cdot dl = vBh. \rightarrow (2)$$

Comparing (1) & (2)

$$\Rightarrow\ \star\ \boxed{\mathcal{E} = -\frac{d\phi}{dt}} \rightarrow \text{FARADAY'S LAW.}$$

Changing magnetic field induces electric field

★ Note :-

$$\mathcal{E} = \oint f \cdot dl = \oint \Big(\underbrace{\frac{J}{\sigma}}_{(\equiv E)}\Big) dl = \int \Big(\frac{I}{A\sigma} dl\Big) = IR.$$

$$\frac{l}{\sigma A} = \frac{\rho l}{A} \equiv R$$

So, ✓ $\mathcal{E} = IR \quad (\equiv V)$

✓ $J = \sigma f \quad (\equiv \sigma E)$

✓ $f = \dfrac{F}{q} \quad (\equiv E)$

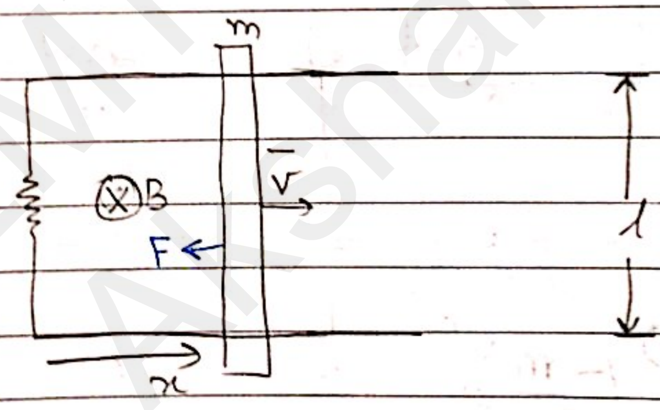
EMT Notes Akshansh

Q. 1.1 A metal bar of mass m slides frictionlessly on 2 parallel conducting rails a distance l apart. A resistor R is connected across the rails & a uniform $\vec{B}$ pointing into the page, fills the entire region.

(a) If the bar moves to the right at speed v, what is current in the resistor? In what dir^n does it flow?

(b) What is magnetic force on the bar? In what dir^n?

(c) If bar starts out with speed v_0 at time t = 0 & is left to slide, what is speed in a later time t?



(a) $\phi = B.dA = B(l x_{(t)})$

$$\mathcal{E} = -\frac{d\phi}{dt} = -Bl\frac{dx}{dt} = -Blv$$

$$I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$$ (anticlockwise)

Induced current will be generated in such a way that it'll try to oppose the change (increasing flux)

Puffin
Date ______
Page ______

So, the I_{ind} is $\circlearrowleft$, B will be $\odot$ & will oppose the change. ✓

(b) $F = I \vec{l} \times \vec{B} = \frac{B^2 l^2 v}{R}$ (left)

($\uparrow l$ $\otimes B$)

($\because$ dir^n of current is $\circlearrowleft$)

(c) Find speed / vel.

Idea :- use eq^n of motion, $F = ma \rightarrow \frac{dv}{dt}$

Integrate & find $\underline{\underline{v_{(t)}}}$

Now, $m\frac{dv}{dt} = \underbrace{-}_{\text{dir}^n \text{ opp.}} \frac{B^2 l^2 v}{R}$

$$\Rightarrow \left(\frac{-mR}{B^2 l^2}\right) \int_{v_0}^{v} \frac{dv}{v} = \int_{t=0}^{t} dt.$$

$\Rightarrow$

$$\Rightarrow \left(-\frac{mR}{B^2 l^2}\right)\left(\ln \frac{v}{v_0}\right) = t$$

$$\Rightarrow \ln\left(\frac{v_0}{v}\right) = \left(\frac{B^2 l^2}{mR}\right) t$$

$$\text{or} \quad v_{(t)} = \left(v_0 e^{-\frac{B^2 l^2 t}{mR}}\right)$$

EMT Notes
Aksharansh

Puffin
Date
Page

(d) Continued ques.

$KE_{initial}$ of rod $= \frac{1}{2} m v_0^2$ → or E lost as heat

Show that energy delivered to the sys $= \frac{1}{2} m v_0^2$.

We know, as $t \to \infty$, $v_{(t)} \to 0$

at $t=0$, $KE = \frac{1}{2} m v_0^2$.

Show :- $P = \frac{W}{t} = \frac{1}{2} m v_0^2$.

$$P = \frac{W}{t} = I^2 R$$

$$= \frac{dW}{dt} = \left(\frac{B^2 l^2 v^2}{R^2}\right) R \quad v \to v_{(t)}$$

$$\Rightarrow \frac{dW}{dt} = \frac{B^2 l^2}{R} v_0^2 e^{-\frac{2B^2 l^2}{mR} t}$$

Energy delivered $= \int dW = \frac{B^2 l^2 v_0^2}{R} \int_0^{\infty} e^{-\frac{2B^2 l^2}{mR} t} dt$

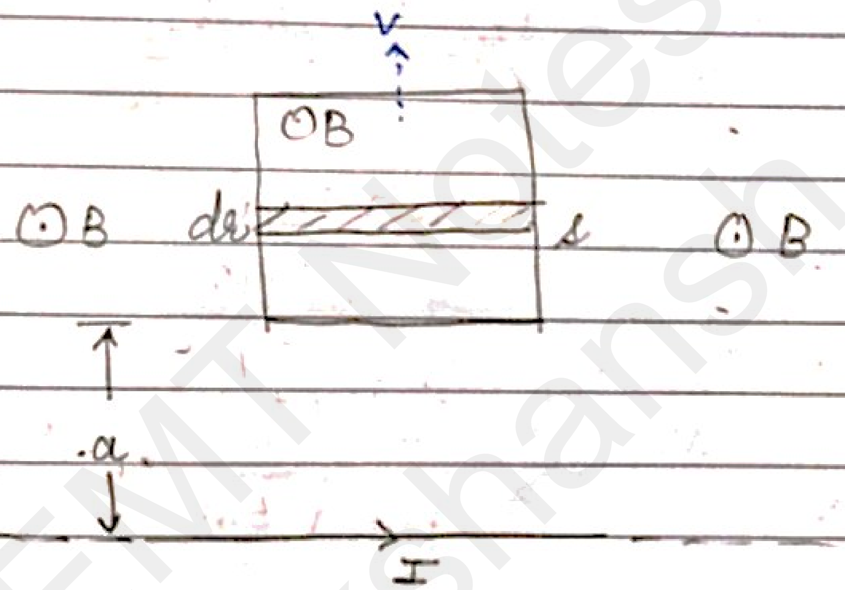
$$\Rightarrow W = \frac{B^2 l^2 v_0^2}{R} \left[\frac{\left(e^{-\frac{2B^2 l^2}{mR} t}\right)}{\left(-\frac{2B^2 l^2}{mR}\right)} \right]_0^{\infty}$$

$$\left(\because \int e^{-\alpha t} dt = \frac{e^{-\alpha t}}{-\alpha}\right)$$

$$= \frac{\cancel{B^2 l^2} v_0^2}{\cancel{R}} \left(\frac{\cancel{-}1}{\cancel{-}2\frac{\cancel{B^2 l^2}}{m\cancel{R}}}\right) = \frac{1}{2} m v_0^2$$

H.P

Q.7.8 Square loop of wire (side s) lies on a table near a very long st. wire which carries a current I
(a) Find the flux of B through the loop.
(b) If someone now pulls the loop away from the wire, at speed v, what emf is generated. In what dir^n does current flow?
(c) What if the loop is pulled to right with speed v instead of away?



EMT Notes
Akshansh

(a) $B_{(\text{at any pt. } r \text{ from wire})} = \dfrac{\mu_0 I}{2\pi r}$ → Keeps changing. So, choose element.

$$d\phi = \left(\frac{\mu_0 I}{2\pi r}\right)(s\,dr)$$

$$\phi = \int d\phi = \frac{\mu_0 I s}{2\pi}\int_a^{a+s}\frac{1}{r}\,dr$$

$$\Rightarrow \phi = \frac{\mu_0 I s}{2\pi}\ln\left(\frac{a+s}{a}\right)$$

Puffin
Date
Page

(b) $\mathcal{E} = -\dfrac{d\phi}{dt}$

$$= -\frac{d}{dt}\left(\frac{\mu_0 I}{2\pi} s \ln\left(\frac{a+s}{a}\right)\right)$$

$$= \left(-\frac{\mu_0 I s}{2\pi}\right)\underbrace{\left(\frac{1}{\left(\frac{a+s}{a}\right)}\right)\left[\frac{a\left(\frac{d}{dt}(a+s)\right) - (a+s)\frac{da}{dt}}{a^2}\right]}$$

$$\left\{\begin{aligned} &\frac{d}{dt}\ln\frac{a(t)}{b(t)} \\ &= \frac{1}{(a_t/b_t)}\frac{d}{dt}\left(\frac{a(t)}{b(t)}\right) \\ &= \left(\frac{1}{\frac{a(t)}{b(t)}}\right)\left[\frac{b(t)\frac{d}{dt}a(t) - a(t)\frac{d}{dt}b(t)}{b^2(t)}\right]\end{aligned}\right.$$

$$= -\frac{\mu_0 I s}{2\pi}\left(\frac{a}{a+s}\right)\left[\frac{aV - (a+s)V}{a^2}\right]$$

$$= -\frac{\mu_0 I s}{2\pi}\left(\frac{a}{a+s}\right)(V)\left[\frac{-s}{a^2}\right]$$

$$\Rightarrow \mathcal{E} = \frac{\mu_0 I s^2}{2\pi a(a+s)} V \quad \text{Ans}$$

Dir^n of current

When loop is pulled away from the wire, $\phi \downarrow$.

So, $B \downarrow$.

By Lenz law, generate current s.t. $I \uparrow$.

EMT Notes Akansh

for increasing I, create it such at B is ⊙ more.

So, I ↺ (anticlockwise)

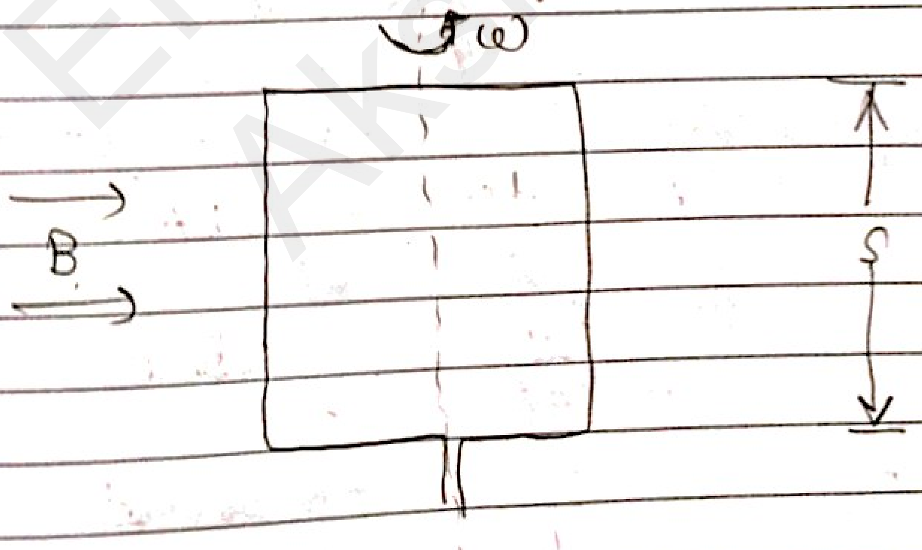
(c) Nothing happens ∵ ϕ remains the same

Note :- a is not changing

Note :- ϕ is not changing

So, $\text{emf}_{\text{induced}} = \boxed{0}$ ★

Q 7.10 A square loop of (side s) is mounted on a vertical shaft & rotated at angular vel. ω. A uniform magnetic field points to the right. Find emf of this alternating current generator.



$$\phi = B \cdot A = B \cdot s^2 \cos\theta \qquad \left(\because \omega = \frac{\theta}{t}\right)$$
$$= B s^2 \cos(\omega t)$$

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt} B s^2 \cos\omega t = + B s^2 \sin\omega t \cdot \omega$$

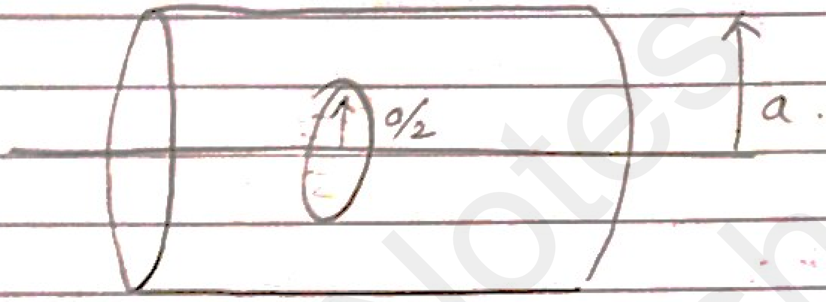
$$\Rightarrow \mathcal{E} = B s^2 \omega \sin\omega t$$

Ans

EM Notes Akshansh

Puffin
Date
Page

Q.7.12 A long solenoid of radius a is driven by an AC s.t field inside is sinusoidal $B(t) = B_0 \cos \omega t \, (\hat{z})$. A circular loop of wire of radius $a/2$ & resistance R is placed inside the solenoid & coaxial with it. Find current induced in the loop.



$B(t) = B_0 \cos \omega t \quad (\hat{z})$

To find : $I_{induced}$

Idea :- $I = \frac{\mathcal{E}}{R}$ $\longrightarrow -\frac{d\Phi}{dt}$ $\longrightarrow$ (B)(A)

Area of smaller loop $= \pi \left(\frac{a}{2}\right)^2$

$B_0 \cos \omega t$.

So, $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(BA)$

$= -\frac{d}{dt}(B_0 \cos \omega t)(A)$

$= -A \, B_0 (-\omega \sin \omega t)$

$= B_0 \omega \sin \omega t \, (\pi a^2/4)$

$\mathcal{E} = B_0 \omega (\pi a^2/4) \sin \omega t$

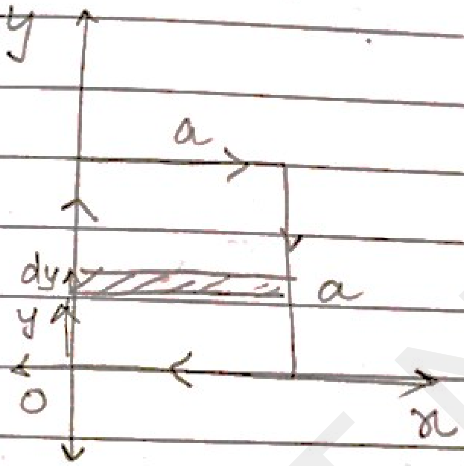
$\Rightarrow I = \frac{\mathcal{E}}{R} = \frac{B_0 \omega (\pi a^2) \sin \omega t}{4 \cdot R}$

Ans

Q.7.13 A sq. loop of wire with sides of length a lies in the first quadrant of x-y plane with one corner at origin. In this region, ~~we~~ non-uniform magnetic field is given by

$$B(y,t) = ky^3t^2(\hat{z}) \quad ; k : \text{const}$$

Find: emf induced in the loop.



B is a f^n of y & t

So, take element accordingly

$$d\phi = B \cdot dA$$
$$= B \cdot (a\,dy)$$
$$= ky^3t^2(a\,dy)$$
$$\Rightarrow \phi = \int d\phi = \int_0^a ky^3t^2 a\,dy$$
$$= kt^2a\int_0^a y^3\,dy$$
$$\Rightarrow \phi = \frac{kt^2a^5}{4}$$

Now, $\varepsilon = -\dfrac{d\phi}{dt} = -\dfrac{k\,a^5}{4}\left(\dfrac{d}{dt}t^2\right)$

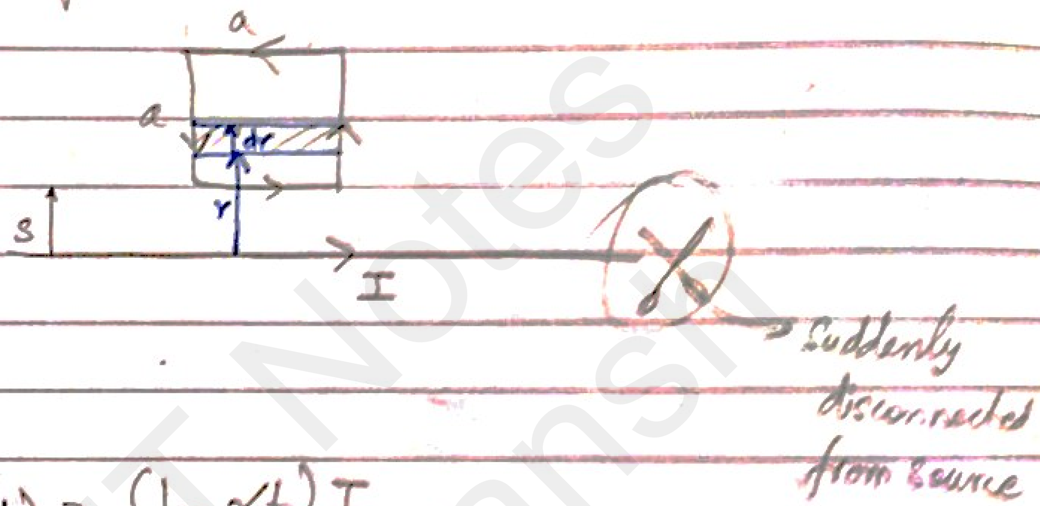
$$\varepsilon = -\frac{ka^5t}{2} \quad \text{Ans}$$

* Dir^n of induced current

↳ Idea:- B is ↑ in $\hat{z}$. So, I s.t it reduces value of B. So, $\hat{z}$, ($\because$ ↻ I $\Rightarrow -\hat{z}$ B)

EMT Notes Akshansh

Q.7.18 A sq. loop, side a, resistance R lies a distance s from infinite st. wire that carries current I. Now, someone cuts the wire s.t I drops to zero. In what dir^n does the induced current in the sq. loop flow. What is the charge passing a given pt in the loop during this time? $I_{(t)} = (1-\alpha t) I$.



$I_{(t)} = (1-\alpha t) I$

↳ as $t \uparrow$, $I \downarrow$

$\Downarrow$

$B \downarrow$ in $\hat{z}$

So, I ↺ to make it ↑ again

Now, $dB = \frac{\mu_0 I}{2\pi h}$ (for at infinite wire at any distance → found before)

$\Rightarrow \int dB = B$

Now, $\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[\int (B \cdot A)\right]$

$= -\frac{d}{dt}\left[\int_s^{s+a} \frac{\mu_0 I_{(t)}}{2\pi r} \cdot (a \cdot dr)\right]$

$= -\frac{\mu_0 a}{2\pi}\left(\frac{d}{dt} I_t \int_s^{s+a} \frac{dr}{r}\right)$

$$= -\frac{\mu_0 a}{2\pi}\left(\frac{d}{dt} I_{tt} \ln\left(\frac{s+a}{s}\right)\right)$$

$$= -\ln\left(\frac{s+a}{s}\right)\frac{\mu_0 a}{2\pi}\left(\frac{d}{dt}(1-\alpha t)I\right) \longrightarrow (1)$$

$$= \cancel{-}\ln\left(\frac{s+a}{s}\right)\frac{\mu_0 a}{2\pi}\left(\cancel{-}\alpha I\right)$$

$$\Rightarrow \varepsilon = \frac{\mu_0 a}{2\pi}(I\alpha)\ln\left(\frac{s+a}{s}\right) \quad \text{Ans}$$

Now,
we know

$$\varepsilon = IR = \frac{dQ}{dt}\cdot R \longrightarrow (2)$$

$$(1) = (2)$$

$$\Rightarrow \frac{dQ}{dt}\cdot R = -\ln\left(\frac{s+a}{s}\right)\frac{\mu_0 a}{2\pi}(-\alpha I)$$

$$\Rightarrow dQ = -\ln\left(\frac{s+a}{s}\right)\frac{\mu_0 a}{2\pi R}(-\alpha I)\,dt$$

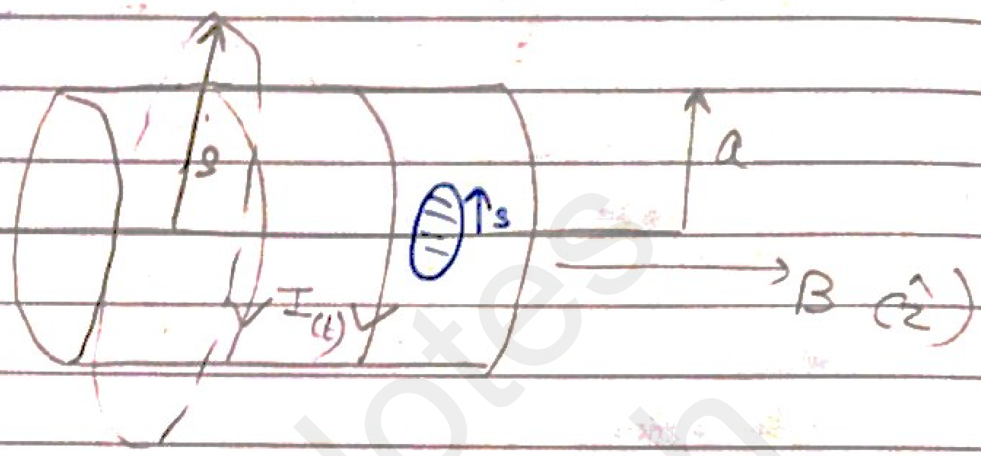
$$\Rightarrow Q = \int dQ = \int \cancel{-}\ln\left(\frac{s+a}{s}\right)\frac{\mu_0 a}{2\pi R}(\cancel{-}\alpha I)\,dt$$

$$\Rightarrow Q(t) = \frac{\mu_0 a}{2\pi R}\ln\left(\frac{s+a}{s}\right)(\alpha I)(t)$$

→ suppose if charge was asked to be found.

EMT Notes
Akshansh

Q.7.15. A long solenoid with radius a & n turns/length carries a time dependent current $I(t)$ in $\hat{\phi}$ dirn. Find electric field at distance s from axis (inside & outside)



$B_{\text{inside solenoid}} = \mu_0 n I_{(t)}$

Suppose → $I_{(t)}$ is ↑. (i.e $\bar{E}$ is ↑)
So, B ($\hat{z}$) is increasing

Lenz → E or I should be $-\hat{\phi}$ to reduce B ($\hat{z}$)

Idea :- $\mathcal{E} = +\int E \, dl = -\frac{d\phi}{dt} = -\frac{d}{dt}(B \cdot A)$
→ ?

$= -\frac{d}{dt}(\mu_0 n I_{(t)})(\pi s^2)$

inside

$\Rightarrow \oint E \, dl = -\mu_0 n \pi s^2 \frac{d}{dt} I_{(t)}$

$\Rightarrow E(2\cancel{\pi}\cancel{s}) = -\mu_0 n \cancel{\pi} s^{\cancel{2}} \frac{d}{dt} I_{(t)}$

induced electric field

$\Rightarrow E_{(s)} = -\frac{\mu_0 n s}{2} \frac{d}{dt} I_{(t)}$ $\hat{\phi}$ or $-\hat{\phi}$.

Outside :- ϕ enclosed will only be within a.
So, area, $A = \pi a^2$

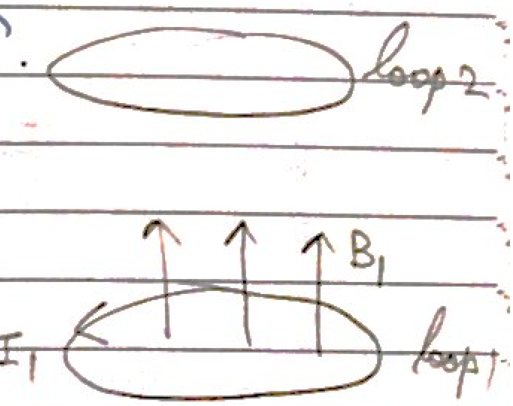
So, $E(2\pi s) = -\mu_0 n \pi a^2 \frac{d}{dt} I_{(t)}$

$$\Rightarrow E_{(s)} = -\frac{\mu_0 n a^2}{2s} \frac{d I_{(t)}}{dt}$$ Ans.

Note :-
Dir^n of I isn't given. Its dir^n is circular.
So, dir^n of $\vec{E}$ will also be circular accordingly.

§ INDUCTANCE

① MUTUAL



$$B_1 = \frac{\mu_0}{4\pi} I_1 \oint \frac{d l_1 \times \hat{r}}{r^2}$$

$$\phi_2 = \int B_1 \cdot da_2$$

We know $\phi_2 \propto I_1$
(observ^n)

$$\Rightarrow \phi_2 = M_{21} \cdot I_1$$

Mutual inductance of 2 loops.

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Puffin
Date
Page

★ Mathematical Derivation

$$\Phi_2 = \int B_1 \cdot da_2 = \int (\nabla \cdot A_1) da_2 = \oint A_1 \cdot dl_2$$

$$\left(\because A_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{dl_1}{r} \right) \text{(sol}^n \text{ of Poisson's eq}^n \rightarrow 1D)$$

$$\Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint \left(\oint \frac{dl_1}{r} \right) dl_2 = M_{21} I_1$$

$$\Rightarrow M_{21} = \frac{\mu_0}{4\pi} \oint\oint \frac{dl_1\, dl_2}{r} \quad \text{(Neumann formula)}$$

$$\Rightarrow \boxed{M_{21} = M_{12}}$$

$$\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M \frac{dI_1}{dt}$$

② SELF Inductance

A changing current induces an emf in the source loop itself.

$$\Phi \propto I \Rightarrow \boxed{\Phi = L I}$$

Self Inductance (unit → Henry)

$$\mathcal{E} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

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Puffin
Date
Page

Energy Stored in a Magnetic field.

$$P = \frac{dW}{dt} = \mathcal{E} I = \left(L\frac{dI}{dt}\right) I \qquad ; \quad \phi = LI$$

$$\mathcal{E} = -\frac{d\phi}{dt}.$$

ignored ∵ magnitude is req^d.

$$\Rightarrow \int dW = \int L(dI) I.$$

$$\Rightarrow \boxed{W = \frac{1}{2} L I^2} \rightarrow (1)$$

Now, $\phi = \int_s B \cdot da = \int_s (\nabla \times A) \cdot da = \oint A \cdot dl$.

$$\Rightarrow \phi = LI = \oint A \cdot dl.$$

So, $W = \frac{1}{2} LI \cdot I = \frac{1}{2} I \left(\oint A \cdot dl\right)$

$$\Rightarrow \boxed{W = \frac{1}{2}\left(\oint A \cdot I\right) dl} \quad \text{(line current)}$$

In terms of vol. current,

$$W = \frac{1}{2}\int_V (A \cdot J)\, d\tau$$

$$\nabla \times B = \mu_0 J \Rightarrow J = \frac{\nabla \times B}{\mu_0}$$

So, $W = \frac{1}{2\mu_0}\int A \cdot (\nabla \times B)\, d\tau$. $\rightarrow$ (a)

We know,

$$\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A(\nabla \times B)$$

So, $A \cdot (\nabla \times B) = B \cdot B - \nabla (A \times B)$

Puffin
Date
Page

$$\Rightarrow W = \frac{1}{2\mu_0}\left[\int B^2 d\tau - \int \nabla\cdot(A\times B)\,d\tau\right]$$

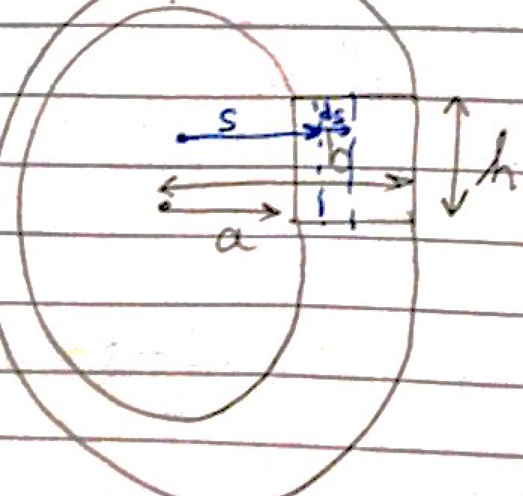
$$= \frac{1}{2\mu_0}\left[\underbrace{\int_V B^2 d\tau}_{\text{from 0 to R}} - \underbrace{\oint_S (A\times B)\cdot da}_{\text{only on surface. At } R\to\infty\text{, term is negligible}}\right]$$

So, V : entire volume
& for large surface, $\oint (A\times B)\cdot dA \to 0$

So, $$\boxed{W = \frac{1}{2\mu_0}\int_{\text{all space}} B^2 d\tau}$$

✱ **Note :-** $\Phi = LI = B.A$ — Idea :- Find B. Then find ϕ using it. So, $L = \frac{\Phi}{I}$ ✓

Q.7.11 Find self inductance of a toroidal coil with rectangular crossection (inner radii a & outer radii b, height h) which has total N turns.



We know
$\Phi = LI = BA$.

Proved before :-
$$B_{(s)} = \frac{\mu_0 N I}{2\pi s}.$$
any pt inside toroid

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Now, s is a f^n of distance. So, take element

So $d\phi = B.dA = \frac{\mu_0 NI}{2\pi s}(h\,ds)$

$$\Rightarrow \phi = \left[\int_a^b \frac{\mu_0 NI}{2\pi s}(h\,ds)\right] \times N$$ ∵ Total flux for all no. of turns.

$$\Rightarrow \phi = \left[\frac{\mu_0 N h I}{2\pi} \ln\frac{b}{a}\right] \times N$$

Now,

$$L = \frac{\phi}{I} \Rightarrow L = \frac{\mu_0 N^2 h}{2\pi} \ln\frac{b}{a}$$

Q.7.27 Continuation of previous problem. Find energy

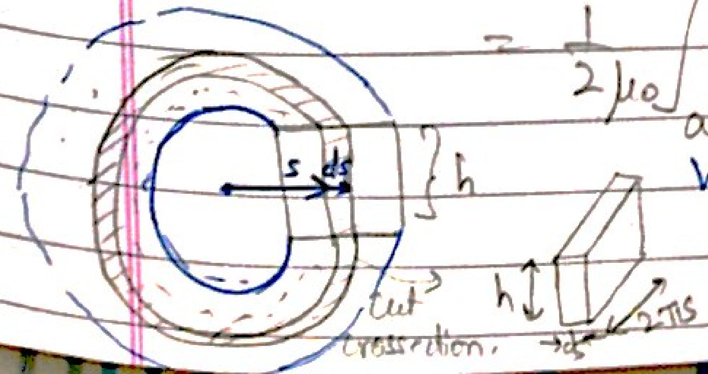
$$W = \frac{1}{2} L I^2$$ → Proved above

$$= \frac{1}{2}\left(\frac{\mu_0 N^2 h}{2\pi} \ln\frac{b}{a}\right) I^2$$

$$W = \frac{\mu_0 N^2 h I^2}{4\pi} \ln\left(\frac{b}{a}\right)$$

Alifer :- $W = \frac{1}{2\mu_0}\int B^2\, d\tau$

$$= \frac{1}{2\mu_0}\int_a^b \left(\frac{\mu_0 NI}{2\pi s}\right)^2 (2\pi s\, ds \times h)$$



$$W = \frac{\mu_0 N^2 I^2 h}{4\pi} \ln\frac{b}{a}.$$

(same as before)

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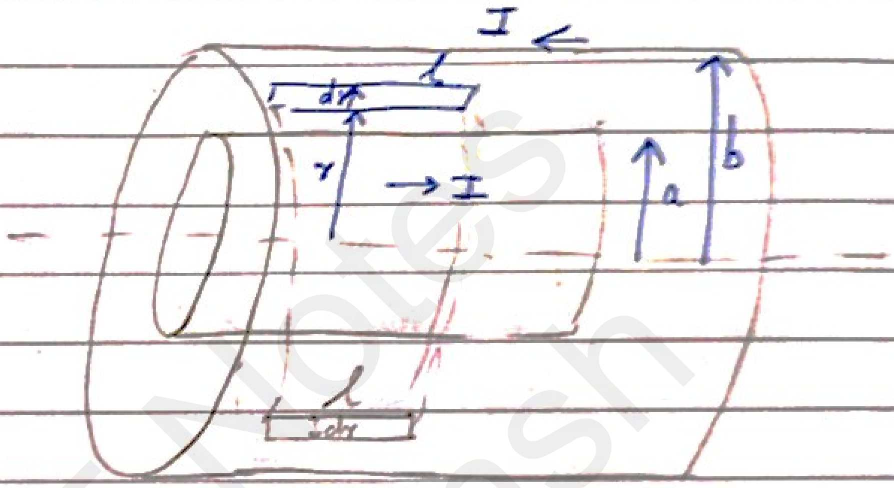
Q.7.13 A long coaxial cable carries current I (the current flows down the surface of the inner cylinder, radius a, & back along outer cylinder b). Find magnetic energy stored in a section of length l.



$$W = \frac{1}{2\mu_0}\int B^2 \, d\tau$$

$$= \frac{1}{2\mu_0}\int_a^b \left(\frac{\mu_0 I}{2\pi r}\right)^2 (2\pi r \, dr \, l)$$

→ B due to an infinite wire (carrying current) at any pt

$$= \frac{1}{2\mu_0}\int_a^b \frac{\mu_0^2 I^2}{(2\pi r)^{\cancel{2}}} (\cancel{2\pi r})(dr\, l)$$

$$= \frac{\mu_0 I^2 (l)}{2(2\pi)}\int_a^b \frac{dr}{r}$$

$$W = \frac{\mu_0 I^2 l}{4\pi} \ln\frac{b}{a}$$

Over a section of length l, W/l

$$= \frac{W}{l} = \frac{\mu_0 I^2}{4\pi}\ln\frac{b}{a} \quad \text{Ans}$$

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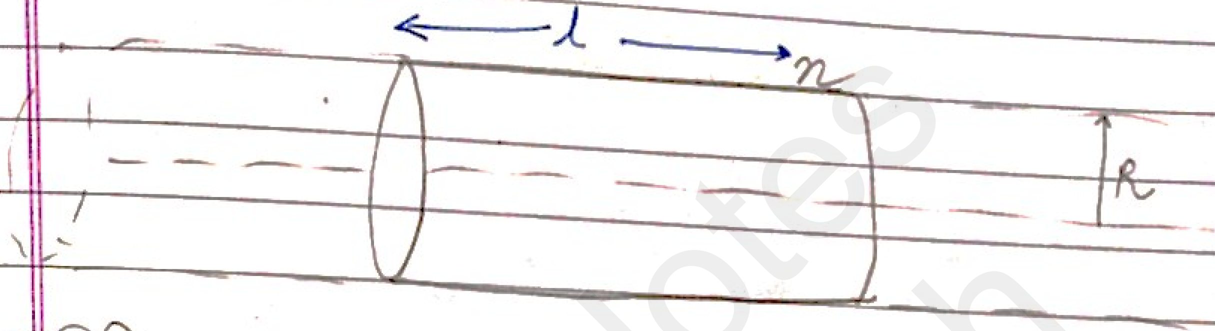
Puffin
Date
Page

Q.7.26 Find the energy stored in a section of length l of a long solenoid (radius R, current I, n turns per unit length).

[Using $W = \frac{1}{2} L I^2$ & $\frac{1}{2\mu_0} \int B^2 d\tau$ both]



M1 $W = \frac{1}{2} L I^2$

we know

$\phi = LI$

& $\phi = BA$

total no. of turns p.u length × length

$= (\mu_0 n I)(\pi R^2) \times (nl)$

$\Rightarrow \phi = \mu_0 n^2 I l (\pi R^2)$

$\Rightarrow L = \frac{\phi}{I} = \mu_0 n^2 l (\pi R^2)$

$\Rightarrow W = \frac{1}{2} \mu_0 n^2 l (\pi R^2) I^2$.

Over a section of length l :-

$$\frac{W}{l} = \frac{\mu_0 n^2 (\pi R^2) I^2}{2}$$ Ans

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Note :- Check in every problem what is asked :- W or $\frac{W}{l}$.

(M2) $W = \frac{1}{2\mu_0}\int B^2 d\tau$.

$= \frac{1}{2\mu_0}\int (\mu_0 n I)^2 \cdot (\pi R^2 l)$

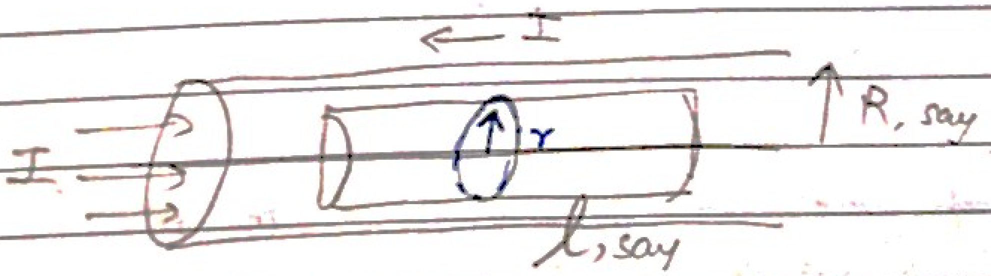
$\int$ not reqd (B : const)

$= \frac{1}{2\mu_0}(\mu_0^2 n^2 I^2)(\pi R^2 l)$

$\Rightarrow W = \frac{1}{2}\mu_0 n^2 I^2 (\pi R^2) l$.

So, $\frac{W}{l} = \frac{1}{2}\mu_0 n^2 I^2 (\pi R^2)$

Q.7.28 A long cable carries current in one dirn distributed over its crossection. The current returns along the surface.
Find (a) Self inductance p.u length
(b) Energy stored using $\frac{1}{2\mu_0}\int B^2 d\tau$



$W = \frac{1}{2\mu_0}\int B^2 d\tau$.

Now $\oint B \cdot dl = \mu_0 I_{encl}$ (A L)

$= \mu_0\left(\frac{I}{\pi R^2}\right)(\pi s^2) = \mu_0 I \frac{s^2}{R^2}$

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$$\Rightarrow B(2\pi r) = \frac{\mu_0 I r^2}{R^2}$$

$$\Rightarrow \bar{B} = \frac{\mu_0 I r}{2\pi R^2} (\hat{\phi})$$

Now, $W = \frac{1}{2\mu_0} \int_0^R \left(\frac{\mu_0 I r}{2\pi R^2}\right)^2 (2\pi r dr . l)$

$$W = \frac{\mu_0 I^2 l}{16\pi}$$

Now, find :- L

$$W = \frac{1}{2} L I^2$$

$$\Rightarrow \frac{\mu_0 \cancel{I^2} l}{16\pi} = \frac{1}{2} L \cancel{I^2}$$

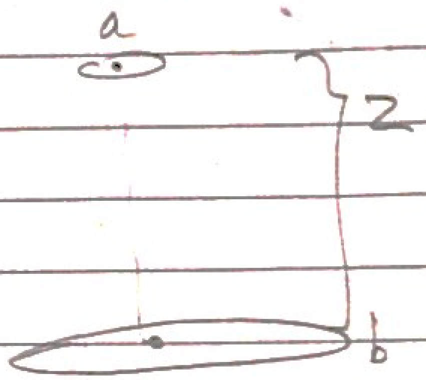
$$\Rightarrow L = \frac{\mu_0 l}{8\pi}$$

So, L p.u length $= \frac{\mu_0}{8\pi}$ Ans

Q.7.2 (a) A small loop of wire (radius a) lies a distance z above the centre of a large loop (radius b). The planes of the 2 loops are parallel & ⊥ to common axis.

(a) Suppose current I flows in big loop. Find flux through little loop.

(b) Find mutual inductance.



$$B(z) = \frac{\mu_0 I b^2}{2(b^2+z^2)^{3/2}}$$ (B above a circular loop)

$$\phi\Big|_{r=a} = B.A = B.\pi a^2$$

$$= \left(\frac{\mu_0 I b^2}{2(b^2+z^2)^{3/2}}\right)\pi a^2$$

Now, find $\underline{M}$

$$\phi = MI \quad (M_{12} = M_{21})$$

So, $$M = \frac{\phi}{I} = \frac{\mu_0 b^2 \pi a^2}{2(b^2+z^2)^{3/2}}$$

Q. 7. 21) A sq. loop of side a lies midway b/w 2 long wires, 3a apart. & in the same plane. A clockwise current I in the sq. loop is gradually increasing. $\frac{dI}{dt} = k$ (a constt.) Find the emf induced in the big loop. What is dir^n of induced current.

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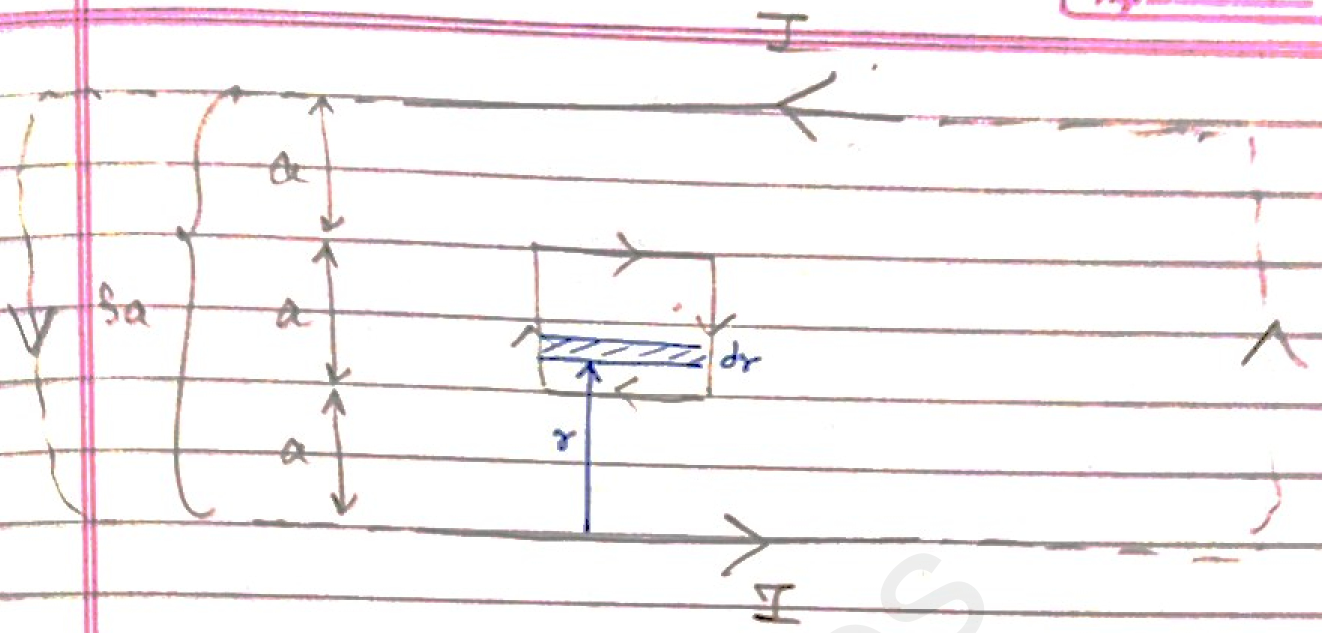

Induced emf $= \mathcal{E} = -\dfrac{d\phi}{dt}$

To find :- flux enclosed within loop (bigger)

→ or, we can also consider current I (say) inside big loop (instead of the loop) & find flux in smaller (Mutual inductance principle)

So, $\mathcal{E} = -\dfrac{d\phi}{dt}$

$$\phi = \int_a^{2a} \left(\frac{\mu_0 I}{2\pi r}\right)(a\,dr) = \frac{\mu_0 I a}{2\pi}\ln 2$$

$$\phi_{Total} = 2\phi = \frac{\mu_0 I a \ln 2}{\pi}$$

(due to I in upper & lower part of bigger loop).

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$$\varepsilon = -\frac{d\phi}{dt}$$

$$= -\frac{\mu_0 a \ln 2}{\pi}\left(\frac{dI}{dt}\right)$$

$$\Rightarrow \varepsilon = \left(-\frac{\mu_0 a \ln 2}{\pi}\right) \underset{\text{(Given)}}{K^2}$$

If M is asked,

$$\phi = MI$$

$$\Rightarrow M = \frac{\phi}{I}$$

$$\Rightarrow M = \frac{\mu_0 \times a}{\pi} \ln 2$$

Q 7.22 Find self inductance p.u length of a long solenoid of radius R, carrying n turns p.u length.

Now,

$$\phi_T = B \cdot dA = (\mu_0 n I)(\pi R^2)\underset{\text{Total}}{(nl)}$$

$$\Rightarrow \phi_T = \mu_0 n^2 I \pi R^2 l$$

Now, $\phi_T = LI$

$$\Rightarrow \frac{L}{l} = \left(\frac{\phi_T}{I}\right)\frac{1}{l} = \mu_0 n^2 \pi R^2$$

Ans.

Puffin
Date
Page

MODIFIED AMPERE'S LAW (By Maxwell)

Before Maxwell,

Ampere's Law, $\nabla \times B = \mu_0 J$

$$\left(\oint B \cdot dl = \mu_0 J_{encl}\right)$$

or $\int (\nabla \times B) \, dA = \mu_0 \int J \, dA$

$$\left(\Rightarrow \nabla \times B = \mu_0 J \rightarrow \text{differential form}\right)$$

Taking Divergence of both sides

$$\Rightarrow \nabla \cdot (\nabla \times B) = \nabla \cdot (\mu_0 J)$$

Now, $\nabla \cdot (\nabla \times B) = 0$ (always)

But, $\nabla \cdot J = -\dfrac{\partial \rho}{\partial t} \Big|_{\text{continuity eq}^n} \neq 0$

So,

LHS $\neq$ RHS. (Mathematical inconsistency)

So, sth should be there on RHS to make it zero.

Maxwell's Modific^n

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t}(\epsilon \nabla \cdot E) = -\nabla \cdot \left(\epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\left[\nabla \cdot E = \frac{\rho}{\epsilon_0} \Rightarrow \int (\nabla E) \, d\tau = \int \frac{\rho \, d\tau}{\epsilon_0} \right.$$

$$\left. \Rightarrow \int E \cdot dA = \frac{q}{\epsilon_0} \right]$$

Gauss law, green's thm.

$-\epsilon_0 \dfrac{\partial E}{\partial t}$

''

If

Differential form

(1) $\boxed{\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \dfrac{\partial E}{\partial t}}$ → added to make zero.

So, divergence on both sides = 0

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Puffin
Date
Page

So, $\epsilon_0 \frac{\partial E}{\partial t} = J_d$ = displacement current density

$\hookrightarrow I_d = \int J_d \cdot dA$

displacement current.

So, change in $\bar{E}$ induces magnetic field.

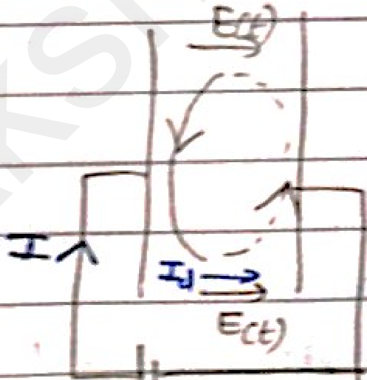
Also,

$$\boxed{\int (\nabla \times B) \cdot da = \oint B \cdot dl = \mu_0 I + \mu_0 \epsilon_0 \int \frac{\partial E}{\partial t} \cdot dA} \quad *$$

$\hookrightarrow$ Integral form of final Ampere's law (modified).

only during charging/discharging

* In charging/discharging process in a capacitor, $\bar{E}$ is a f^n of time. So, changing E will create $\bar{B}$ b/w 2 plates, circular in nature.



Note :- In b/w plates, physical current, I is zero.

So,

$$\oint B \cdot dl = \mu_0 I + \mu_0 \epsilon_0 \int \frac{\partial E}{\partial t} da$$

$$\Rightarrow B(2\pi R) = \mu_0 (0) + \underbrace{\mu_0 \epsilon_0 \int \frac{\partial E}{\partial t} da}_{I_d}$$

known b/w plates $= \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$

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So, basically, Modified AL :-

$$\oint B \cdot dl = \mu_0 (I + I_d)$$

physical current   displacement current

So, $\mu_0 I_d = \mu_0 \epsilon_0 \int \frac{\partial}{\partial t}\left(\frac{Q}{A\epsilon_0}\right) \cdot da$

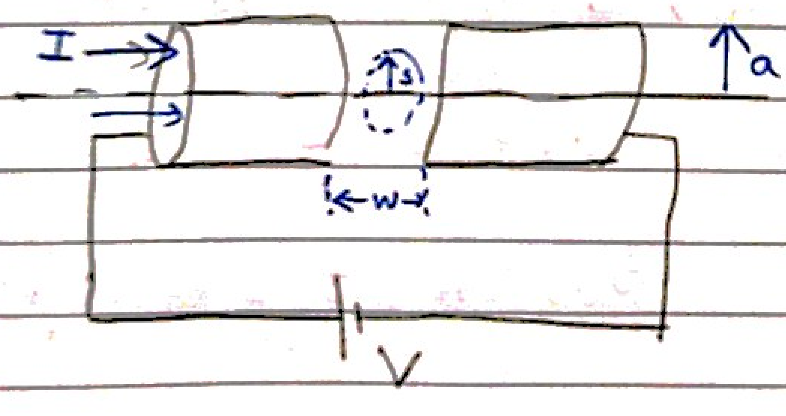
Now $\frac{\partial E}{\partial t} = \frac{1}{\epsilon_0 A} \frac{dQ}{dt} = \frac{I}{\epsilon_0 A}$

$\Rightarrow I = \epsilon_0 A \frac{\partial E}{\partial t}$

Also, $I = \int J_d \cdot dA = I_d$

So, physical current = displacement current.

Q.7.31 A flat wire of radius a carries a constt current I uniformly distributed over its crosssection. A narrow gap in the wire of width w (<<a) forms a ‖ plate capacitor. Find $\vec{B}$ in the gap at a distance s (<a) from axis.



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☆ Note $|I_d| = |I_d|$

i.e if we see in a capacitor, current flowing outside the plates (I) = current b/w plates (I_d)

Puffin Date Page

AL in integral form

$$\oint B \cdot dl = \mu_0 (I + I_d)$$
$$= \mu_0 I + \mu_0 \epsilon_0 \int \frac{\partial}{\partial t} E \cdot da$$

B/w plates (or wires), $I = 0$.

But $I_d \neq 0$.

$$\Rightarrow \oint B \cdot dl = 0 + \mu_0 \epsilon_0 \int \frac{\partial E}{\partial t} \cdot da$$

$$\Rightarrow B \cdot 2\pi s = \mu_0 \left[\left(\frac{I}{\pi a^2} \right) \times \pi s^2 \right]$$

→ sth like q_{encl} in electro statics

$$q_{en} = \frac{Q}{V} (dV)$$

$$\Rightarrow B(s<a) = \frac{\mu_0 I s}{2\pi a^2} \hat{\phi}$$

(I along z axis. So By RHTR, B along $\hat{\phi}$)

Ans

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Q. 7.33 If $E(s,t) = \frac{\mu_0 I_0 \omega}{2\pi} \sin \omega t \ln \left(\frac{a}{s} \right) \hat{z}$.

Find (i) J_d : Displacement current density

(ii) I_d $\left(= \int J_d \cdot dA \right)$

(i) We know :- $J_d = \epsilon_0 \frac{\partial E}{\partial t}$

$$\Rightarrow J_d = \epsilon_0 \left[\frac{\mu_0 I_0 \omega}{2\pi} \ln \left(\frac{a}{s} \right) \right] (\omega \cos \omega t)$$

Puffin
Date
Page

(ii) $I_d = \int J_d \cdot da$

$$= \int_0^a J_d\,(2\pi s\, ds) \qquad (\because J_d \text{ is a } f^n \text{ of } s)$$

$$= \left(\mu_0 \epsilon_0 I_0 \omega^2 \cos\omega t \cdot \frac{\cancel{2\pi}}{\cancel{2\pi}}\right)\left(\underbrace{\int_0^a \underset{I}{\ln\left(\frac{a}{s}\right)}\, \underset{II}{s}\, ds}_{ILATE}\right)$$

$$= \left(\mu_0 \epsilon_0 I_0 \omega^2 \cos\omega t\right)\left[\ln\left(\frac{a}{s}\right)\left(\frac{a^2}{2}\right) - \int_0^a \left(\frac{1}{a/s}\right)\frac{a^2}{2}\, ds\right]$$

$$= \left(\mu_0 \epsilon_0 I_0 \omega^2 \cos\omega t\right)\left[\ln\left(\frac{a}{s}\right)\cdot\frac{a^2}{2} - \frac{a}{2}\left(\frac{s^2}{2}\right)_0^a\right]$$

$$I_d = \left(\mu_0 \epsilon_0 I_0 \omega^2 \cos\omega t\right)\left[\ln\left(\frac{a}{s}\right)\cdot\frac{a^2}{2} - \frac{a^3}{4}\right]$$

$$= \left(\mu_0 \epsilon_0 I_0 \omega^2 \cos\omega t\right)\left(\frac{a^2}{2}\right)\left[\ln\frac{a}{s} - \frac{a}{2}\right]$$

Ans

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✡ MAXWELL'S EQUATIONS.

(i) $\nabla \cdot E = \frac{\rho}{\epsilon_0}$ (Gauss law)

(ii) $\nabla \cdot B = 0$

(iii) $\nabla \times E = \frac{-\partial}{\partial t}(B)$ (Faraday's Law)

(iv) $\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$ (Ampere's Law with Maxwell Correction)

Self Test

Q. Write all above eq^{ns} in integral form (Using Green's & Stoke's thm)

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f : amplitude

Puffin Date 17/12/12 Page

Chapter - 9

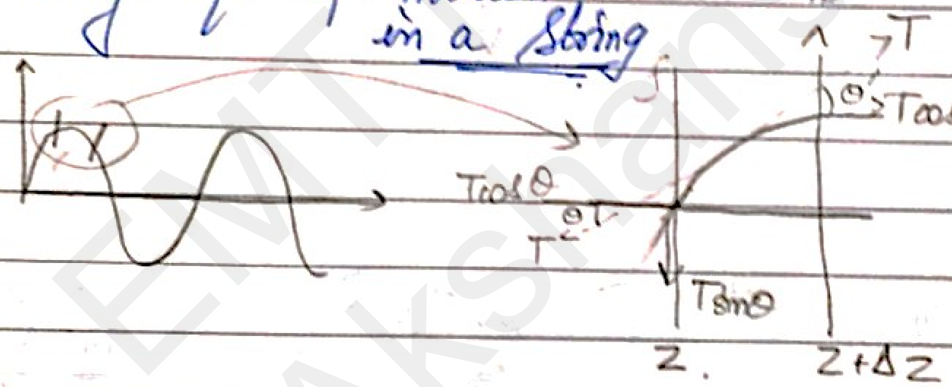
ELECTROMAGNETIC WAVES

→ Transverse

↳ wave amplitude is ⊥ to dirⁿ of flow

§ MOTION IN A STRING

★ • Deriving eqⁿ of motion in a String



$$\sum F_x = 0$$

$$\sum F_y = T\sin\theta' - T\sin\theta = ma = m\frac{\partial^2 f}{\partial t^2}$$

$$\sum F_y = T\underbrace{(\tan\theta' - \tan\theta)} = T\left(\left.\frac{\partial f}{\partial z}\right|_{z+\Delta z} - \left.\frac{\partial f}{\partial z}\right|_{z}\right)$$

for small θ, $\sin\theta \simeq \theta \simeq \tan\theta$

$$= T\left[\left(\cancel{\frac{\partial f}{\partial z}} + \Delta z \cdot \frac{\partial^2 f}{\partial z^2} + \dots\right) - \left(\cancel{\frac{\partial f}{\partial z}}\right)\right]$$

$$\Rightarrow ma = m\frac{\partial^2 f}{\partial t^2} = T\left[(\nabla z)\frac{\partial^2 f}{\partial z^2}\right]$$

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For element $\underline{\Delta z}$

$\left(\frac{m}{l}\right) \times \Delta l \to \mu$

$$(\mu \Delta z) \frac{\partial^2 f}{\partial t^2} = T \cdot \Delta z \frac{\partial^2 f}{\partial z^2}$$

$$\Rightarrow \frac{\partial^2 f}{\partial t^2} = \frac{T}{\mu} \frac{\partial^2 f}{\partial z^2}$$

$$\Rightarrow \frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial z^2}$$

$$\Rightarrow \boxed{\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}} \longrightarrow (1)$$

- → classical wave eq^n
- → $v = \sqrt{\frac{T}{\mu}}$
- → f is a f^n of z & t
- → $f \equiv y$ (displacement in y dir^n)

★ Solution of wave eq^n :- DISCUSSION FOR MOTION IN A STRING ★

for the form → $f(z,t) = g(z - vt) \longrightarrow$ (A)   → any f^n

(self test :- Show eq^n (1) is satisfied using eq^n (A))

General sol^n →

→ to make dimensionally correct

★ $$\boxed{f(z,t) = A\cos[k(z - vt) + \delta]}$$

- → $\lambda = \frac{2\pi}{k}$; $T = \frac{2\pi}{\omega}$, $\nu = \frac{1}{T} = \frac{v}{\lambda}$
- → $\omega = 2\pi\nu = kv$
- → δ = phase const ($\equiv \phi$) ie if $t=0$, what is starting pt. in terms

phase

phase const.

$\Rightarrow \boxed{f(z,t) = A\cos[kz - \omega t + \delta]}$ $(\longrightarrow)(+\hat{z})$

other forms of same eq^n.

or $f(z,t) = A\cos[kz + \omega t - \delta]$ $(\longleftarrow)(-\hat{z})$

dir^{ns}.

eq^{ns} of normal transverse wave.

• Complex notation $(e^{i\theta} = \cos\theta + i\sin\theta)$

$$f(z,t) = \underbrace{R_e}_{\text{Real part}}\left[Ae^{i(kz-\omega t+\delta)}\right]$$

$$\underbrace{\bar{f}(z,t)}_{\text{complex wave } f^n} = \tilde{A}e^{i(kz-\omega t)}$$

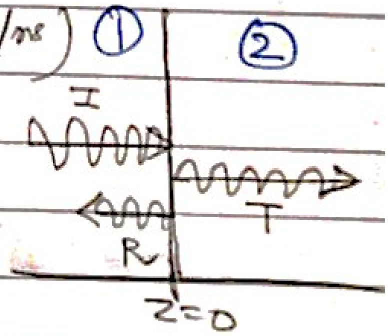
$\boxed{\tilde{A} = Ae^{i\delta}}$: complex amplitude.

◇ Reflection & Transmission (Boundary cond^{ns})

Notation : Incident wave f^n f_I

Transmitted " f_T

Reflected " f_R



f^{ns} of an incident, reflected & transmitted wave.

$$\bar{f}_I(z,t) = \tilde{A}_I e^{i(k_1 z - \omega t)} \;;\; z<0.$$

k_1 : wave no. in region ①, as shown

$$\bar{f}_R(z,t) = \tilde{A}_R e^{i(-k_1 z - \omega t)} \;;\; z<0.$$

$$\bar{f}_T(z,t) = \tilde{A}_T e^{i(k_2 z - \omega t)} \;;\; z>0.$$

k_2 : wave no. in region ②, as shown.

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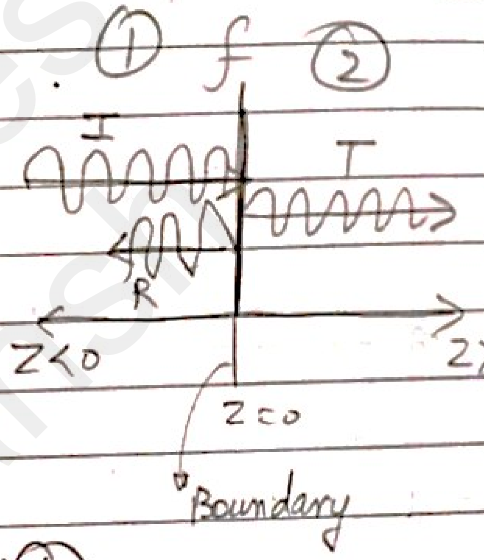
Puffin
Date
Page

So,

$$\bar{f}(z,t) = \begin{cases} \tilde{A}_I e^{i(k_1 z - \omega t)} + \tilde{A}_R e^{i(-k_1 z - \omega t)} & ; z<0 \\ \tilde{A}_T e^{i(k_2 z - \omega t)} & ; z>0 \end{cases} \quad \text{③}$$

True + waves (for any kind of wave)

- **Boundary cond^n**



- $f(0^-,t) = f(0^+,t)$
- $\left.\dfrac{\partial f}{\partial z}\right|_{0^-} = \left.\dfrac{\partial f}{\partial z}\right|_{0^+}$ ④

Apply Boundary cond^{ns} (eq^n ④) in eq^n ③, we get

- $\tilde{A}_I + \tilde{A}_R = \tilde{A}_T$ ⑤
- $k(\tilde{A}_I - \tilde{A}_R) = k_2 \tilde{A}_T$ ⑥

after solving

2 eq^{ns}, 3 unknowns.
So, find $\tilde{A}_T$ & $\tilde{A}_R$ in terms of $\tilde{A}_I$

$k_1 \times$ ⑤ + ⑥

$$\Rightarrow 2k_1 \tilde{A}_I = k_1 \tilde{A}_T + k_2 \tilde{A}_T$$

$$\Rightarrow \tilde{A}_I = \left(\frac{k_1 + k_2}{2k_1}\right)\tilde{A}_T$$

or

$$\boxed{\tilde{A}_T = \left(\frac{2k_1}{k_1+k_2}\right)\tilde{A}_I}$$

||^{ly},

$$\boxed{\tilde{A}_R = \left(\frac{k_1-k_2}{k_1+k_2}\right)\tilde{A}_I} \rightarrow k_1 \text{ (5)} - \text{(6)}$$

Now, we know

$$v = \nu\lambda = \nu \cdot \frac{2\pi}{k}$$

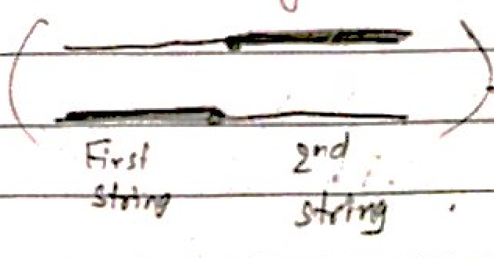
$$\Rightarrow \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{k_2}{k_1} \left.\right\} \rightarrow \begin{array}{l} v \propto k \\ v \propto 1/\lambda \end{array}$$

So, $\tilde{A}_R = \left(\frac{v_1 - v_2}{v_1 + v_2}\right)\tilde{A}_I$; $\tilde{A}_T = \left(\frac{2v_2}{v_1+v_2}\right)\tilde{A}_I$

or

$$\boxed{A_R e^{i\delta_R} = \left(\frac{v_1-v_2}{v_1+v_2}\right)A_I e^{i\delta_I} \; ; \; A_T e^{i\delta_T} = \left(\frac{2v_2}{v_1+v_2}\right)A_I e^{i\delta_I}}$$

→ Amplitude eq^{ns} of any wave
→ Rel^{ns} true in general.
→ Intensity $\propto$ (Amplitude)^2.
→ Self
  → See their modific^{ns} in formula



→ PTO

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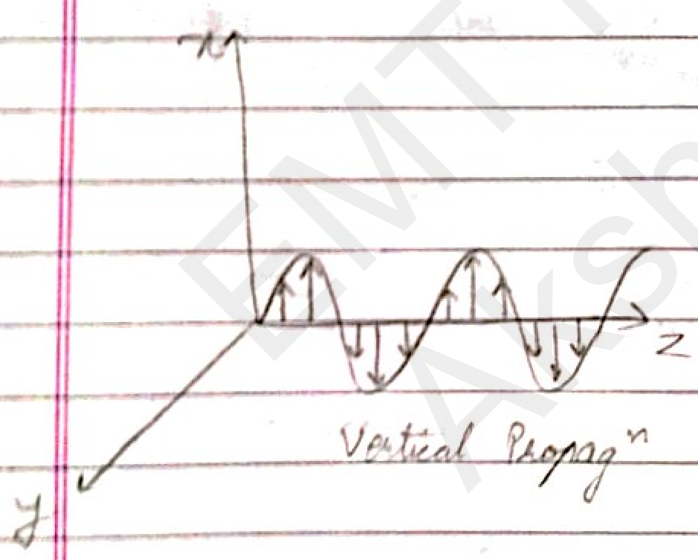
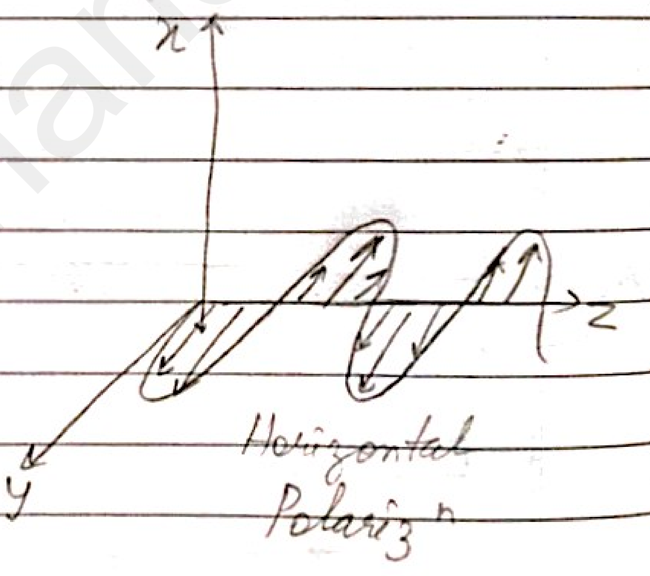
eg Simplify A_R & A_T when

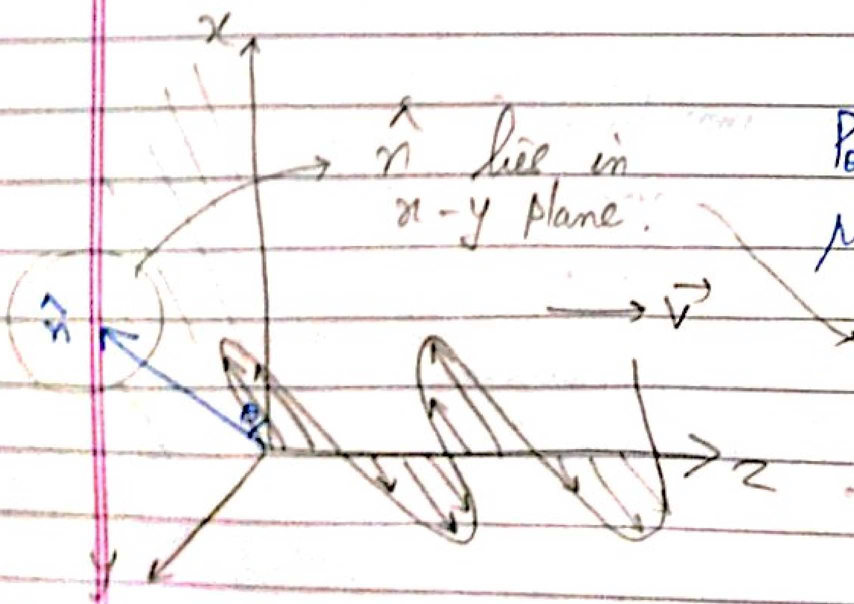
① Second string is heavier than 1st one
② 1st string is heavier than 2nd one

(do from book)

✫ POLARIZATION







Polarizn vector defines the plane of vibrn.

$\hat{n}$ is ALWAYS $\perp$ to dirn of propagn.

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So, $\hat{n}$ is $\perp$ to dir^n of propag^n in horizontal & vertical polariz^n.

* Convention of writing amplitude f^{ns} in polariz^n.

✓ $\tilde{f}_v(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{x}$: vertical polariz^n.

✓ $\tilde{f}_h(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{y}$: horizontal polariz^n

✓ $\tilde{f}(z,t) = \tilde{A} e^{i(kz-\omega t)} \hat{n}$: along any other dir^n in x-y plane.

$\hat{n} \to \cos\theta\, \hat{x} + \sin\theta\, \hat{y}$

$$\tilde{f}(z,t) = (\tilde{A}\cos\theta) e^{i(kz-\omega t)} \hat{x} + (\tilde{A}\sin\theta) e^{i(kz-\omega t)} \hat{y}.$$

* Wave eq^n for E & B in vacuum. $\Rightarrow$ no charge or current

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \quad (\text{wave eq}^n)$$

here, f is replaced by E or B.

Deriv^n: Maxwell's eq^n in vacuum

In vacuum
no charge,
no current

(i) $\nabla \cdot E = 0 \quad \left(\frac{\rho}{\epsilon_0} = 0 ;\ \because q = 0\right)$

(ii) $\nabla \cdot B = 0$

(iii) $\nabla \times E = -\frac{\partial B}{\partial t}$

(iv) $\nabla \times B = I_D = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \quad (\nabla \times B = \overset{0}{I} + I_D)$

∴ **For E :**

Using (iii), in

$$\nabla \times (\nabla \times E) = \nabla \times \left(-\frac{\partial B}{\partial t}\right)$$

$$= \nabla(\nabla \cdot E) - \nabla^2 E = \nabla \times \left(-\frac{\partial B}{\partial t}\right)$$

0 (eqn (1)) (Using vector product rule)

$$\Rightarrow \nabla^2 E = \nabla \times \left(\frac{\partial B}{\partial t}\right)$$

$$= \frac{\partial}{\partial t}(\nabla \times B)$$

$$= \frac{\partial}{\partial t}\left(\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\Rightarrow \nabla^2 E = (\mu_0 \epsilon_0)\frac{\partial^2 E}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 E = \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2}}$$

$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ → speed of light $= c$

eq^n for electric field.

→ valid only in vacuum.

→ Solution of the eq^n :- $E(z) = \tilde{E}_0 e^{i(kz - \omega t)}$

∴ **For B :**

$$\nabla \times (\nabla \times B) = \nabla \times \left(\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\Rightarrow \nabla(\nabla \cdot B) - \nabla^2 B = \nabla \times \left(\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right)$$

0

$$\Rightarrow -\nabla^2 B = \mu_0 \epsilon_0 \left(\nabla \times \frac{\partial E}{\partial t}\right)$$

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Akshansh

Puffin
Date
Page

$$\Rightarrow -\nabla^2 B = (\mu_0 \epsilon_0)\left[\frac{\partial}{\partial t}(\nabla \times E)\right]$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\Rightarrow \nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 B = \frac{1}{v^2}\frac{\partial^2 B}{\partial t^2}}$$

↳ eq^n for magnetic field.
↳ valid only in vacuum
↳ sol^n :- $\tilde{B}(z) = \tilde{B}_0 e^{i(kz-\omega t)}$

☆ Both eq^{ns} for B & E follow this :-

$$\boxed{\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

↳ sol^n :- $\tilde{f}_{(z,t)} = \tilde{A} e^{i(kz-\omega t)}$

☆ E & B are ⊥ to each other.

Derivation (Self) (PTO)

We know, $\nabla \times E = -\frac{\partial B}{\partial t}$.

Also, $(E_0)_z = (B_0)_z = 0$. (E & B have no component in z dir^n)

Now, using previous rel^{ns},

Prove :- $\tilde{B}_0 = \frac{k}{\omega}(\hat{z} \times \tilde{E}_0)$ $(\equiv E_0 \perp B_0)$

in terms of magnitude → $B_0 = \frac{k}{\omega} E_0 = \frac{E_0}{c} \Rightarrow \boxed{E_0 = cB_0}$ ☆

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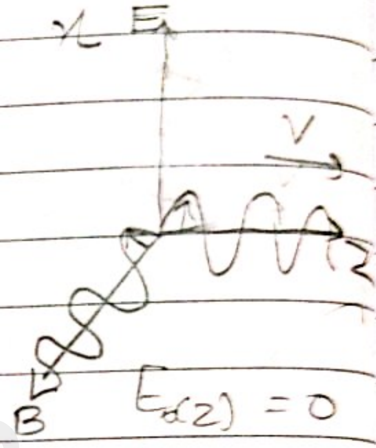
Puffin
Date
Page

✯ Proof : $E_0 = C B_0$

We know $\nabla \times E = -\dfrac{\partial B}{\partial t}$



$$\nabla \times E = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & E_y & \cancel{E_z} \end{vmatrix}$$

$E_{x}(z) = 0$

$B_0(z) = 0$

✯ Note :-
E is along x dir^n
So, $\dfrac{\partial}{\partial z} E_y = 0$

$\dfrac{\partial}{\partial x} E_y = 0$

$$= \hat{i}\left[-\frac{\partial E_y}{\partial z}\right] - \hat{j}\left[\frac{-\partial E_x}{\partial z}\right] + \hat{k}\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]$$

Now

$$-\frac{\partial B}{\partial t} = \tilde{B}_0 \omega \, e^{i(kz-\omega t)} = \omega \tilde{B}_y \, \hat{y} \longrightarrow (1)$$

assuming B is along y only

$$\Rightarrow \nabla \times E = \left(\frac{\partial E_x}{\partial z}\right)\hat{j} - \left(\frac{\partial E_x}{\partial y}\right)\hat{k}$$

$$= \frac{\partial}{\partial z}\left[E_0 e^{i(kz-\omega t)}\right]\hat{j} - (0)\hat{k}$$

$$= k\left[\tilde{E}_0 e^{i(kz-\omega t)}\right]\hat{y}$$

$\left(\dfrac{\partial}{\partial y} E_0 e^{i(kz-\omega t)}\right)$ const w.r.t y

$$\Rightarrow \nabla \times E = k\left(\tilde{E}(z)\right)_x \hat{y} \longrightarrow (2)$$

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$(1) = (2)$

$$\Rightarrow \omega \tilde{B}_y \hat{y} = k (\tilde{E}_{(z)})_x \hat{y}.$$

$$\Rightarrow \omega \left[\tilde{B}_0 \cancel{e^{i(kz-\omega t)}}\right] \hat{y} = k \left[\tilde{E}_0 \cancel{e^{i(kz-\omega t)}}\right] \hat{y}$$

$$\Rightarrow \boxed{\omega (\tilde{B}_0)_y = k (\tilde{E}_0)_x} \longrightarrow (3)$$

↳ when we assumed
B is along y & E is along x.
So, v is along +z.

Self-Phone:- $\boxed{-k(\tilde{E}_0)_y = \omega (\tilde{B}_0)_x}$. $\longrightarrow (4)$.

↳ Assume:-
B is along x & E is along y.
So, v is along −z.

Combined eq^n of (3) & (4).

$$\boxed{\tilde{B}_0 = \frac{k}{\omega} (\hat{z} \times \tilde{E}_0)}$$

↳ dir^n of $\hat{z}$, $\tilde{E}_0$ & $\tilde{B}_0$ can be seen accordingly by RHTR

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